deep imaging

Lecture 9: Theoretical basics of machine learning

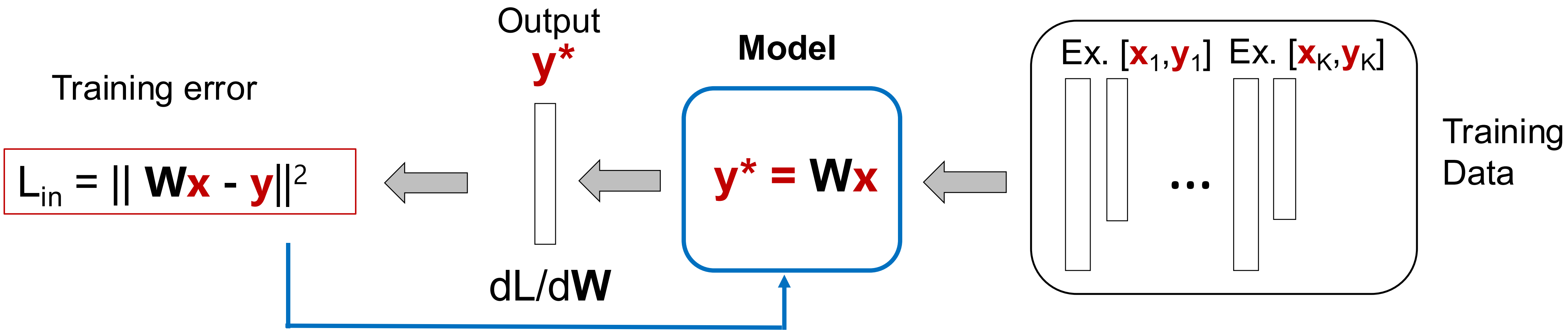
Machine Learning and Imaging

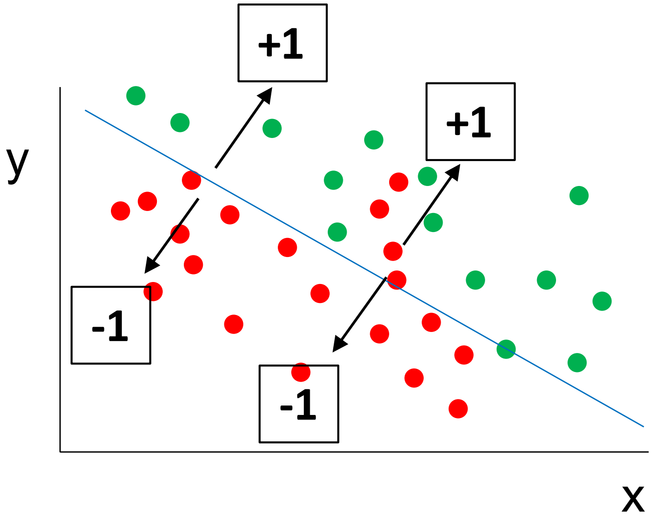
BME 548L
Roarke Horstmeyer

Announcements

- HW1 due *TODAY,* 2/11 at 11:59pm
  - Submit via Canvas
- Lab workbooks due today
- HW2 will be posted soon, will be due ~**two weeks after**

The linear classification model – what's not to like?



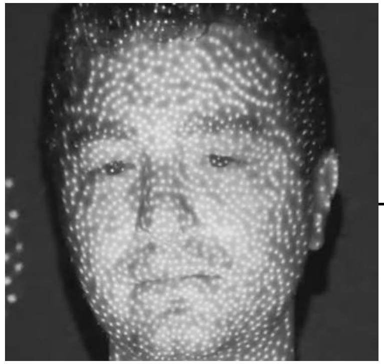


1. Can only separate data with lines (hyper-planes)…
2. We only allowed for binary labels (y = +/- 1)
3. Error function L_{in} inherently makes assumptions about statistical distribution of data

Cost functions matter: a simple example

→

x



→ f(x, W) →

+1 = You

-1 = Bad guy

What if you're a CIA agent?

$$L_{in} = \mathbf{100{,}000}\ \mathrm{ReLU}[f(x, W)-y] + \mathrm{ReLU}[y-f(x, W)]$$

BIG penalty for intruder

Don't mind about annoyance…

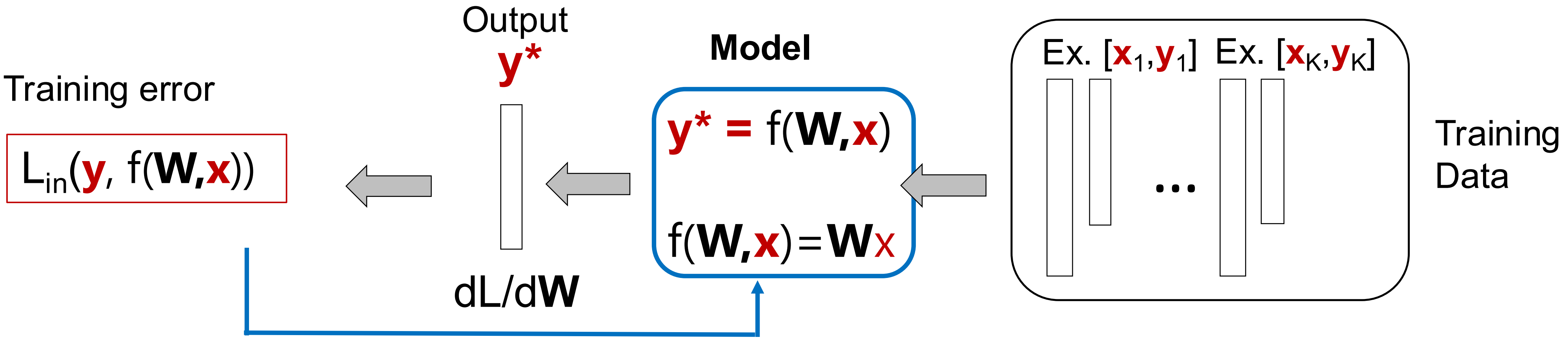
f(x, W)

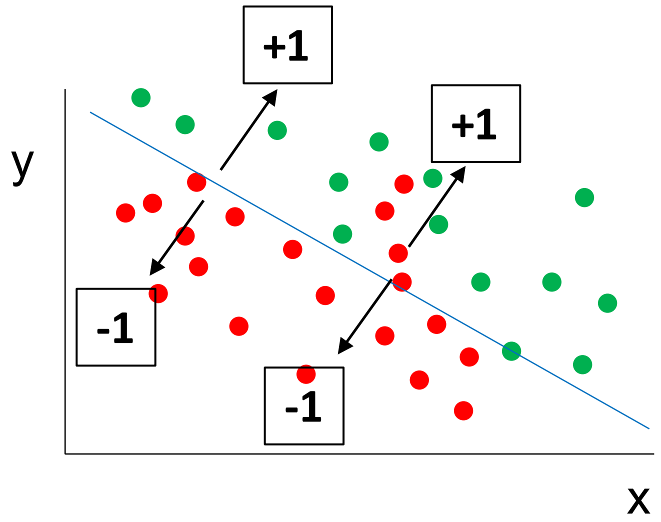
y \ f(x, W)	+1	-1
+1	No Error	False reject
-1	False accept	No Error

It's you, but you can't get in…

Letting an intruder in

The linear classification model – what's not to like?





1. Can only separate data with lines (hyper-planes)…
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Deriving cost function for logistic classification for probabilistic outputs

Similar to the linear classification case, the likelihood of observing *N* independent outputs is given by,

$$P(y_1, y_2 \ldots y_N \mid \mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_N) = \prod_{n=1}^{N} P(y_n \mid \mathbf{x}_n)$$

$$= \prod_{n=1}^{N} \theta(y_n \, \mathbf{w}^T \mathbf{x}_n)$$

The Logistic Function *θ*

$$\theta(x) = \frac{e^x}{1+e^x}$$

Also called Sigmoid function

This is the probability of the labels, given the data. We'd like to maximize this probability!

*Like the linear regression case, but now the probability of classes given the data is not Gaussian distributed, but instead follows the sigmoid curve (is bound to [0,1], which is more realistic)

Maximize $P(y_1, y_2 \ldots y_N \mid \mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_N) = \prod_{n=1}^{N} \theta(y_n \, \mathbf{w}^T \mathbf{x}_n)$

Deriving cost function for logistic classification for probabilistic outputs

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Maximize $P(y_1, y_2 \ldots y_N \mid \mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_N) = \prod_{n=1}^{N} \theta(y_n \mathbf{w}^T \mathbf{x}_n)$

Minimize $-\frac{1}{N} \ln\left(\prod_{n=1}^{N} \theta(y_n \mathbf{w}^T \mathbf{x})\right)$

Minimize $\frac{1}{N} \sum_{n=1}^{N} \ln\left(\frac{1}{\theta(y_n \mathbf{w}^T \mathbf{x})}\right)$

Use relationship $\theta(a) = \frac{1}{1 + e^{-a}}$

Minimize $L_{in}(\mathbf{w}) = \frac{1}{N} \sum_{n=1}^{N} \ln(1 + e^{-y_n \mathbf{w}^T \mathbf{x}})$

$L_{in}(\mathbf{w}) = \frac{1}{N} \sum_{n=1}^{N} (y_n - \mathbf{w}^T \mathbf{x})^2$

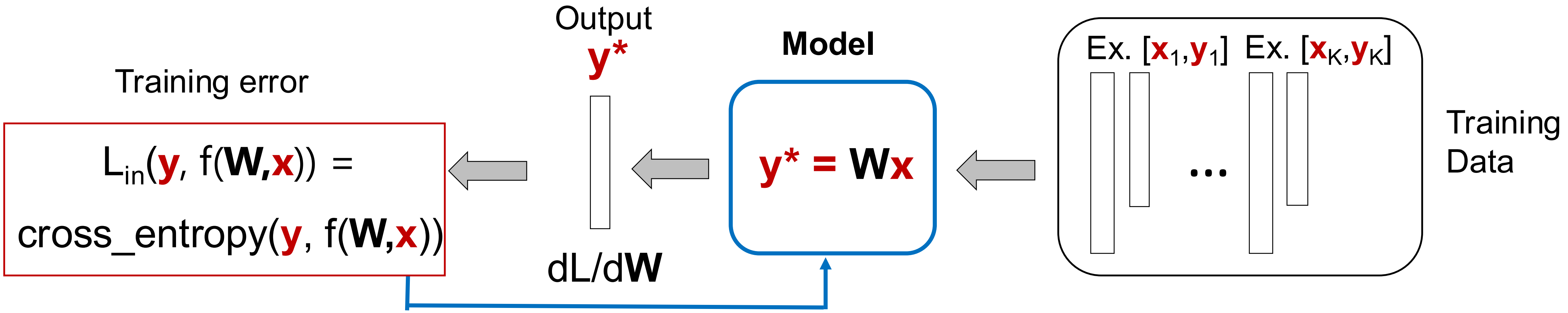
Cross entropy error for logistic classification

Typically requires iterative solution to minimize

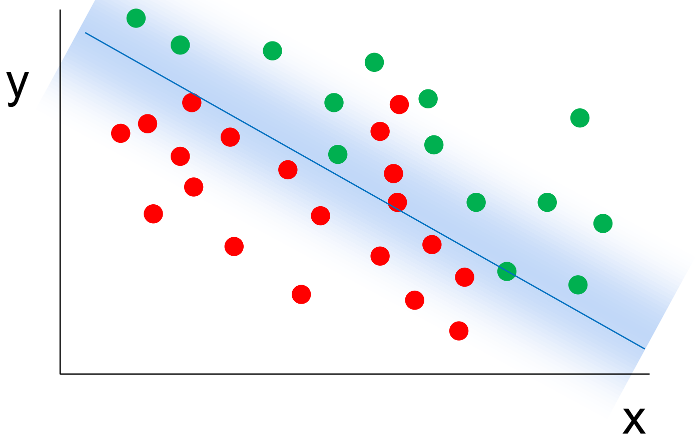
Mean-square error for linear classification

Closed form solution available

The linear classification model – what's not to like?



Probabilistic mapping to y



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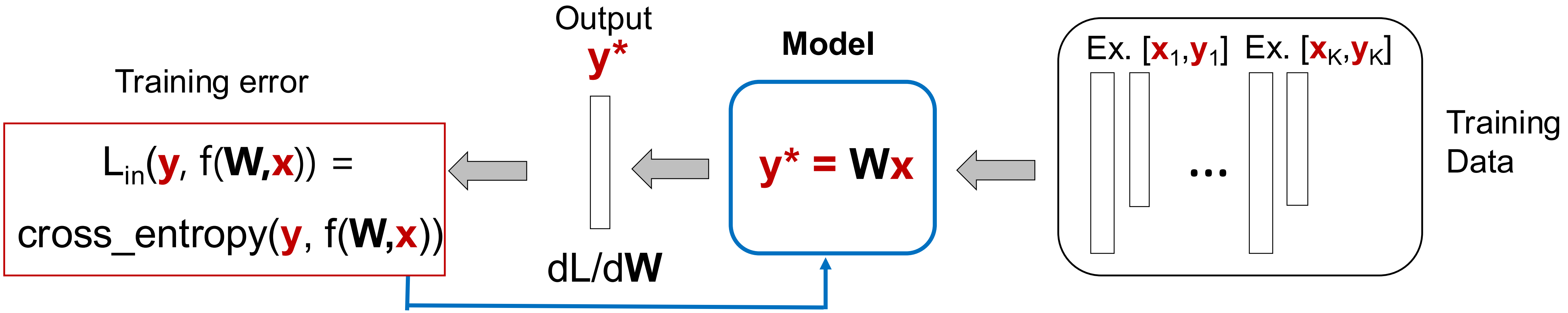

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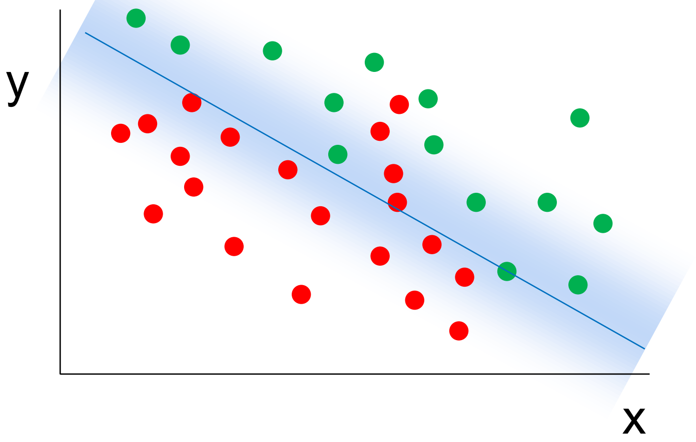

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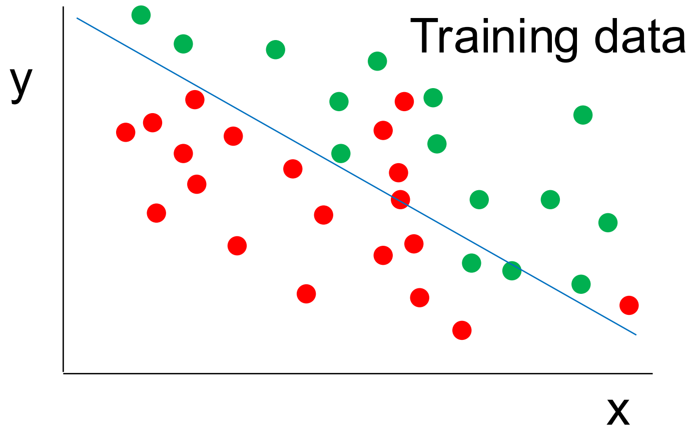
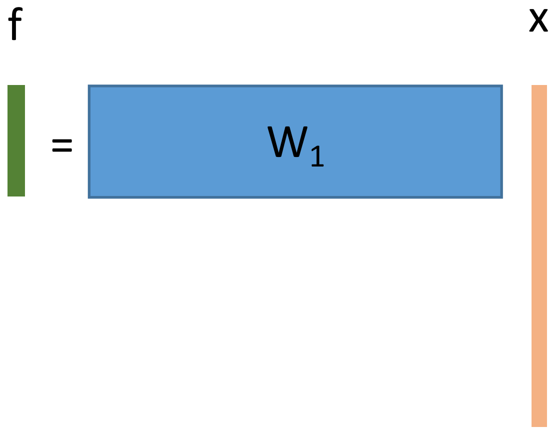


$$f = W_1 x$$

Learned *f:* not flexible



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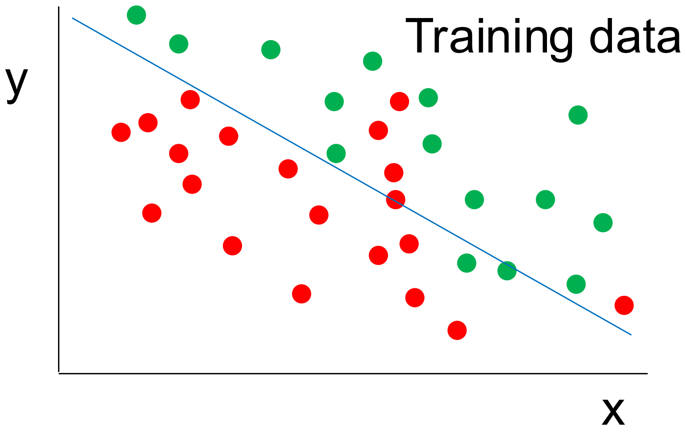

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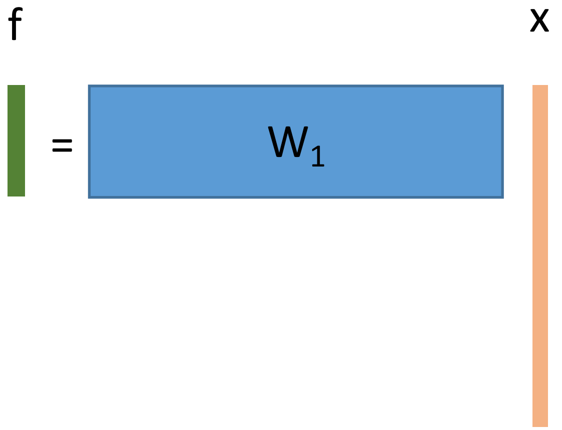
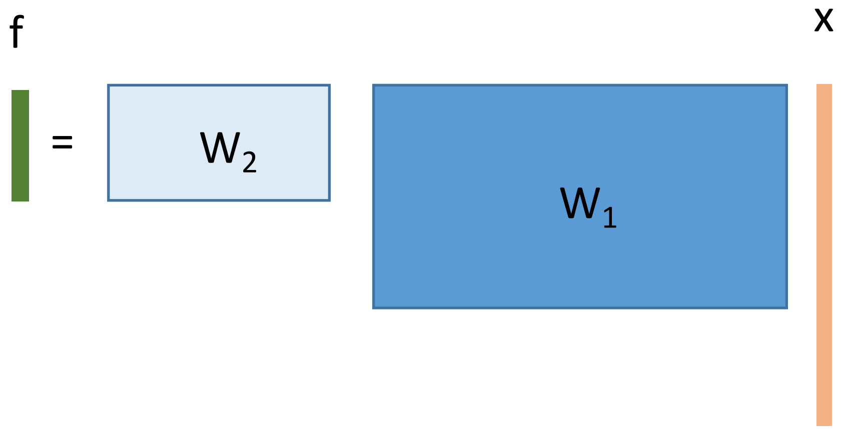


Can we add flexibility by multiplying with another weight matrix?

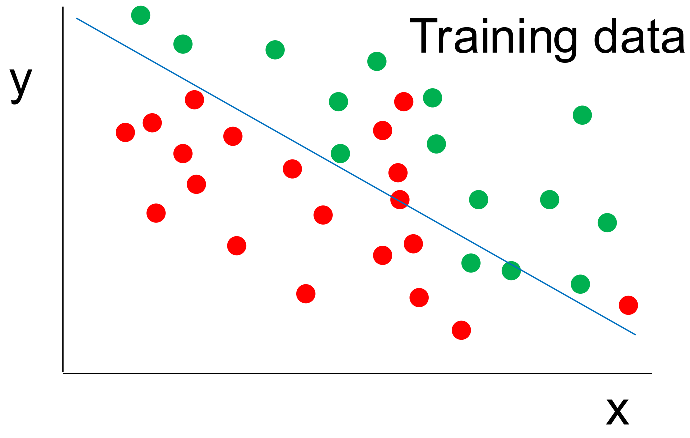
$$\begin{cases} f_1 = W_1 x + b_1 \\ f_2 = W_2 f_1 + b_2 \end{cases}$$



$$f_2 = W_2(W_1 x + b_1) + b_2$$

$$f_2 = W' x + b'$$

Unfortunately not…



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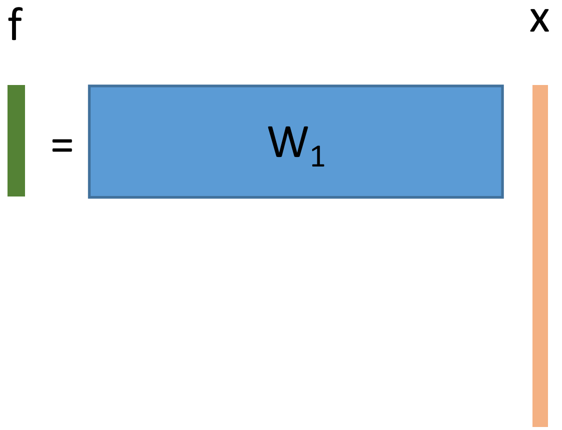

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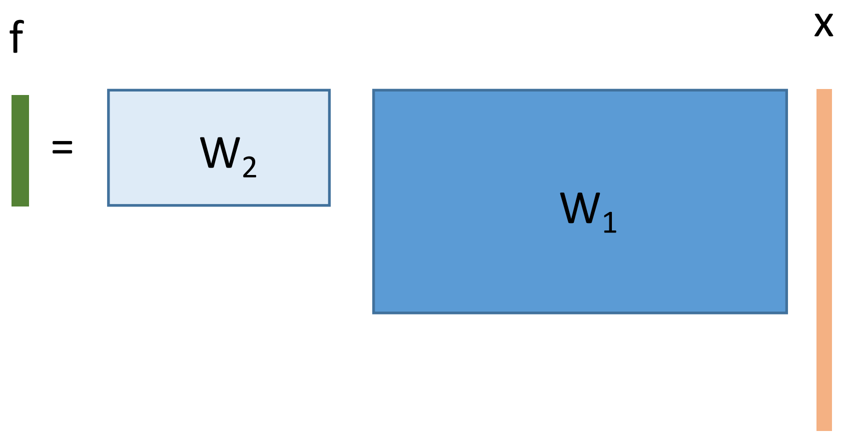

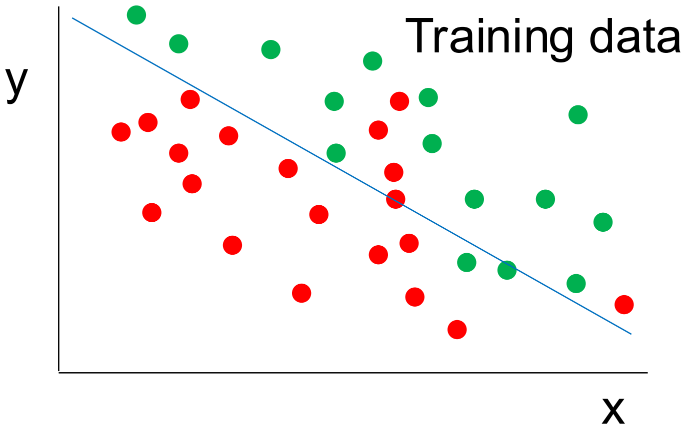

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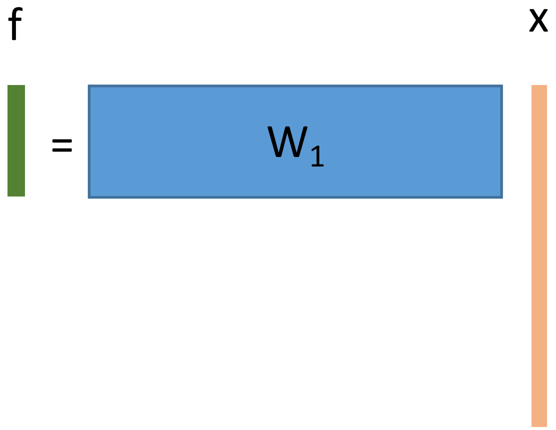
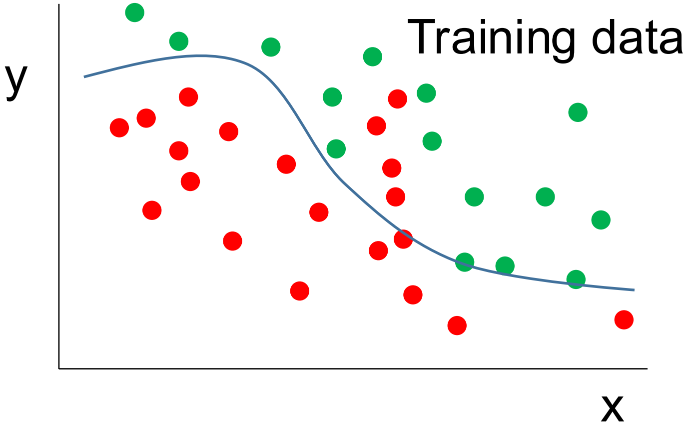
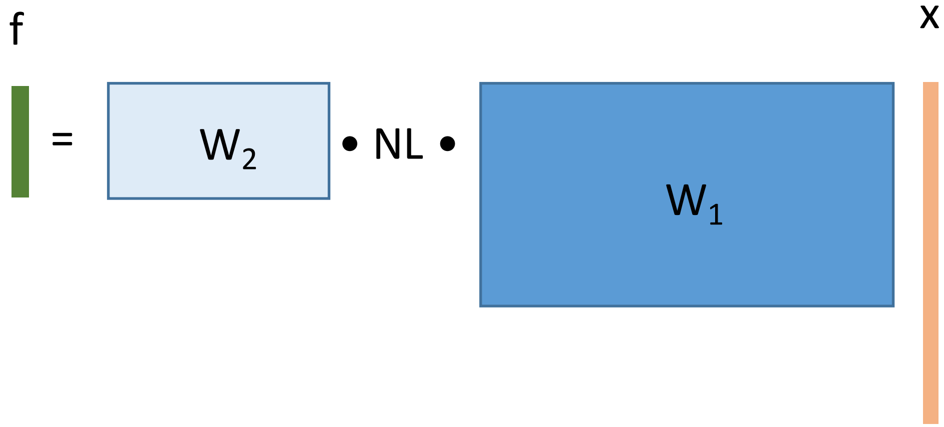


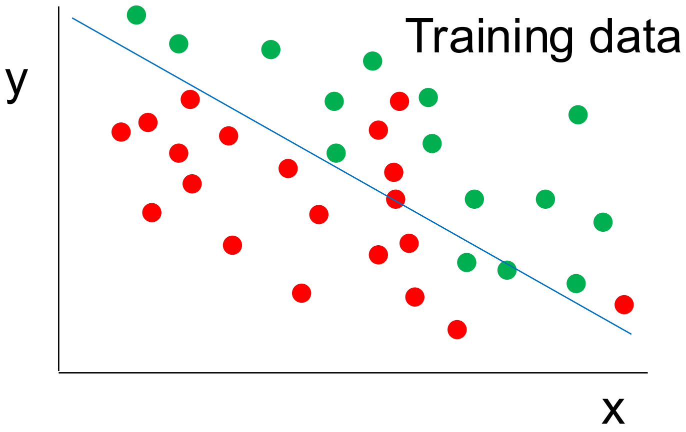


Add a non-linearity!

$$f = W_2 \max(W_1 x, 0)$$

Learned *f:* a bit flexible





$$f = W_1 x$$

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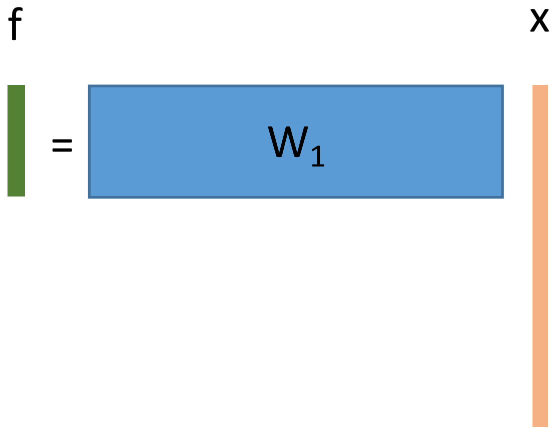

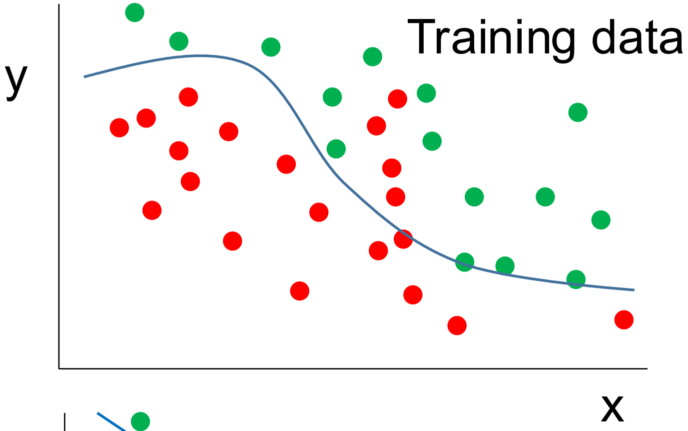

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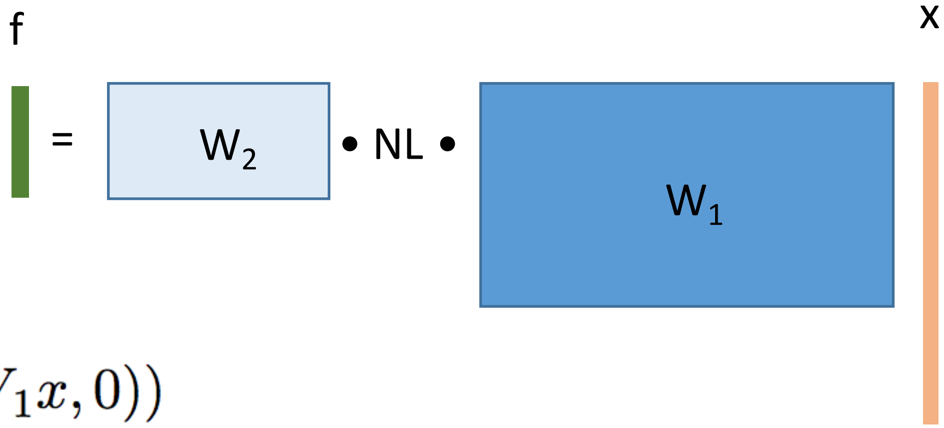
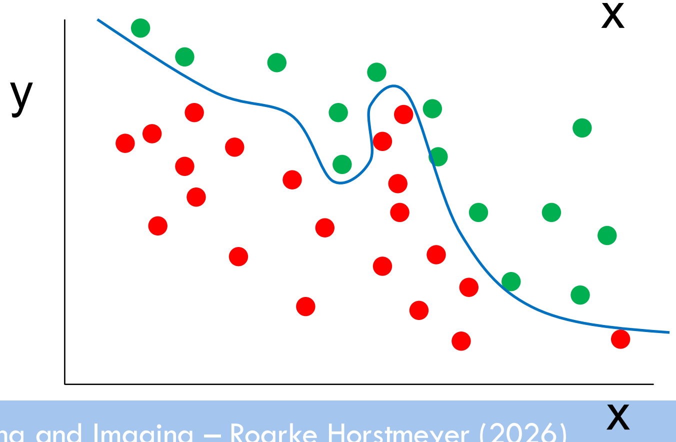




$$f = W_3 \max(0, W_2 \max(W_1 x, 0))$$

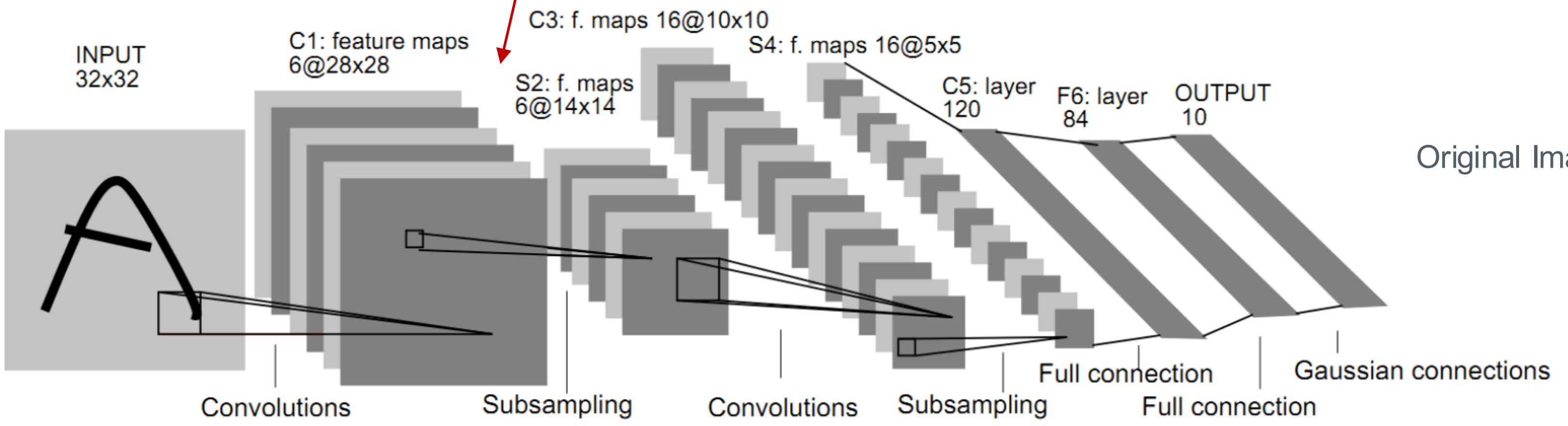
Learned *f:* more flexible

Does it generalize???

We can keep adding these "layers"…

Getting us to Convolutional Neural Networks

After, apply non-linearity and sub-sampling



Original Image published in [LeCun et al., 1998]

Each matrix W is a convolution matrix

Repeat a few times

At the end, use a full W for a final matrix multiplication

Getting us to Convolutional Neural Networks

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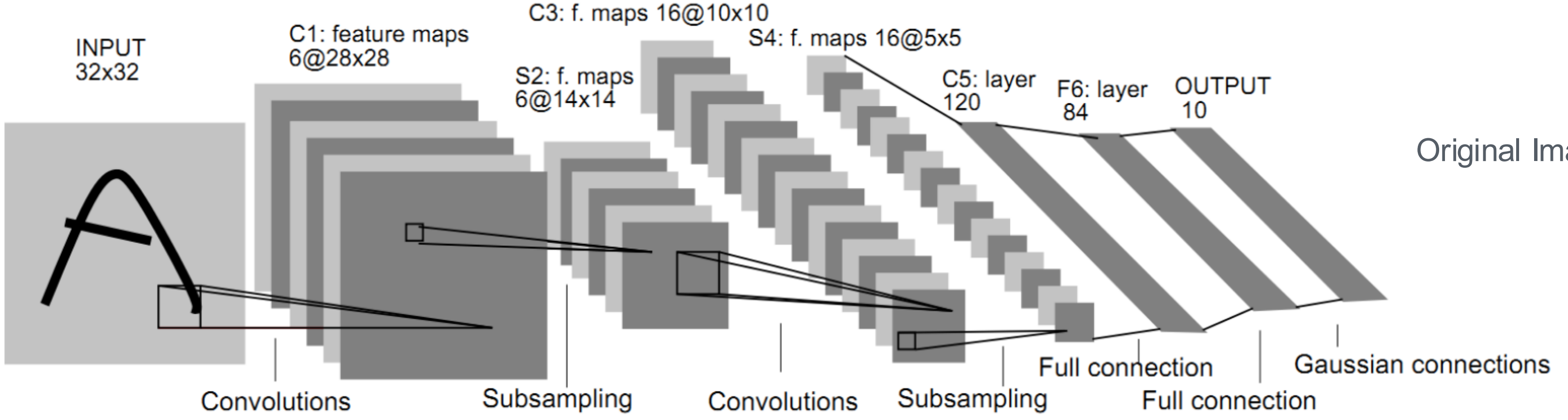
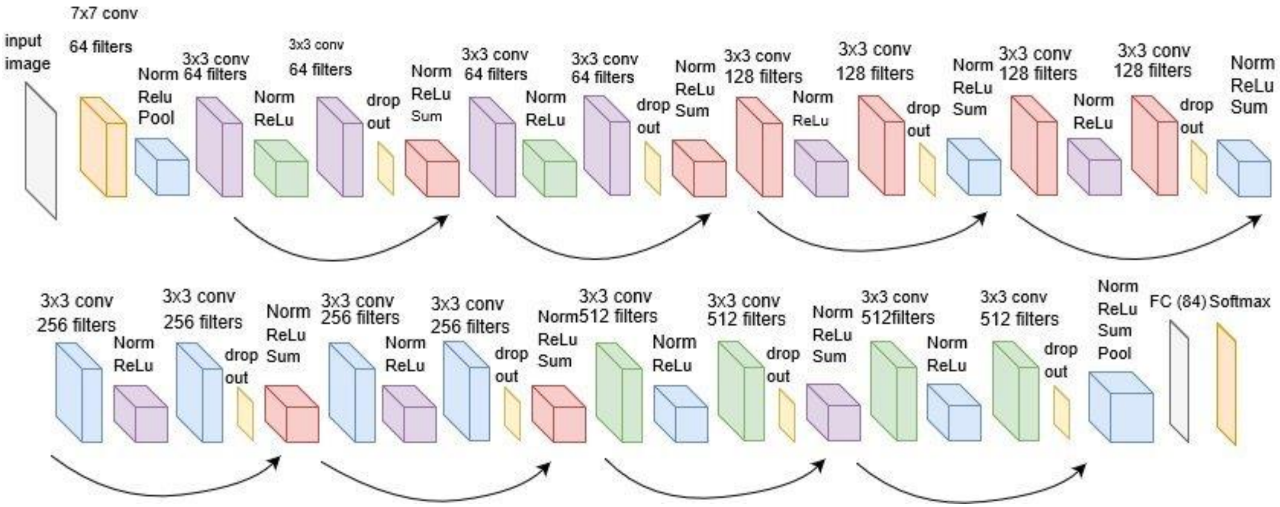


Original Image published in [LeCun et al., 1998]

In practice, this process is repeated many times:



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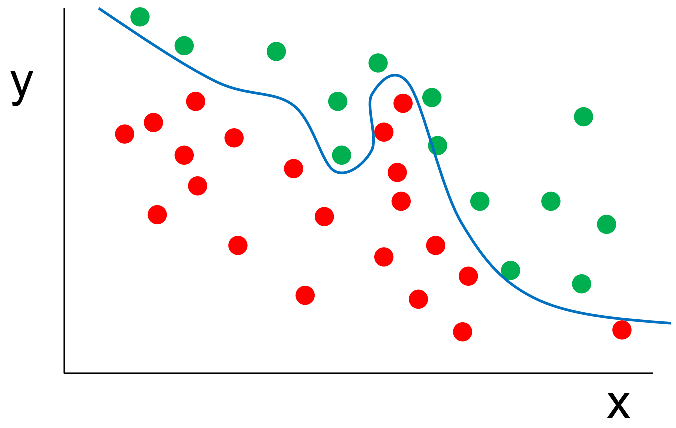
Aside #1 before convolutional neural network details

Q: Can we try to avoid making these learning models too complicated?



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Does it generalize???

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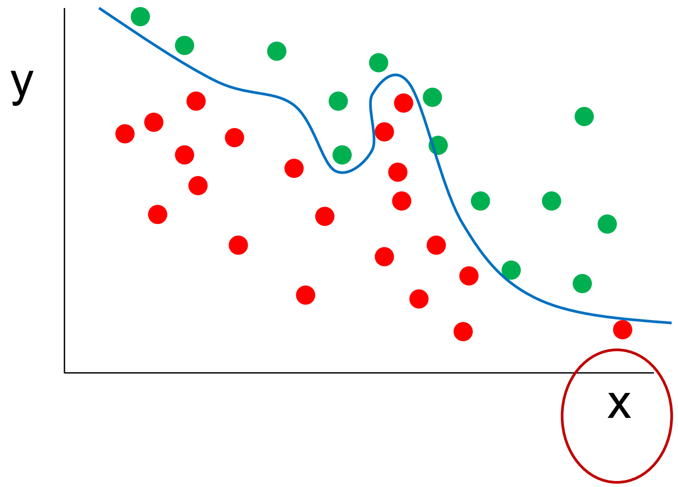
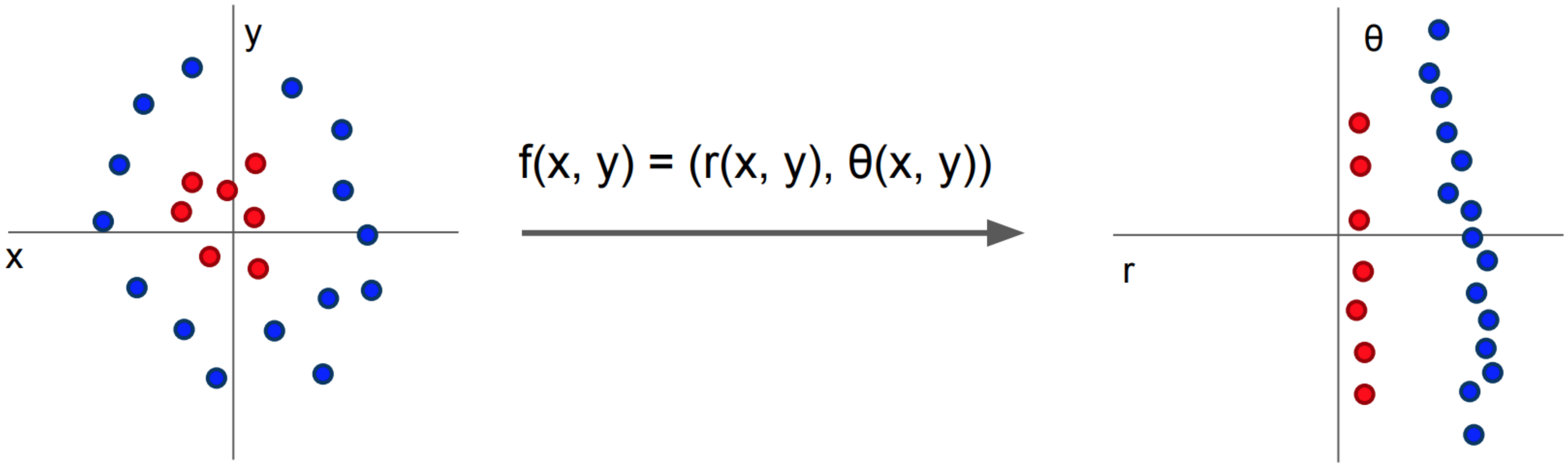


Learned *f:* more flexible

Does it generalize???

A: Yes, by transforming the data coordinates *before* classification

Image Features: Motivation

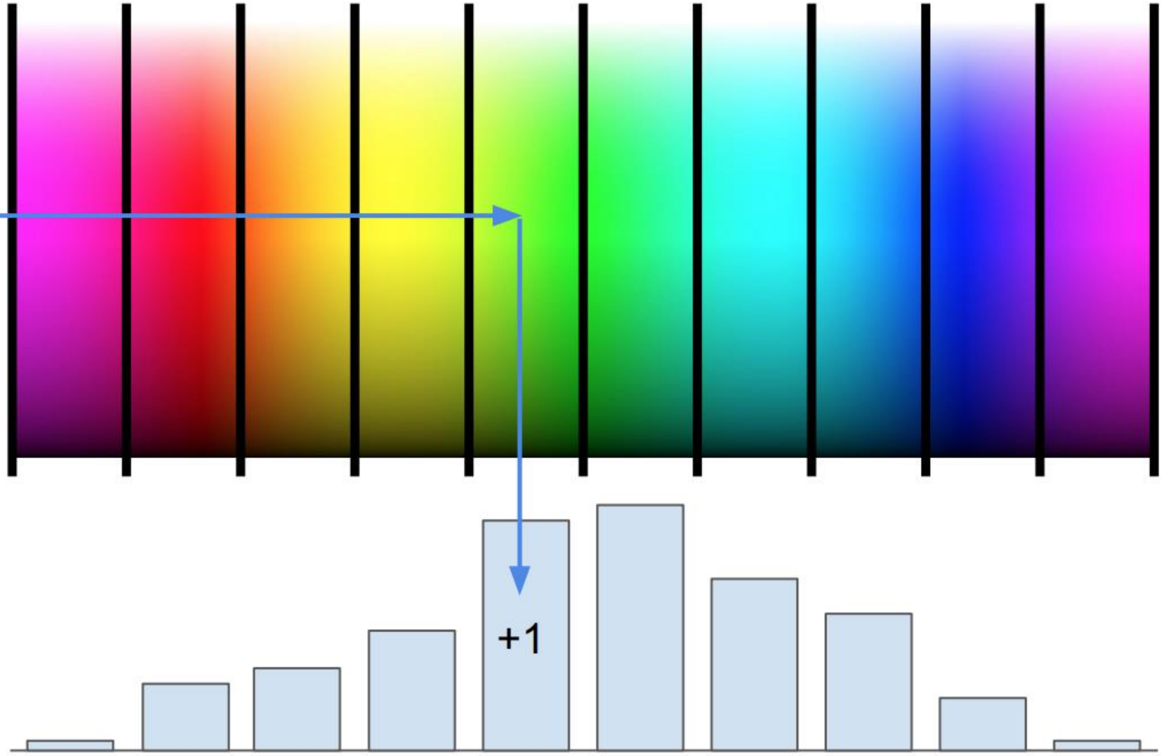


Cannot separate red and blue points with linear classifier

After applying feature transform, points can be separated by linear classifier

From Stanford CS231: http://cs231n.stanford.edu/

Example: Color Histogram



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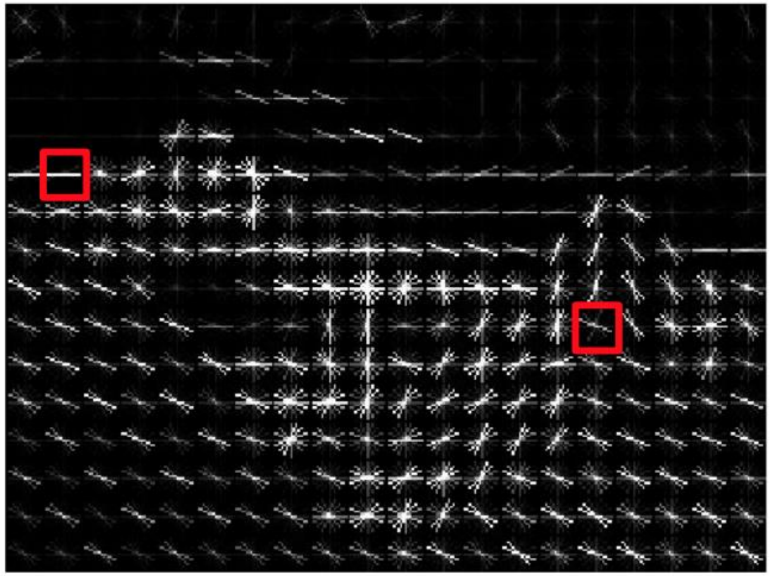
Example: Histogram of Oriented Gradients (HoG)

Divide image into 8x8 pixel regions
Within each region quantize edge direction into 9 bins



Example: 320x240 image gets divided into 40x30 bins; in each bin there are 9 numbers so feature vector has 30*40*9 = 10,800 numbers

Lowe, "Object recognition from local scale-invariant features", ICCV 1999
Dalal and Triggs, "Histograms of oriented gradients for human detection," CVPR 2005

From Stanford CS231: http://cs231n.stanford.edu/

Example: Bag of Words

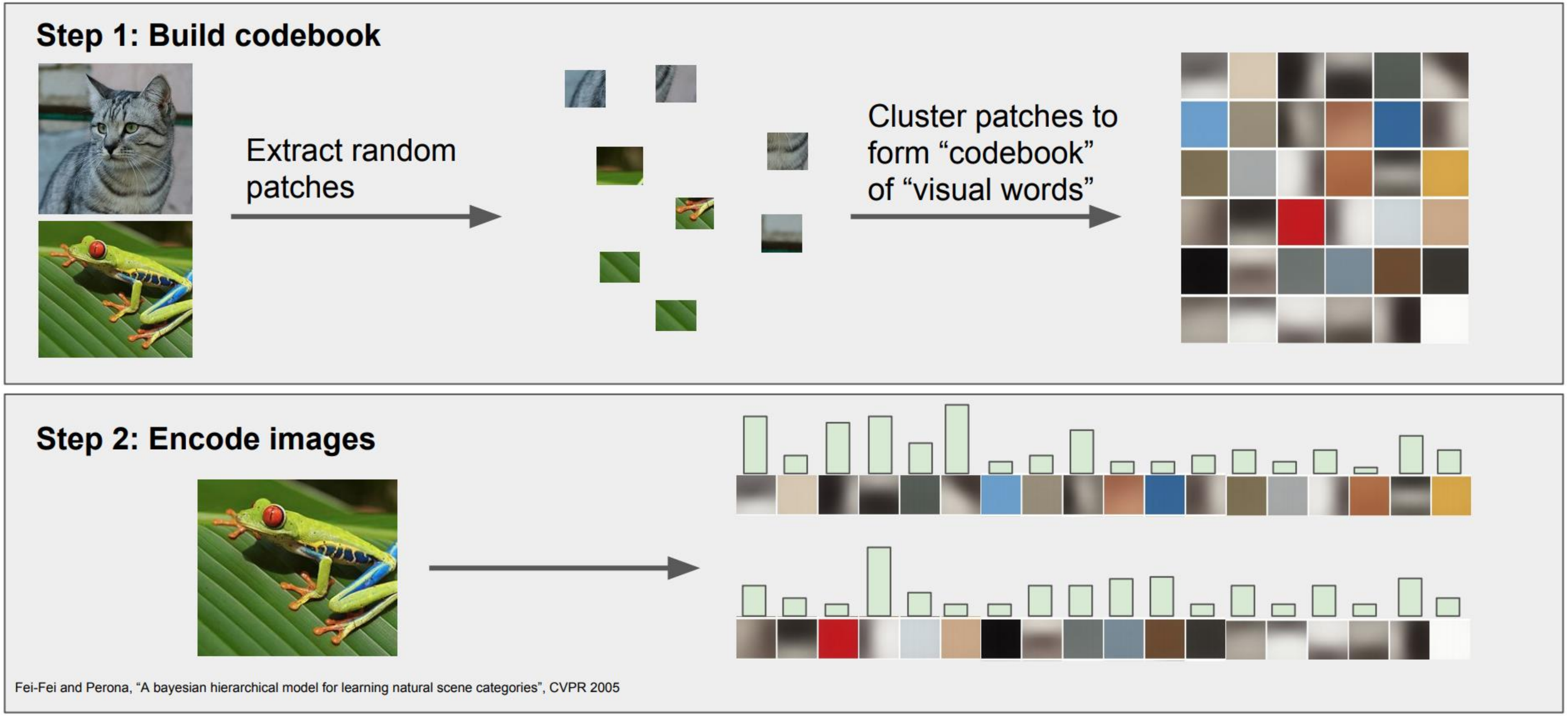


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Image features vs ConvNets

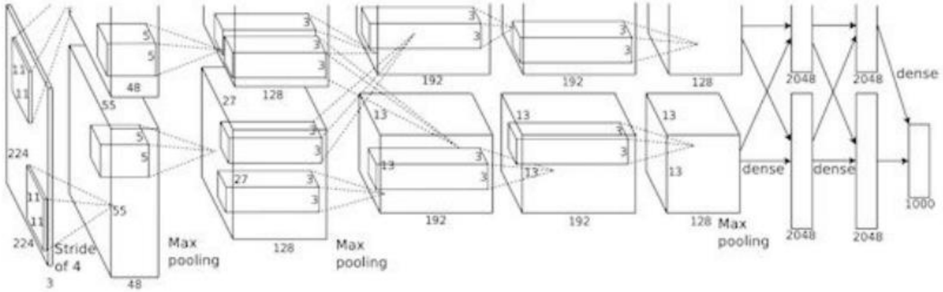
“Hand crafted”

Feature Extraction

f

10 numbers giving scores for classes

training



Krizhevsky, Sutskever, and Hinton, “Imagenet classification with deep convolutional neural networks”, NIPS 2012.

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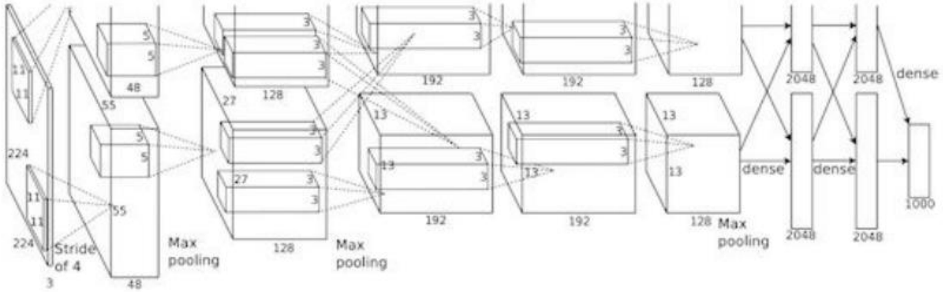

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10 numbers giving scores for classes

training

History has now proven – bottom approach works better!

From Stanford CS231: http://cs231n.stanford.edu/

Statistical Machine Learning in ~30 minutes

Two competing goals in machine learning:

1. Can we make sure the in-sample error $L_{in}(y, f(x,W))$ is small enough during network training?
    - Appropriate cost function
    - "complex enough" model

Statistical Machine Learning in ~30 minutes

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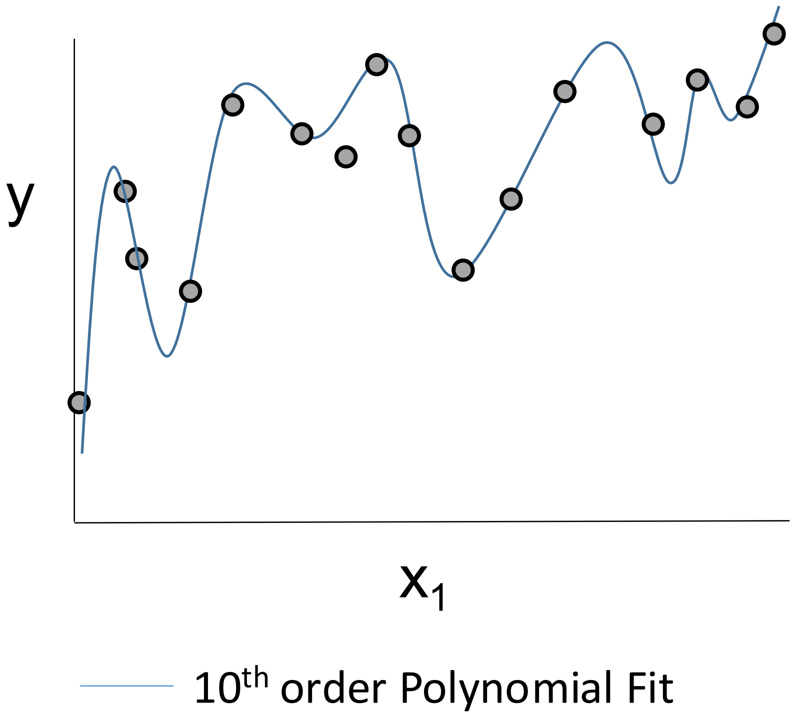
Two competing goals in machine learning:

1. Can we make sure the in-sample error $L_{in}(y, f(x,W))$ is small enough during network training?
   - Appropriate cost function
   - “complex enough” model

2. Can we make sure that $L_{out}(y, f(x,W))$ is close enough to $L_{in}(y, f(x,W))$ during network testing?
   - Probabilistic analysis says yes!
   - $|L_{in} – L_{out}|$ bounded from above
   - Bound grows with model capacity (i.e., complexity - bad)
   - Bound shrinks with # of training examples (good)

Model overfitting versus underfitting – a thought exercise

Let's fit these "training" data points:



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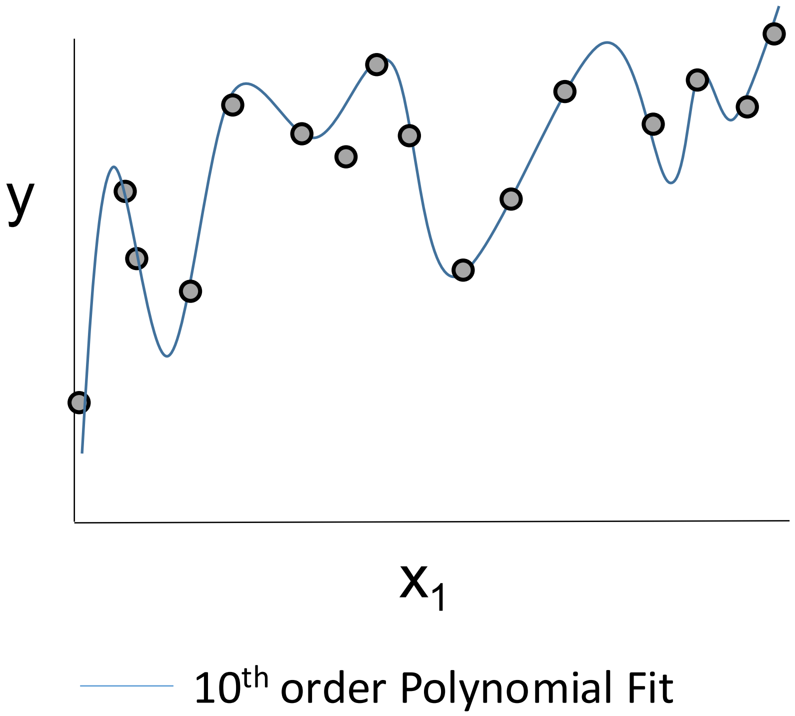
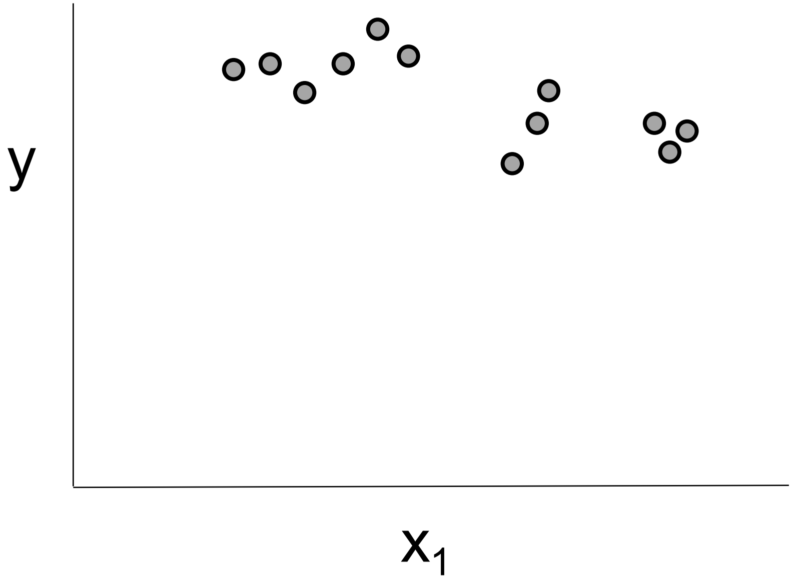


And then here's our testing dataset – good?



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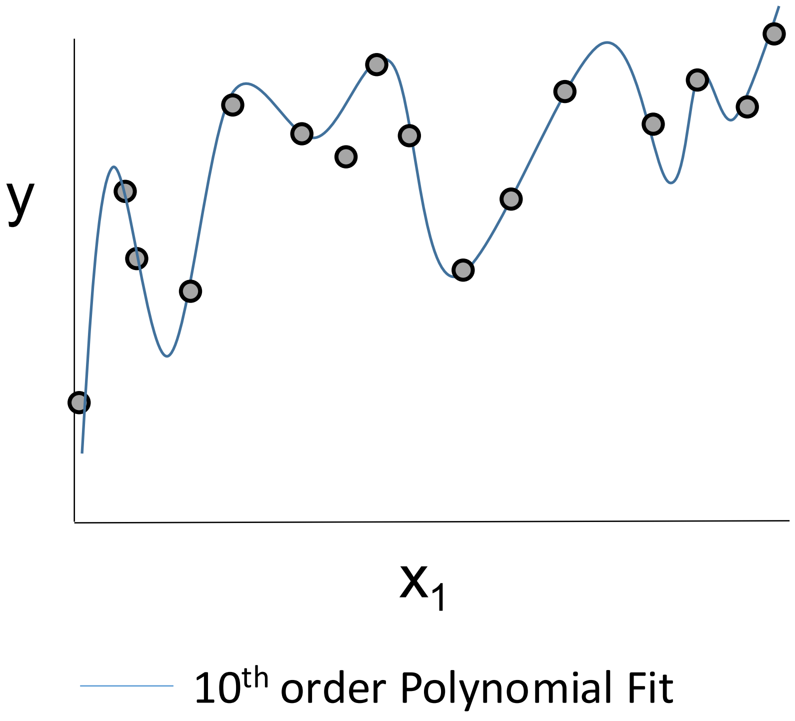
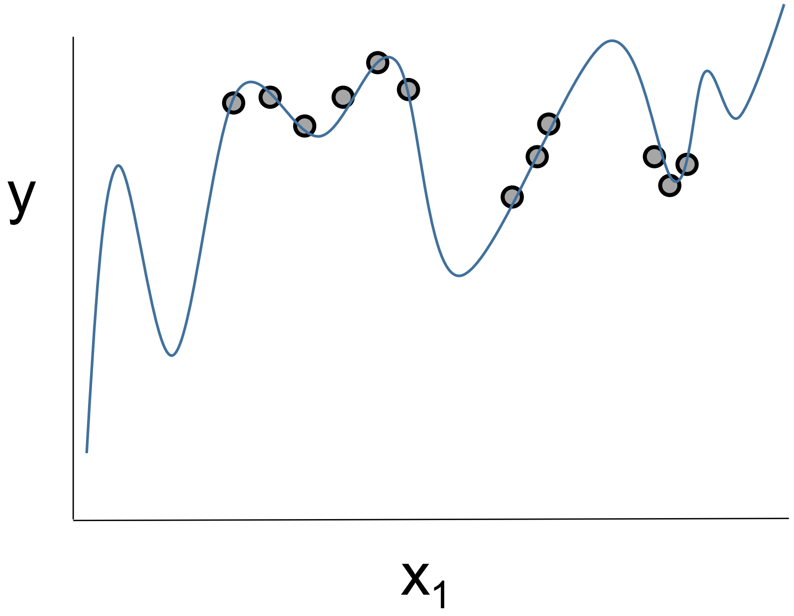


And then here's our testing dataset – good?

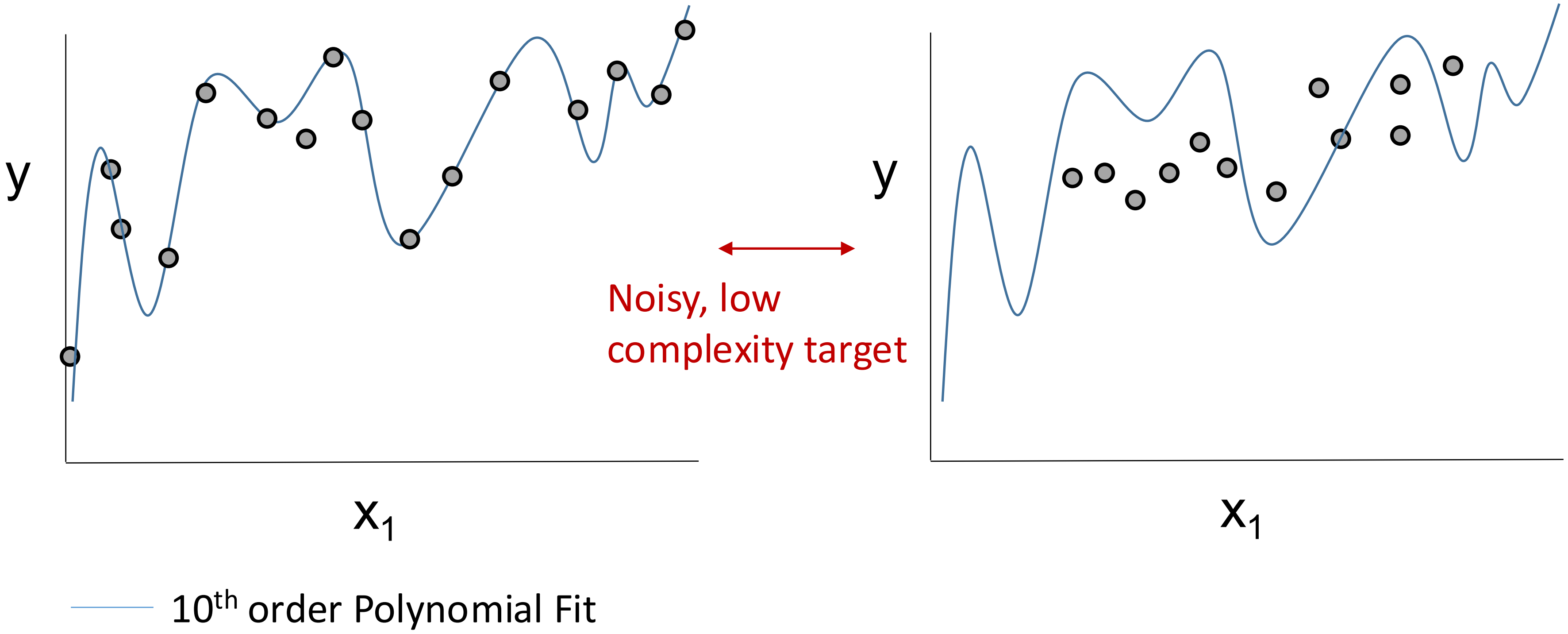
Perfect! But lucky….



Model overfitting versus underfitting – a thought exercise

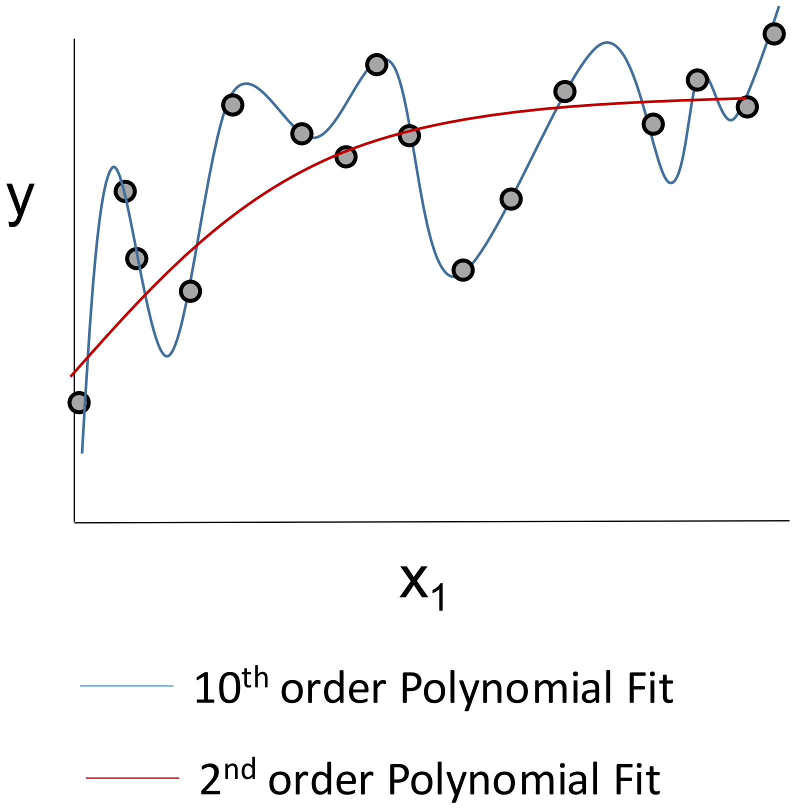
Let's fit these "training" data points:

What if our test dataset was this :



Model overfitting versus underfitting – a thought exercise

Let's fit these "training" data points:



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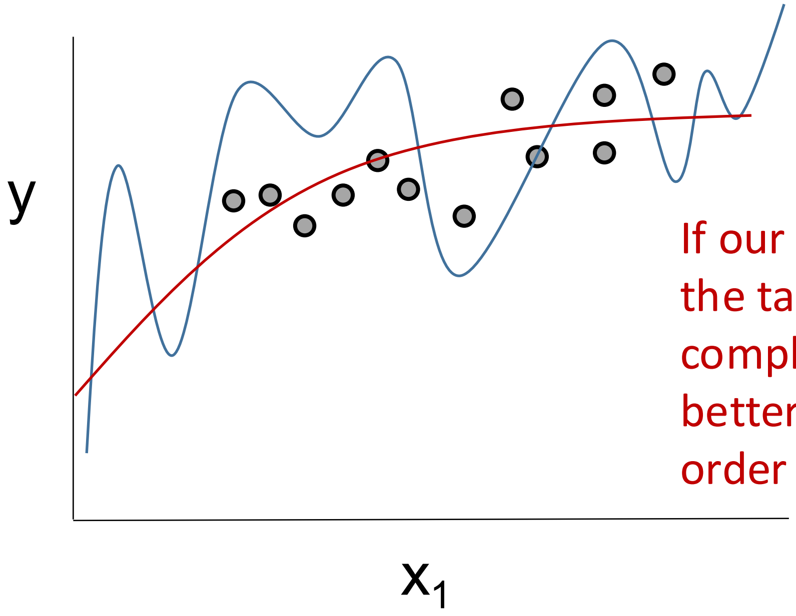
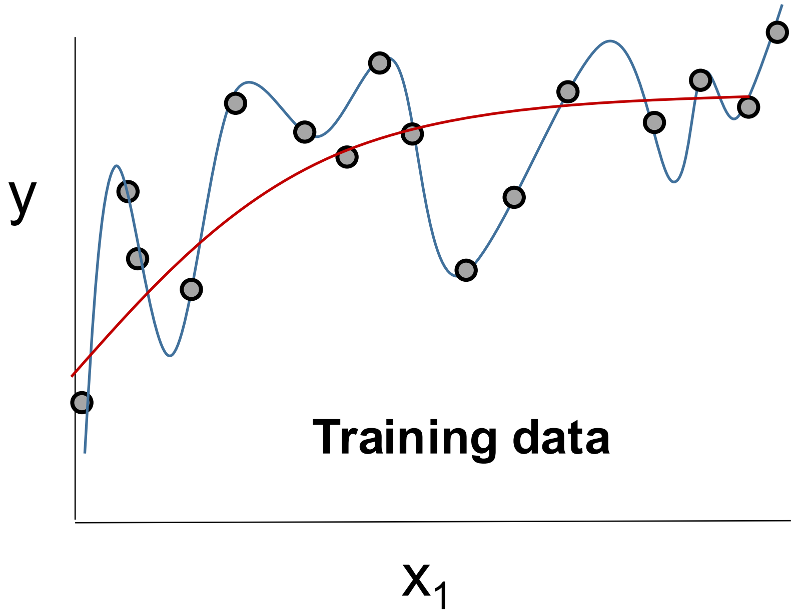
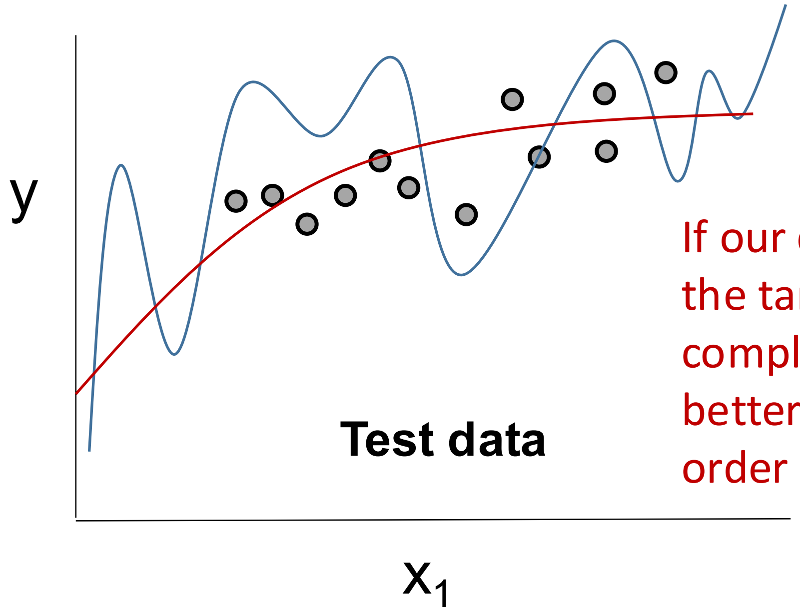


If our data was noisy and the target followed a low-complexity model, we'd be better off with a second order fit!

Model overfitting versus underfitting – a thought exercise





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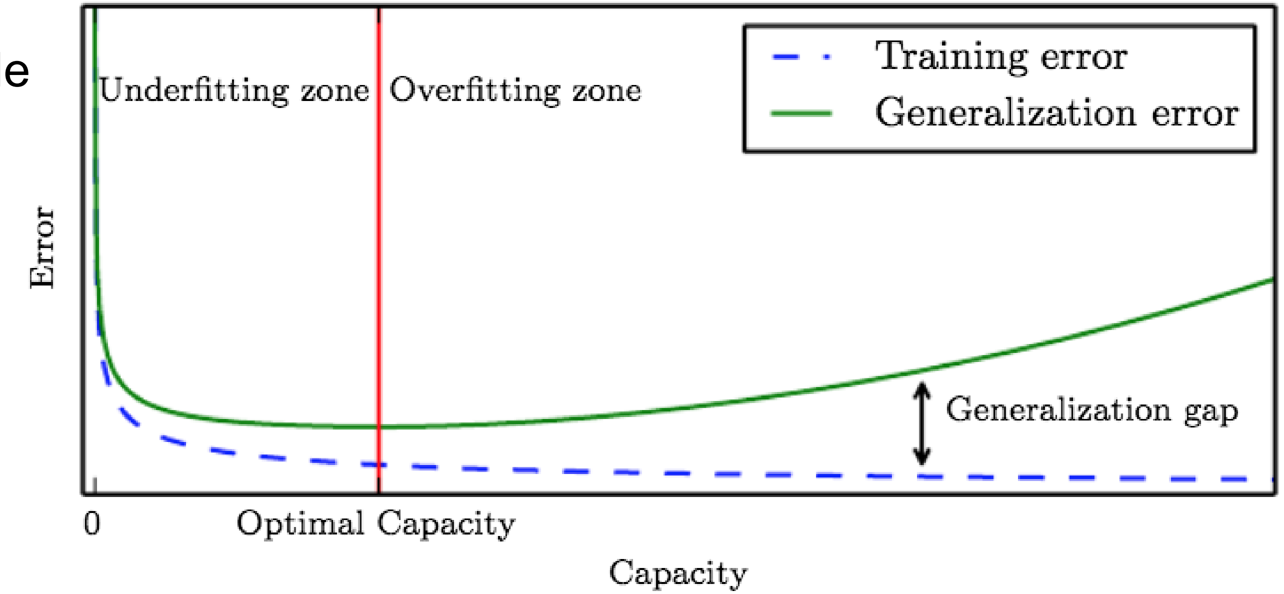
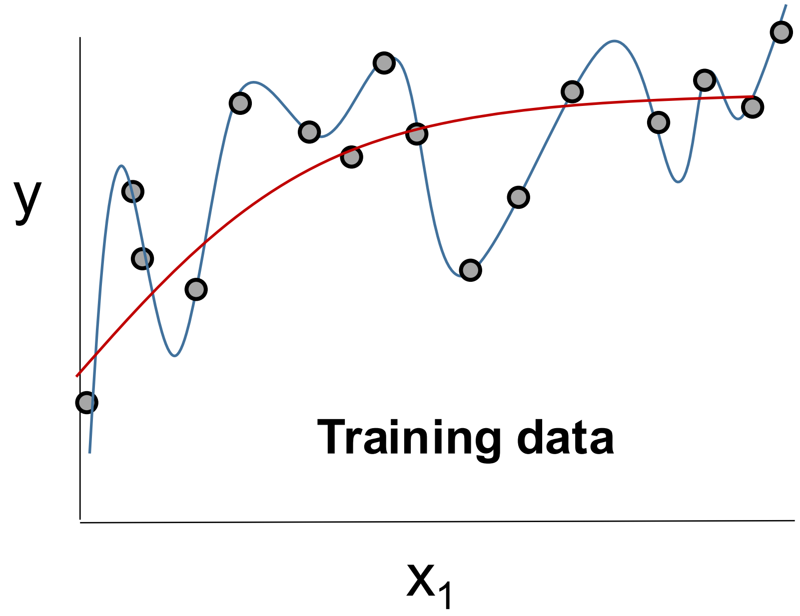
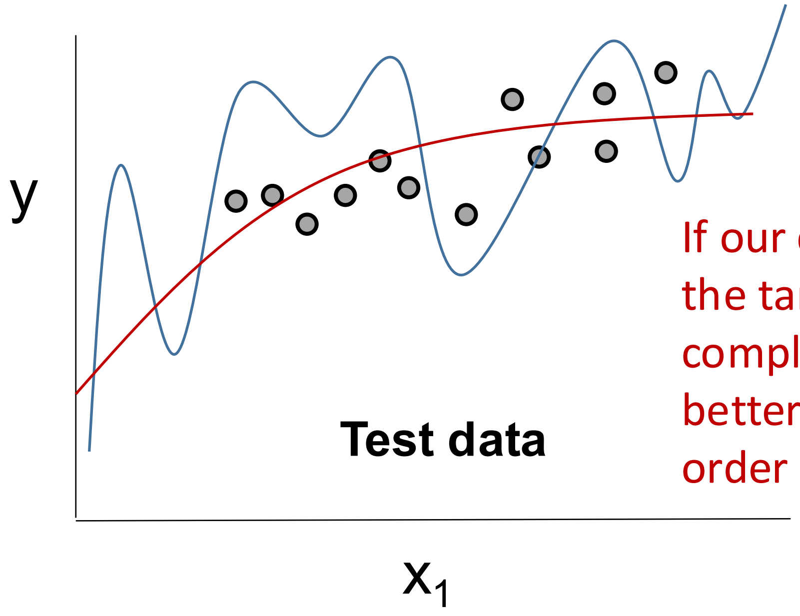
Model capacity: ability to fit a wide range of functions



Deep Learning, I. Goodfellow et al., Fig. 5.3

Model overfitting versus underfitting – a thought exercise



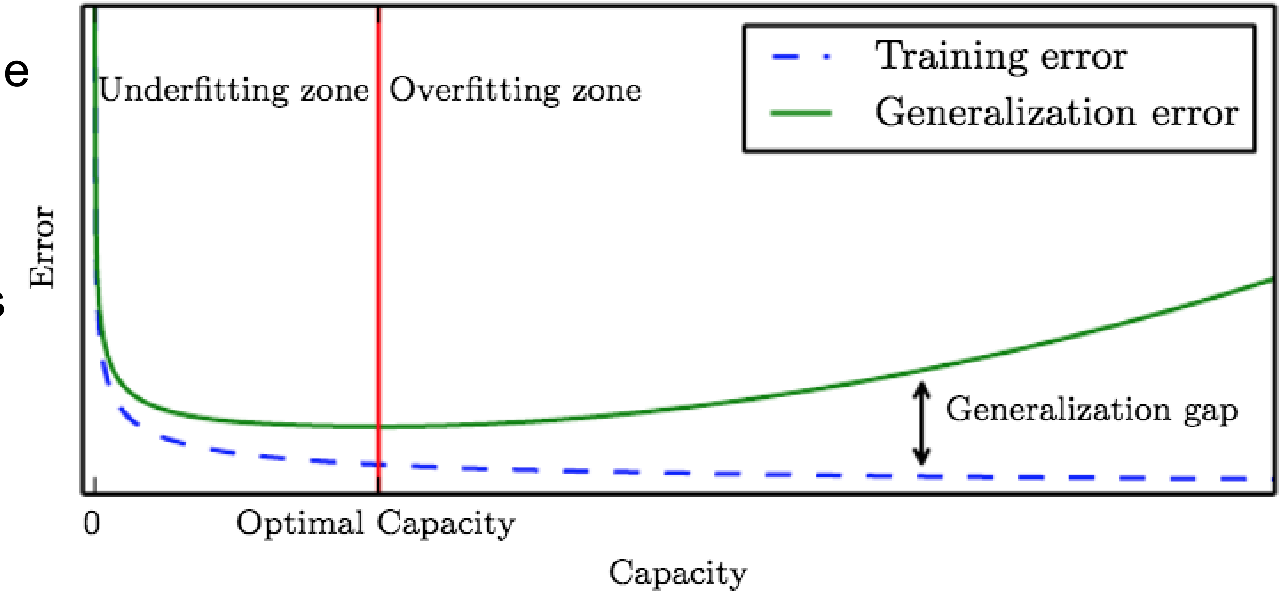


If our data was noisy and the target followed a low-complexity model, we'd be better off with a second order fit!

Model capacity: ability to fit a wide range of functions

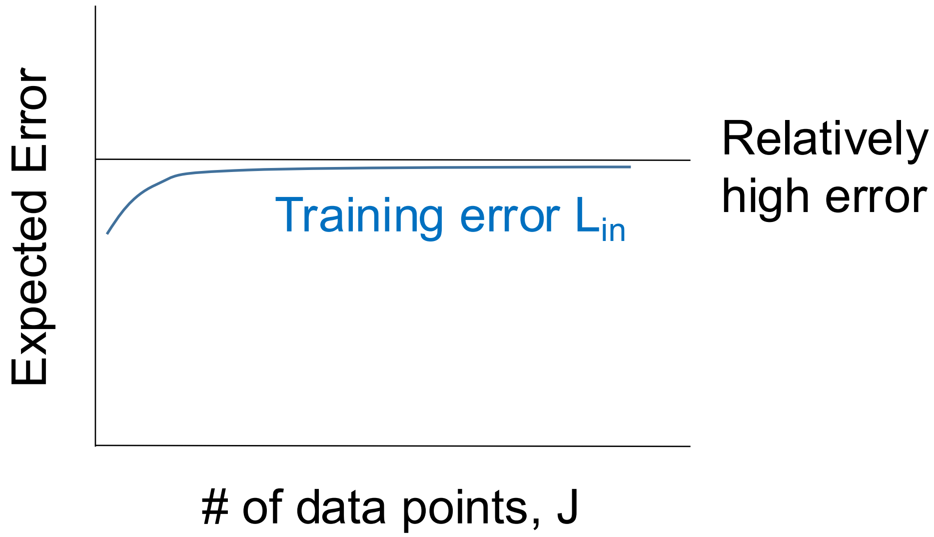
Control capacity through model's hypothesis space (set of functions model can take)

Hard to know ahead of time!



Deep Learning, I. Goodfellow et al., Fig. 5.3

Low Capacity Model



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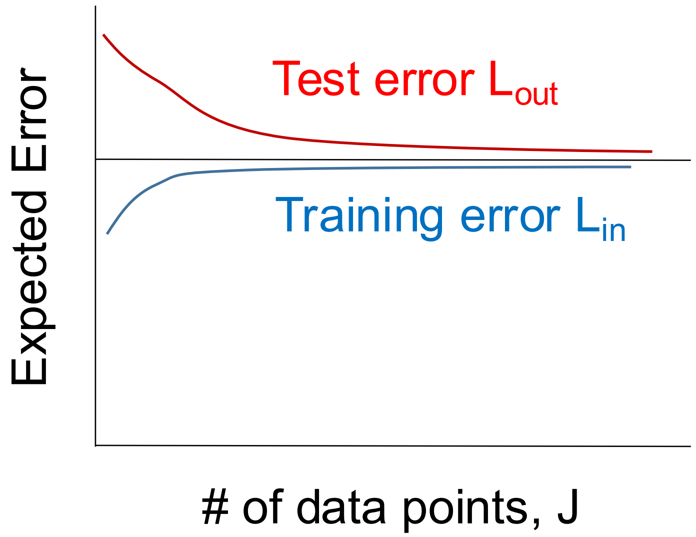
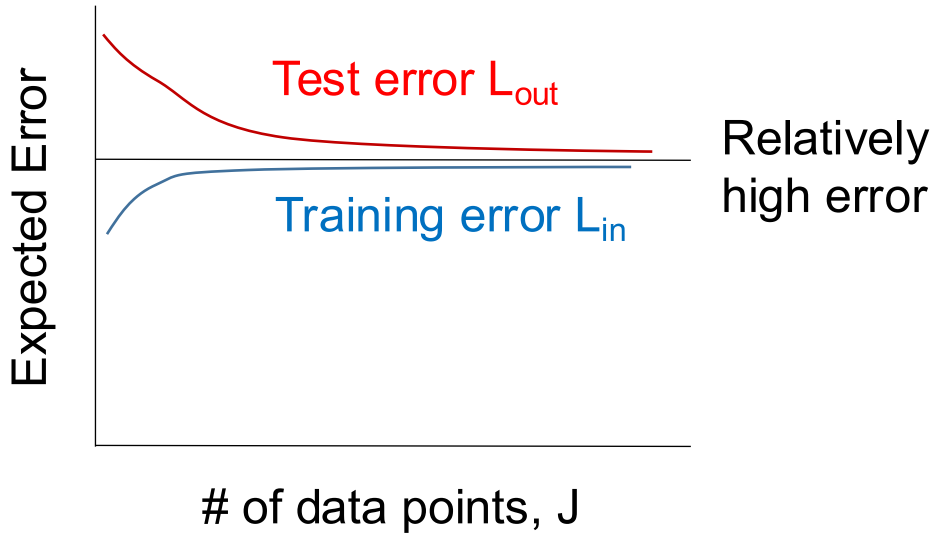


Relatively high error

Low Capacity (complexity) Model



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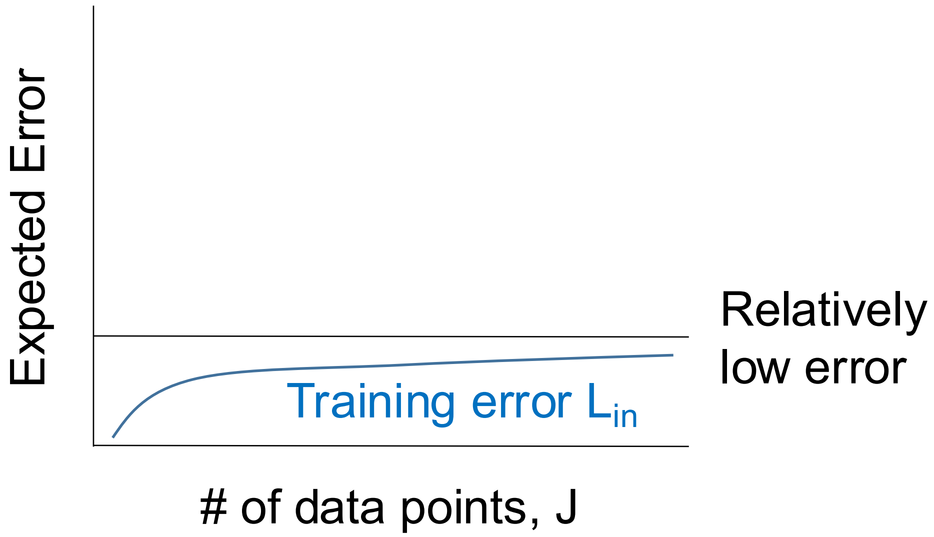

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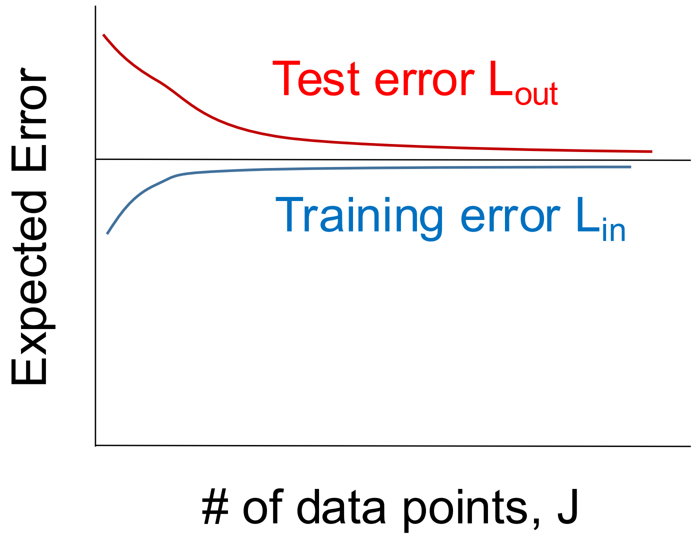

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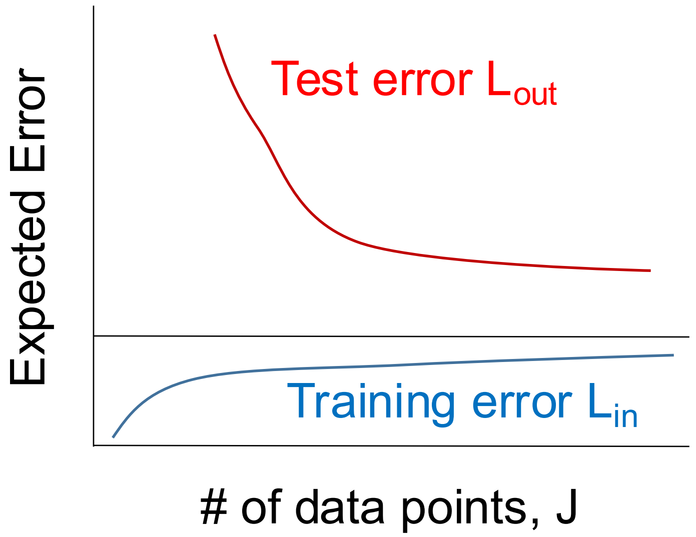
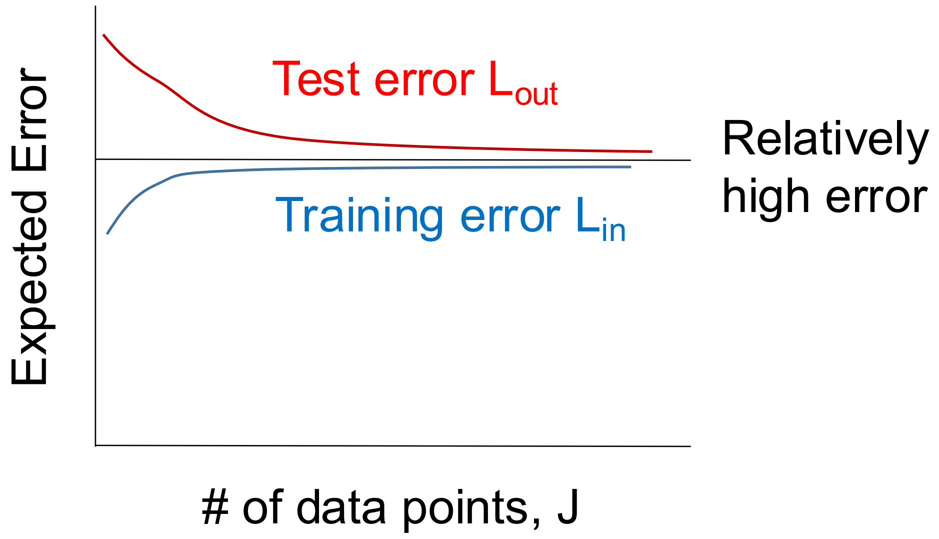


But harder to get test error down…

Relatively low error

Low Capacity (complexity) Model



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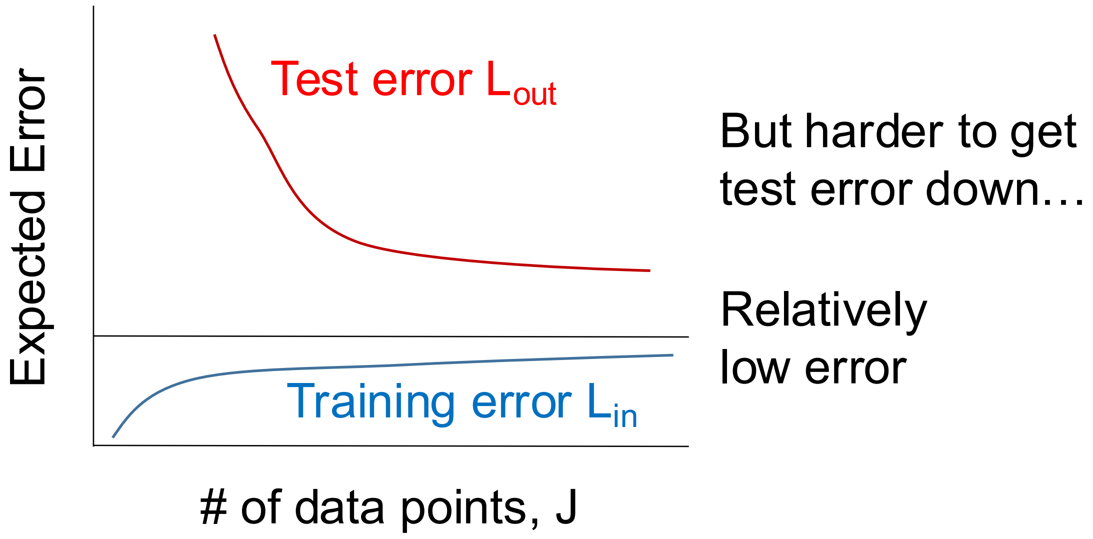
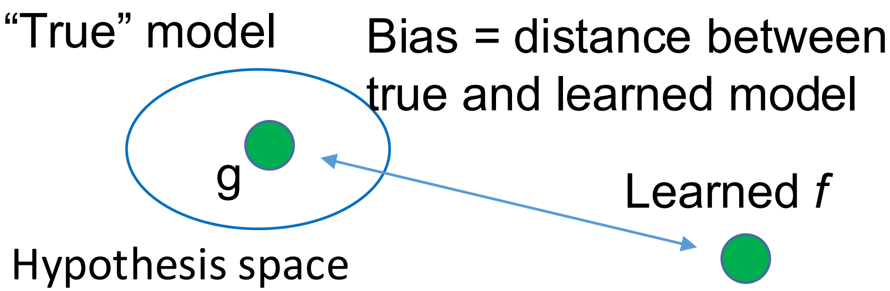
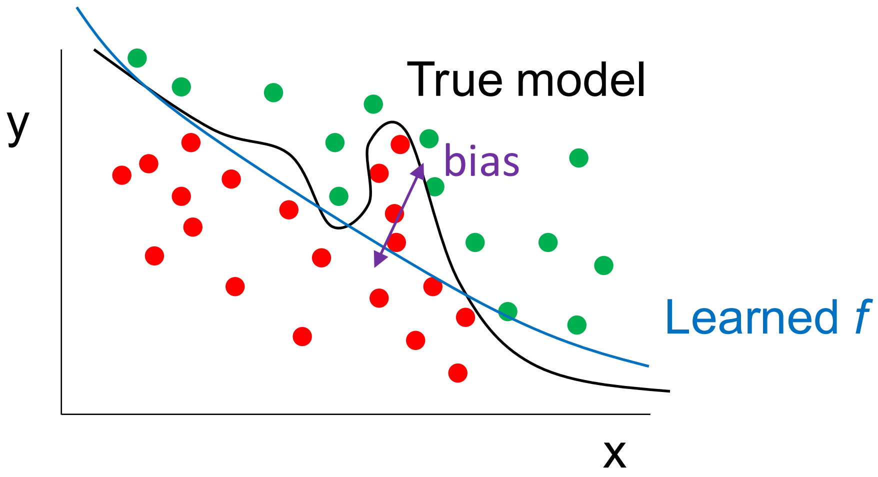


Take away concepts:

- Can’t ever really expect test error to be less than training error
- Complicated models tend to appear to “do better” during training, before trying test data
- When the model gets complicated and you don’t have enough data, challenging to get test error down

Model bias versus variance





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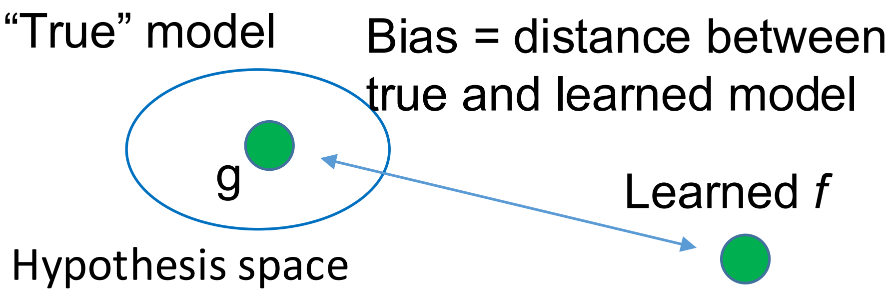
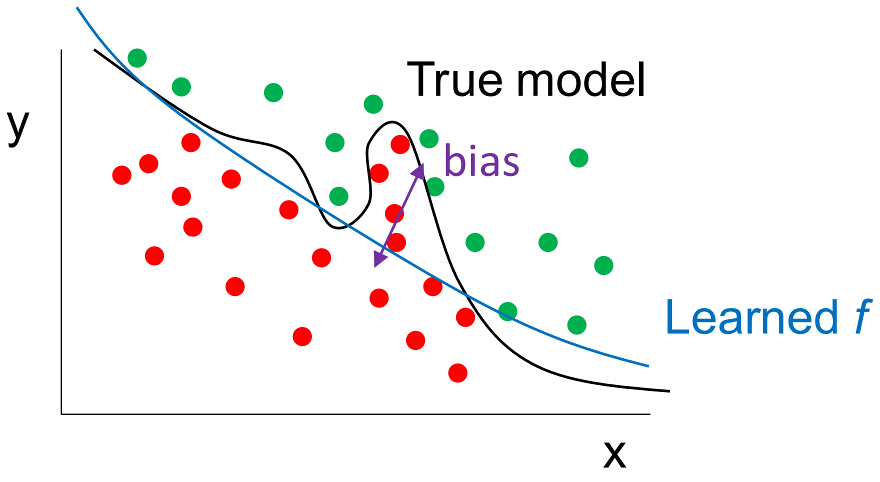


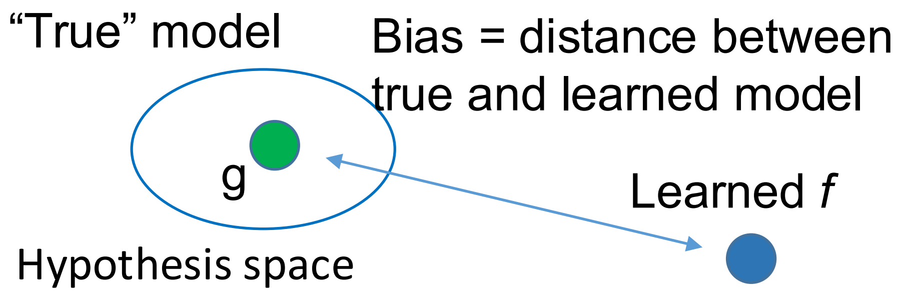


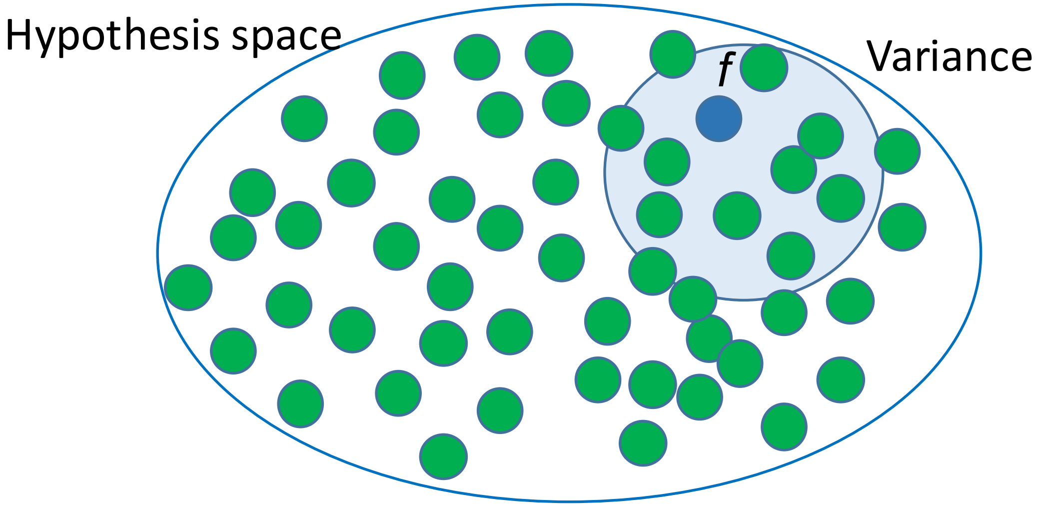
Models that tend to be "a bit too simple" are biased away from "true" model

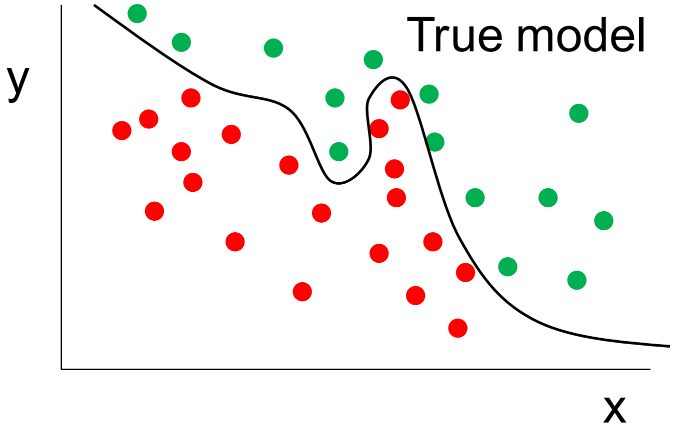
$$\text{Bias} = (g(\mathbf{x}) - f(\mathbf{x}))^2$$

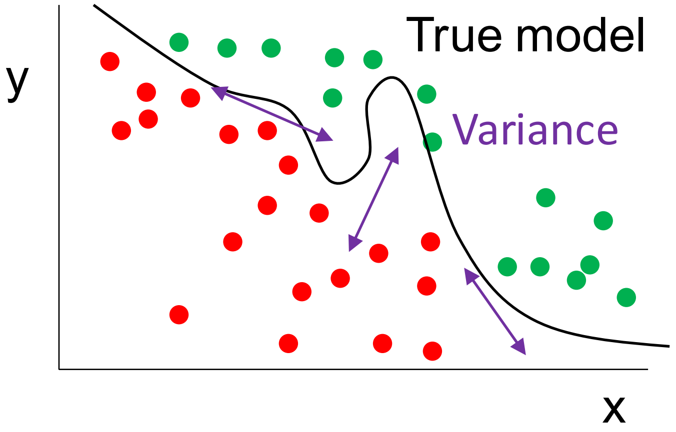
Measures how far our learning model f is biased away from target function g (for perfect training data classification)

Model bias versus variance





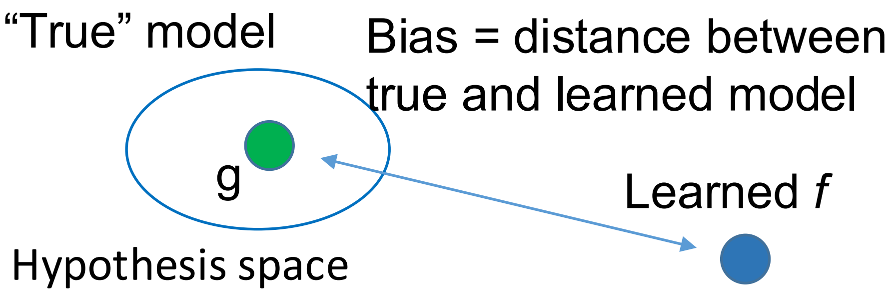


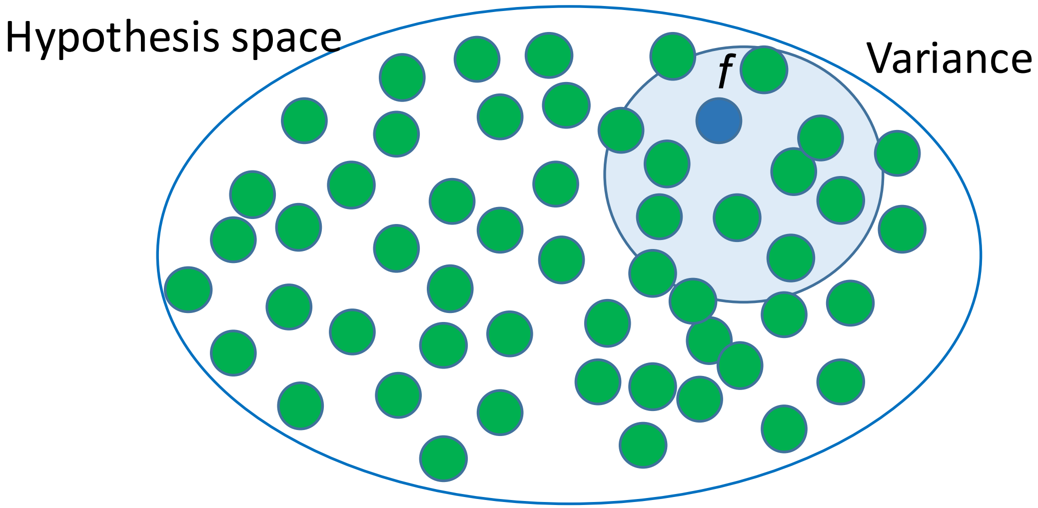


Variance = Var[g(**x**)]

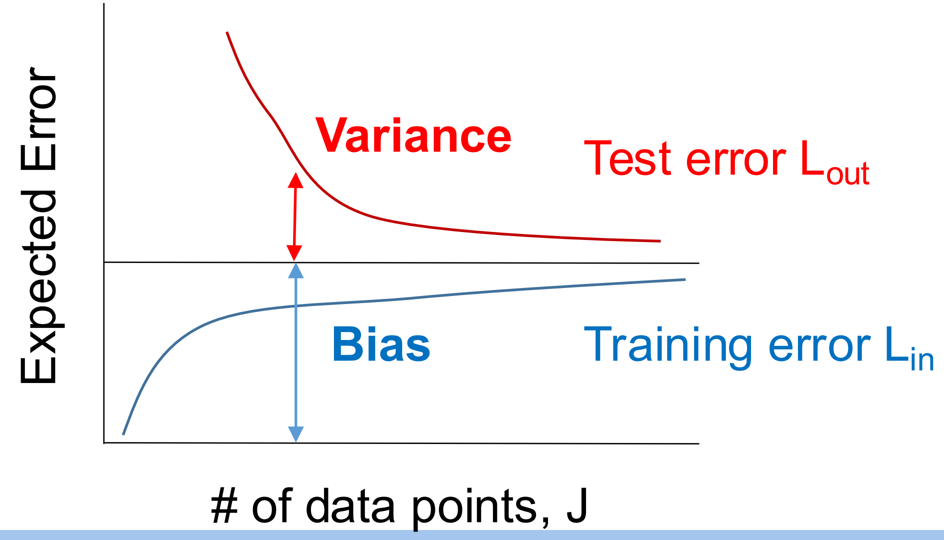
More complicated datasets exhibit lots of variance between ideal boundary for training and testing

Model bias versus variance





Error vs. # data points



Test Error is sum of model bias and variance!

Goal is to find a model *f* that balances between these two quantities for a given dataset

How to formally define capacity and complexity?

- Short answer: it's complicated…
- Related to something called the *VC Dimension*
    - Can provide theoretical bounds on performance
    - Dimensional bounds rather than scalar bounds…
- I decided not to go into it, but please do take a look at the following lecture material to learn more!

Learning From Data (Caltech, Prof. Y Abu-Mostafa)
https://www.youtube.com/watch?v=Dc0sr0kdBVI#t=3m24s

Conclusions from statistical machine learning

deep imaging

- Conclusion: you want a model that is complex enough to capture variations within high-dimensional space, but not too complex such that it overfits the data

- Want a model with a high capacity, but can still *generalize* to data outside training set
  - More data -> less overfitting, complex target -> more overfitting

- For simple models, we can measure complexity via degrees of freedom, the VC bound and so-on to help us nail down ideal models that can generalize well

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- **For DL models**: this will get too hard…here's a few counter-intuitive properties:

1. A fixed DL *architecture* exhibits data-dependent complexities
   - e.g., "good" DL networks achieve 0 training error on images with random labels, so cannot generalize at all in this case, and are too complex

2. DL networks with more hidden units leads to *better* generalization (the main finding of the last few years). So deeper models tend to be less complex, actually…

3. Complexity depends upon loss function and optimization method…

Important to remember: "No Free Lunch Theorem"

- *"Averaged over all possible data-generating distributions, every classification algorithm has the same error rate when classifying previously unobserved points."*
- The most sophisticated DL algorithm has same average performance (averaged over all possible tasks) as the simplest.

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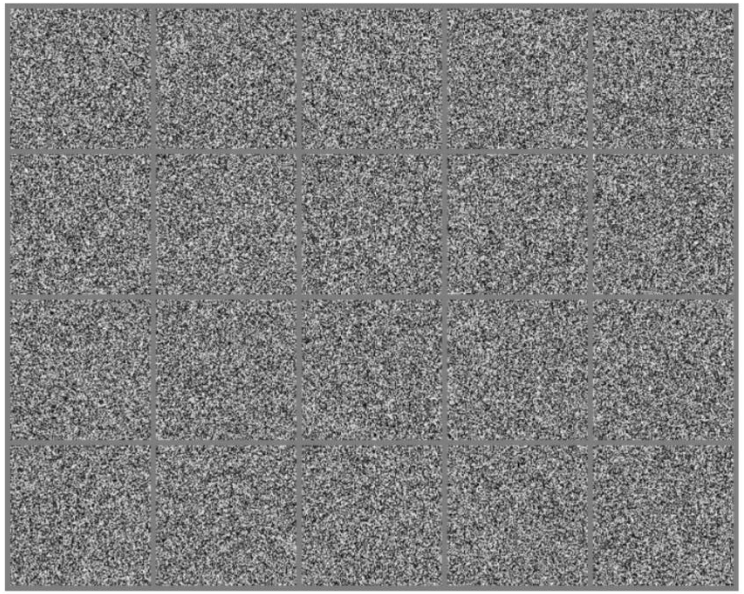
- Must make assumptions about probability distributions of inputs we’ll encounter in real-world

Set of 20 “images”, random Gaussian distribution



Face at different orientations = manifold n-D space

Deep Learning, I. Goodfellow et al., Fig. 5.12-13

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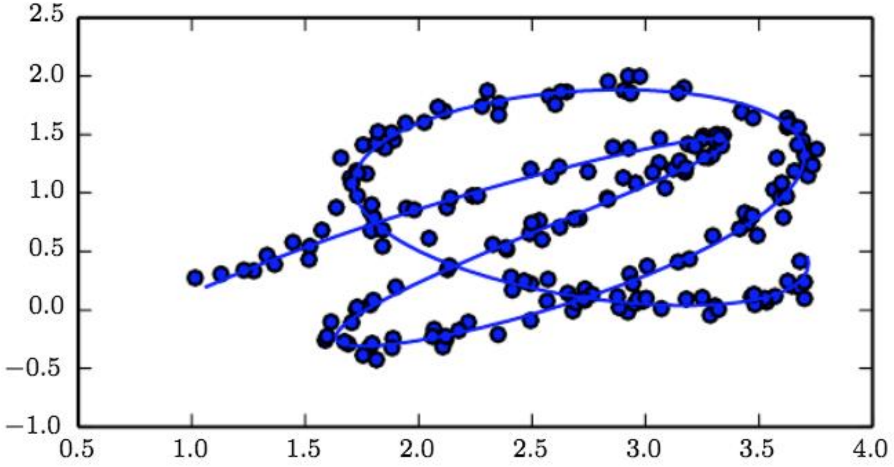
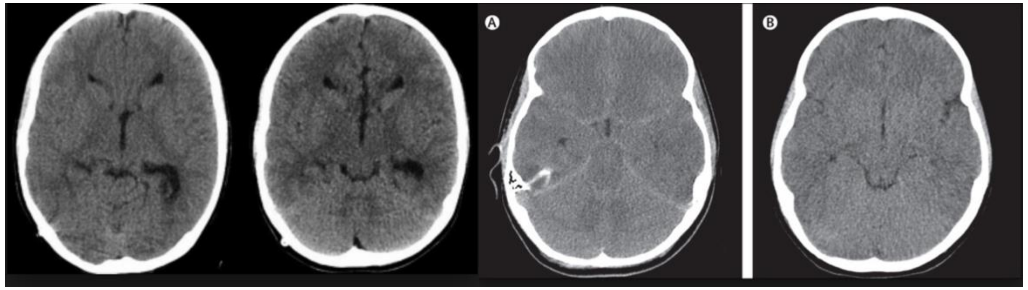
1D Manifold in 2D space



Deep Learning, I. Goodfellow et al., Fig. 5.11

Manifold Hypothesis ⟺

CT reconstructions of every brain in the world = kD manfold in nD space?



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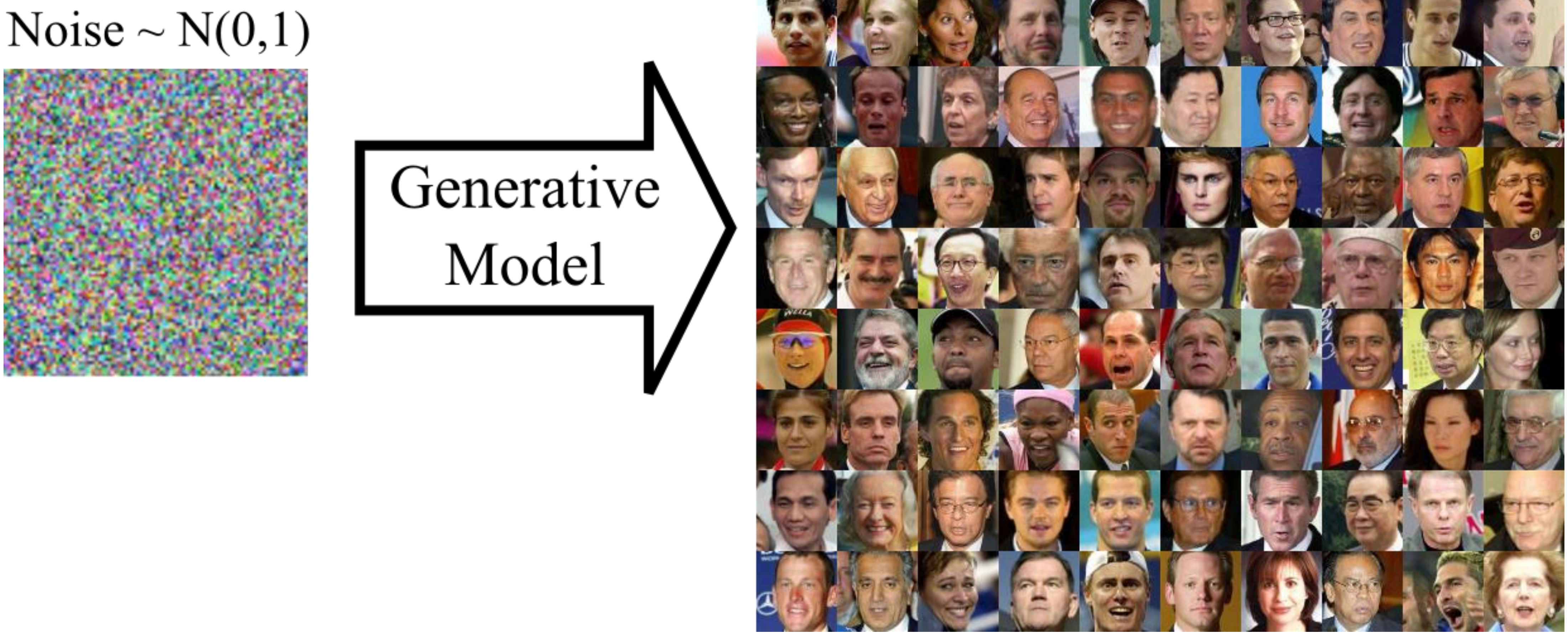
Noise ~ N(0,1)
Generative Model