deep imaging

Lecture 24: Reinforcement Learning

Machine Learning and Imaging

BME 548L
Roarke Horstmeyer

Resources for this lecture

Stanford CS231n, Lecture 17

Berkeley CS 294: Deep Reinforcement Learning
http://rail.eecs.berkeley.edu/deeprlcourse-fa17/f17docs/lecture_3_rl_intro.pdf

V. Mnih et al., “Human-level control through deep reinforcement learning,” Nature (2016)

Technical note: Q-Learning
http://www.gatsby.ucl.ac.uk/~Dayan/papers/cjch.pdf

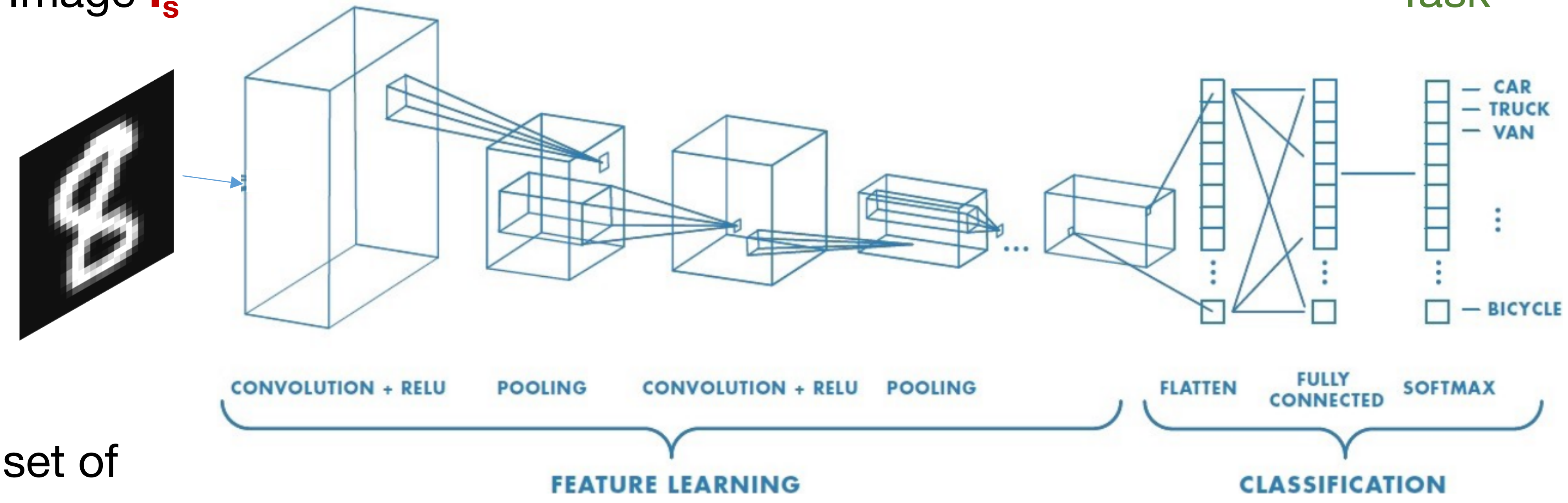
Reinforcement learning - in a nutshell

- So far, we've looked at:
  1) Decisions from fixed images (classification, detection, segmentation)

CNN's

Image $\mathbf{I_s}$

Task



Fixed set of training images

$$\text{Task} = \mathbf{W_n} \ldots \text{ReLU}[\mathbf{W_1}\, \text{ReLU}[\mathbf{W_0}\, \mathbf{I_s}]\ldots]$$

Reinforcement learning - in a nutshell

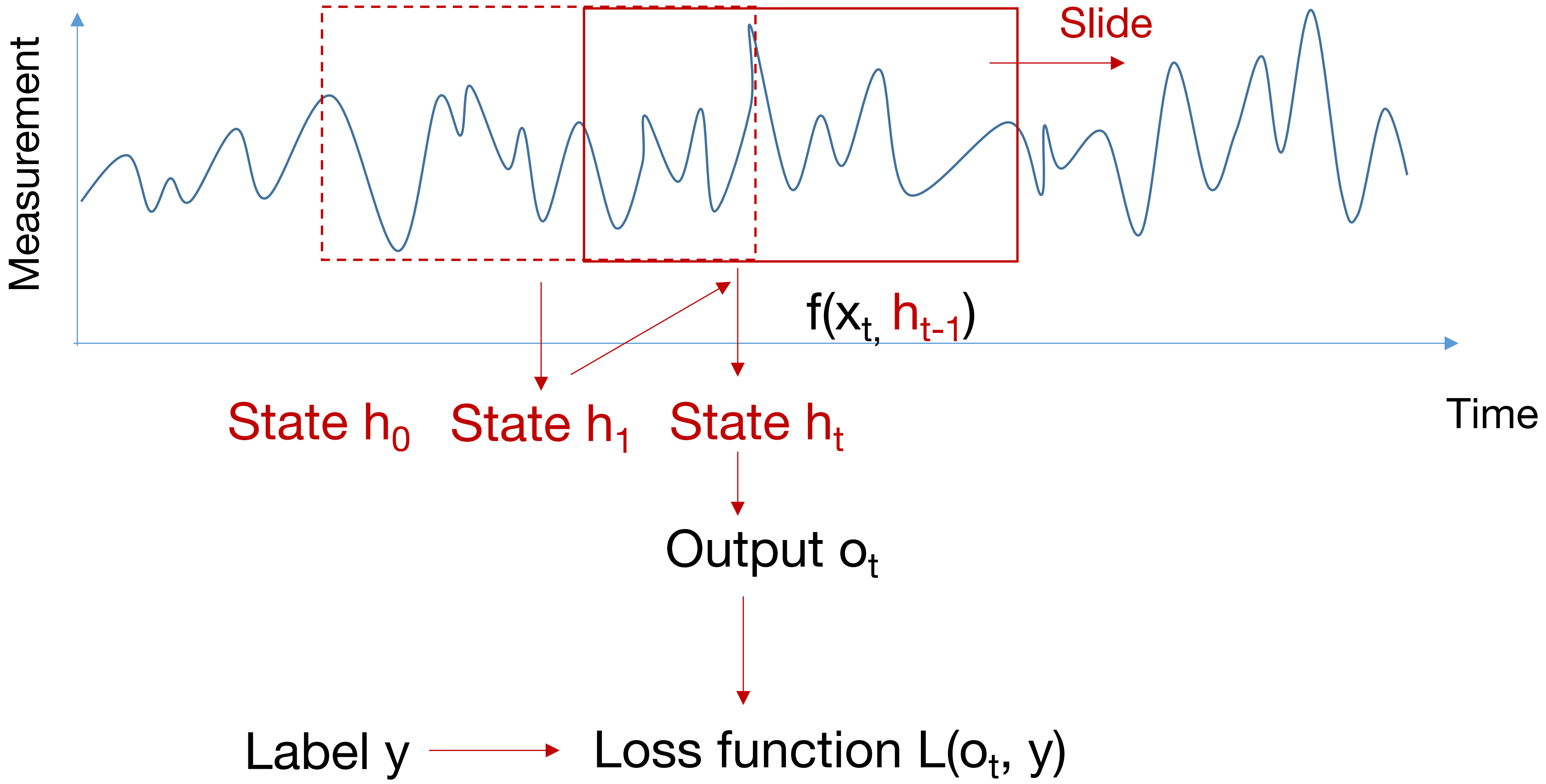
- So far, we've looked at:
  1) Decisions from fixed images (classification, detection, segmentation)

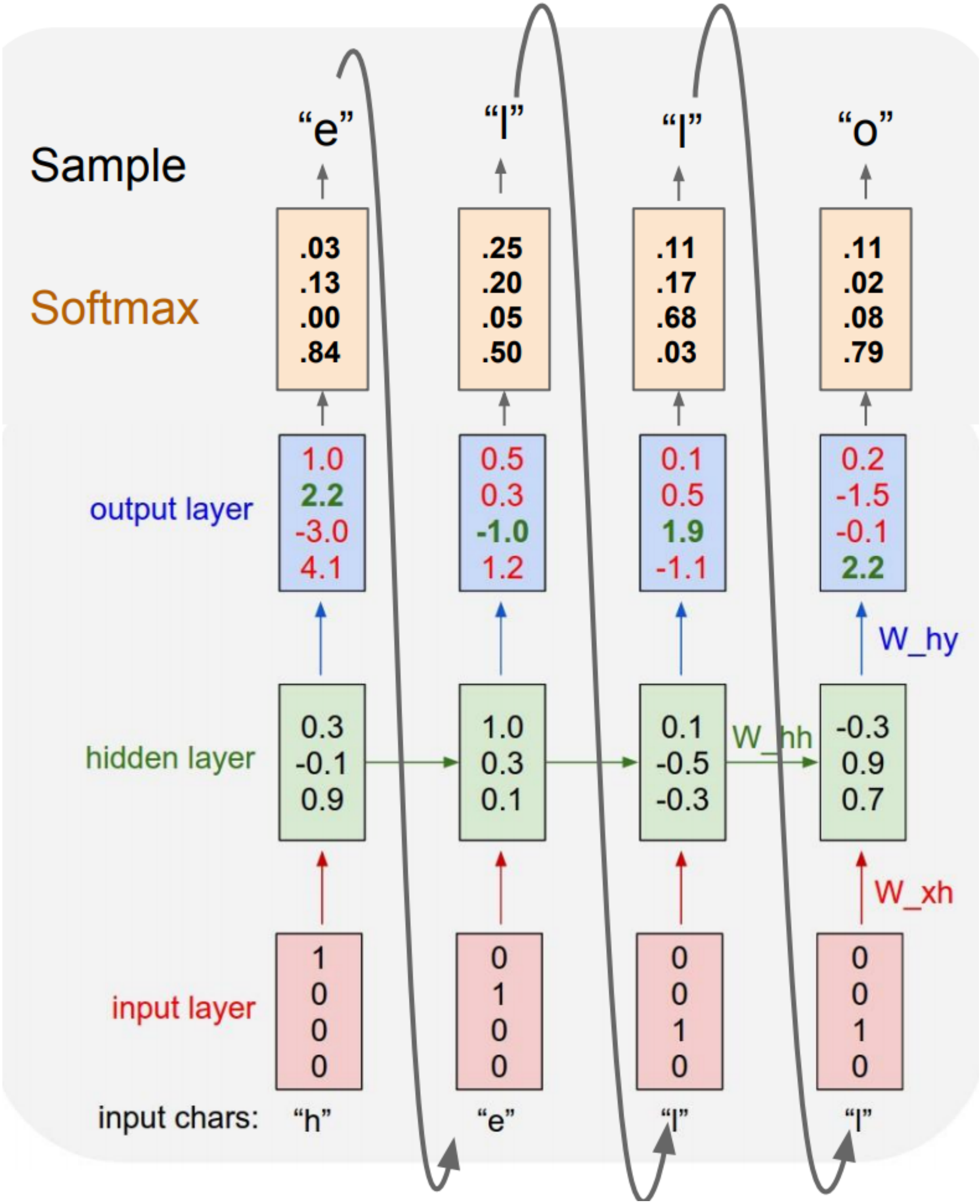
     CNN's

  2) Decisions from time-sequence data (captioning as classification, etc.)

     RNN's

Fixed set of
temporal sequences





From Stanford CS231n Lecture 10 slides

deep imaging

Reinforcement learning - in a nutshell

- So far, we've looked at:
  1) Decisions from fixed images (classification, detection, segmentation)

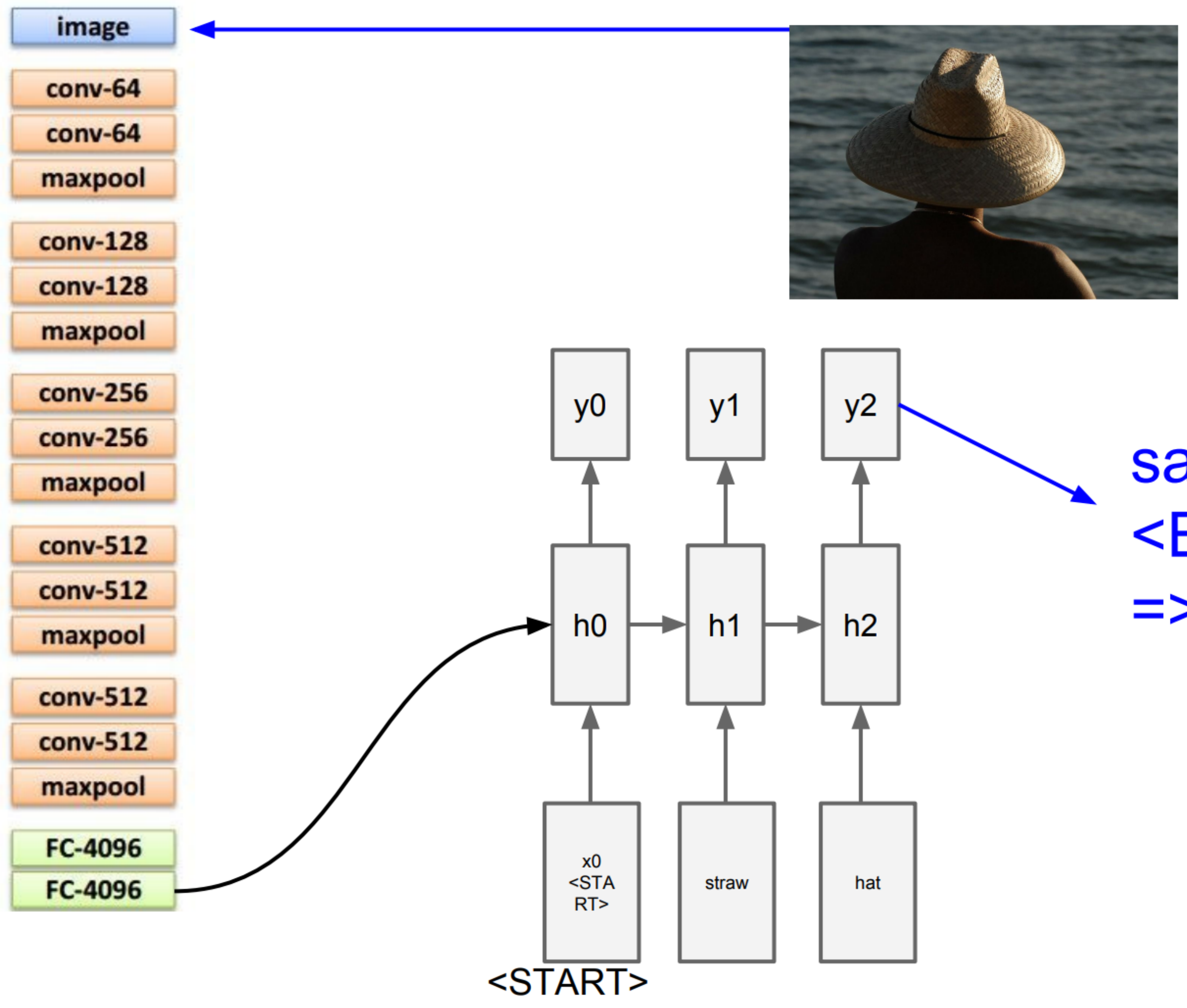
CNN's

2) Decisions from time-sequence data (captioning as classification, etc.)

Decisions from images and time-sequence data (video classification, etc.)

RNN's

Example: Image captioning



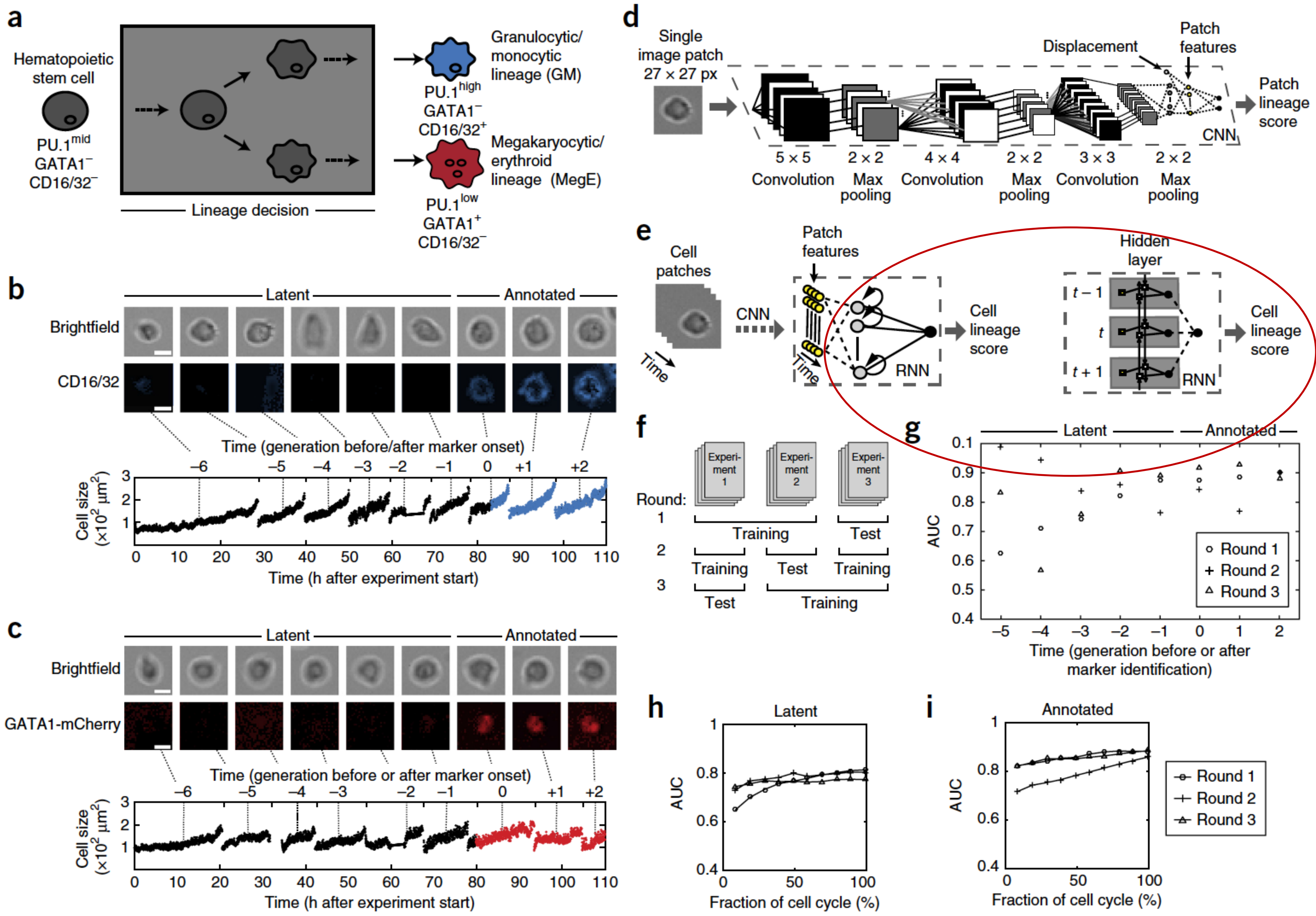
test image

sample <END> token => finish.

From Stanford CS231n Lecture 10 slides

Prospective identification of hematopoietic lineage choice by deep learning

Felix Buggenthin[1,6], Florian Buettner[1,2,6], Philipp S Hoppe[3,4], Max Endele[3], Manuel Kroiss[1,5], Michael Strasser[1], Michael Schwarzfischer[1], Dirk Loeffler[3,4], Konstantinos D Kokkaliaris[3,4], Oliver Hilsenbeck[3,4], Timm Schroeder[3,4], Fabian J Theis[1,5] & Carsten Marr[1]



deep imaging

Reinforcement learning - in a nutshell

- So far, we've looked at:
  1) Decisions from fixed images (classification, detection, segmentation)

CNN's

2) Decisions from time-sequence data (captioning as classification, etc.)

Decisions from images and time-sequence data (video classification, etc.)

RNN's

- Now, we're going to consider decisions for *dynamic data*
  - Most successful application: dynamic image data
    e.g.: video games, images of a Go game, car turning through obstacles

Reinforcement Learning

The step from fixed video to dynamic video

Outcome:
Cell type B

Goal: examine *all* data to make final decision

The step from fixed video to dynamic video

deep imaging

Outcome:
Cell type B

Goal: examine *all* data to make final decision

“jump”

block

forward

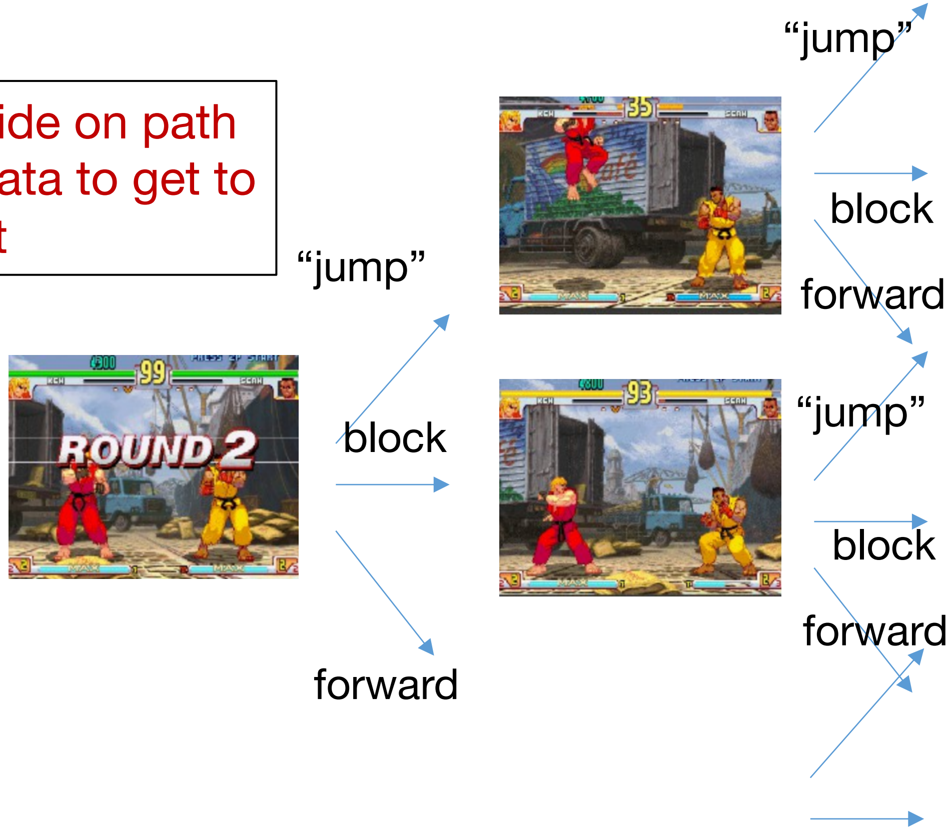
The step from fixed video to dynamic video

deep imaging

Outcome:
Cell type B

Goal: examine *all* data to make final decision

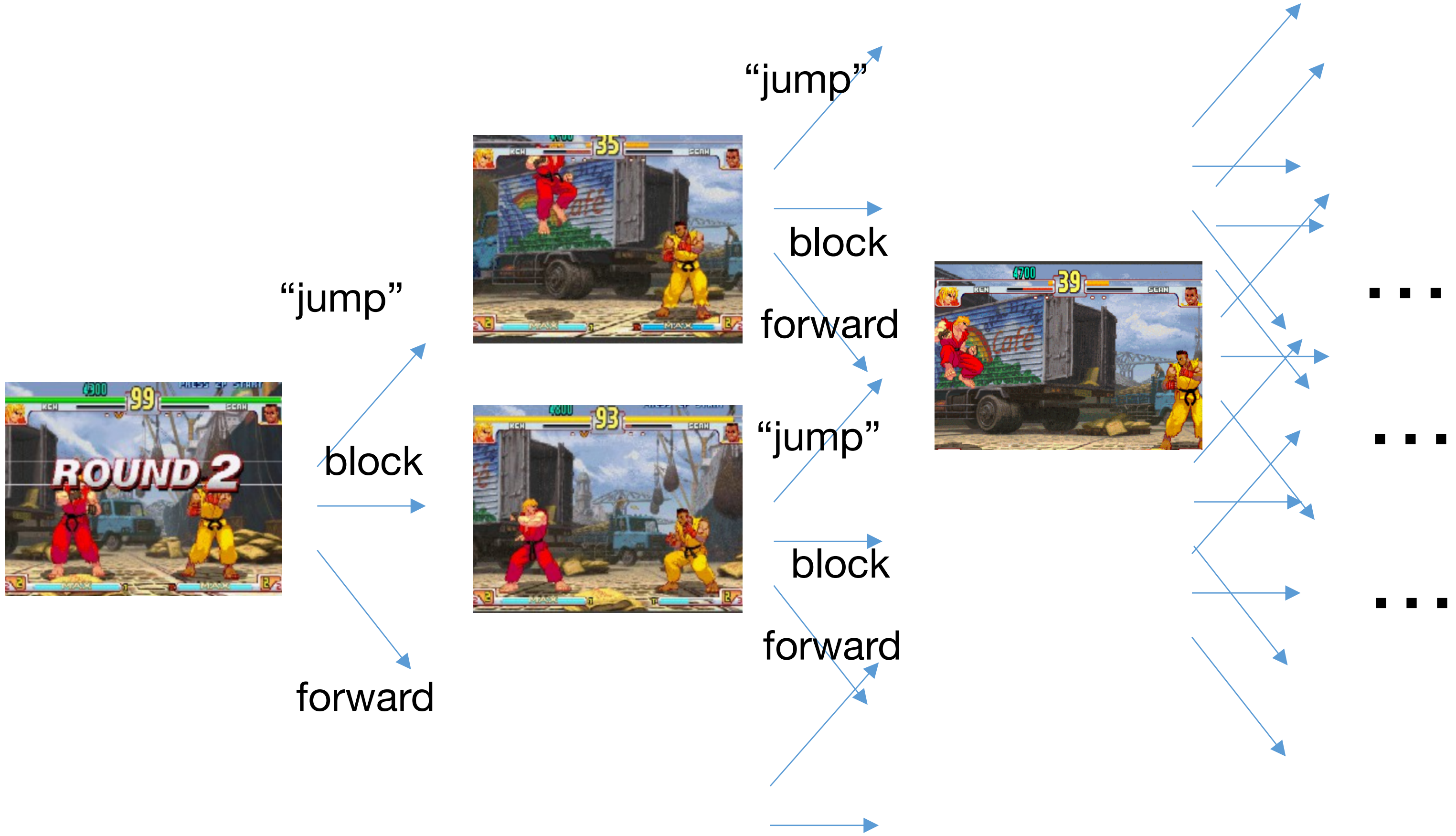
Goal: decide on path through data to get to final result



The step from fixed video to dynamic video

Outcome:
Cell type B

Goal: examine *all* data to make final decision



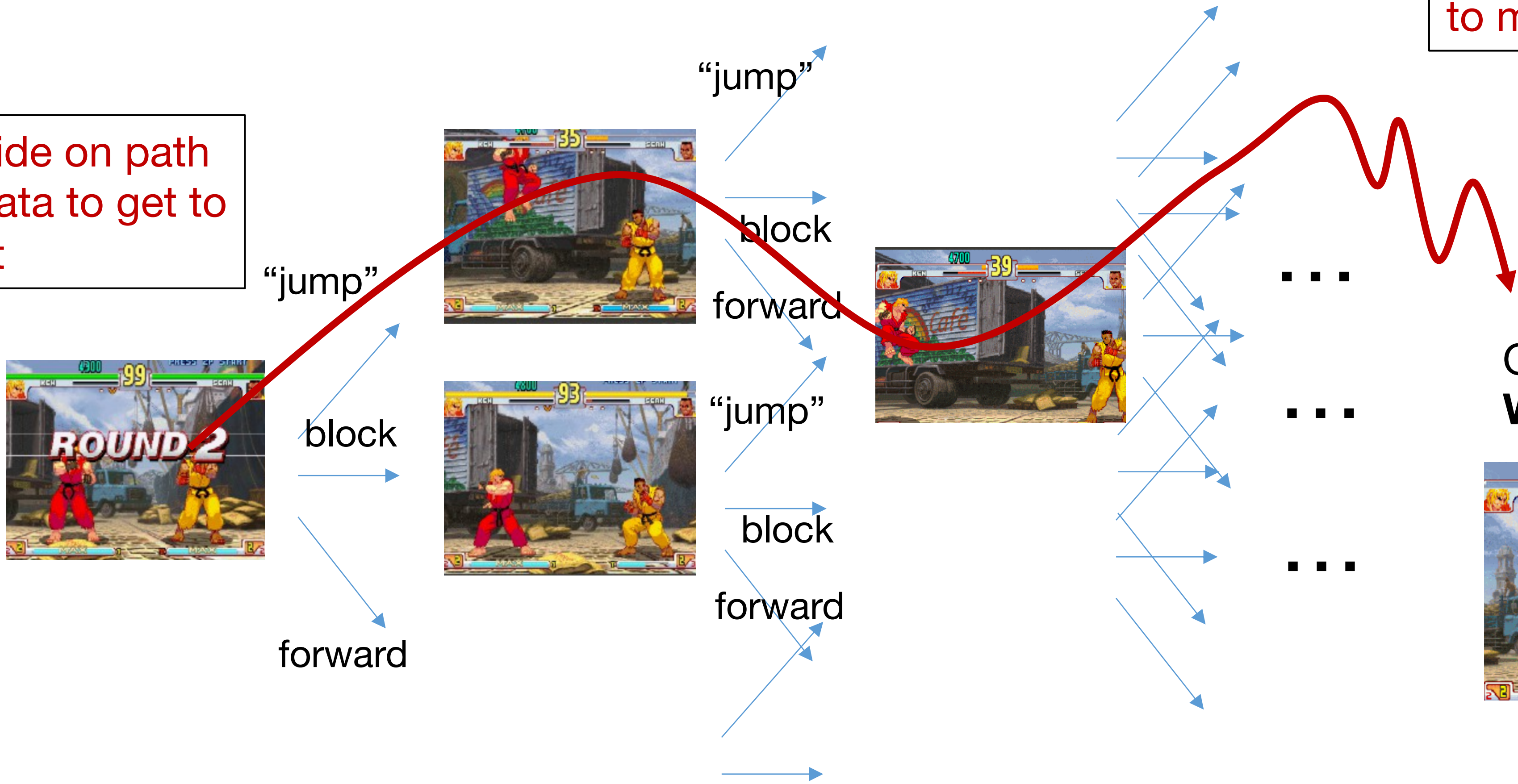
Outcome:
Win the game!

The step from fixed video to dynamic video

Outcome:
Cell type B

Goal: examine *all* data to make final decision

Goal: decide on path through data to get to final result



Outcome:
Win the game!

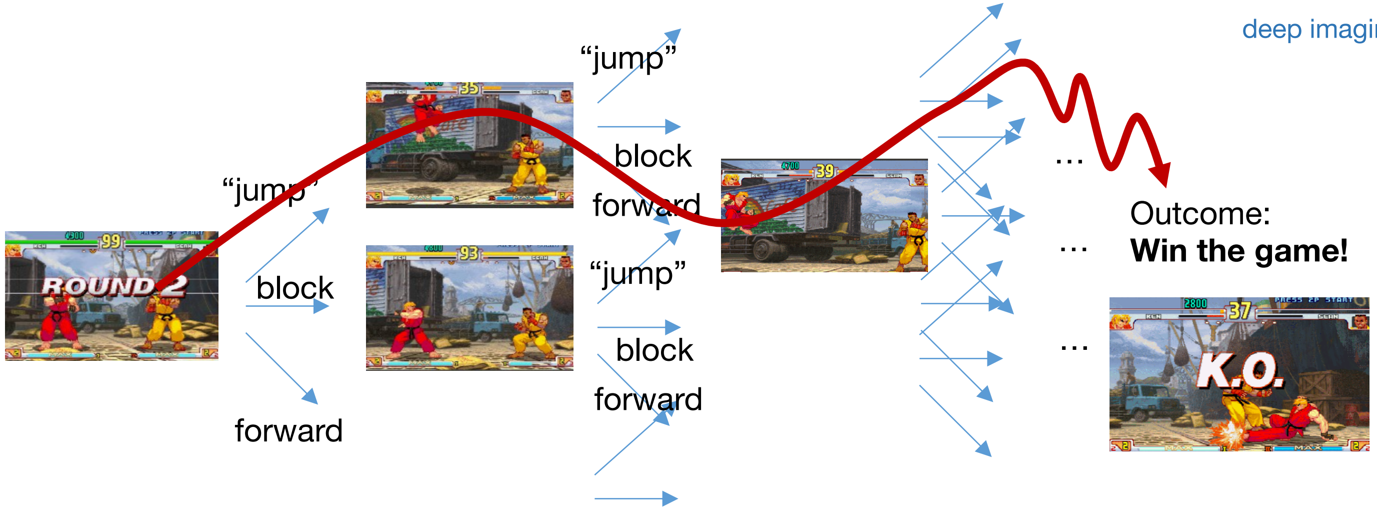
deep imaging

Supervised ML

Outcome:
Cell type B

- Fixed image sequence
- Goal: match to known label
  (large labeled dataset needed)
- Output: label
- Examines all data

Reinforcement learning



- *Dynamic/active* image sequence
- Goal: get to known desired outcome
  (no labels needed, really…)
- Output: sequence of actions
- Not possible to examine *all* data

Terms and notation

Example situation: Preparing for surgery using a robotically controlled instrument with an endoscope camera. You want endoscope to guide itself to tumor as quickly as possible

Movement choices:

First image
(of skin)

Initial state s_0

Up

Down

Left

Right

Action a_0

Terms and notation

deep imaging

Example situation: Preparing for surgery using a robotically controlled instrument with an endoscope camera. You want endoscope to guide itself to tumor as quickly as possible

Movement choices: How'd we do?

First image (of skin) → Up / Down / Left / Right → +1 for darker image / -1 for lighter image → Second image → …

Initial state s_0

Action a_0

Reward r_0

state s_1

Continue until final state (see tumor)

Terms and notation

Example situation: Preparing for surgery using a robotically controlled instrument with an endoscope camera. You want endoscope to guide itself to tumor as quickly as possible

Movement choices: How'd we do?

First image (of skin)

Initial state s_0

Policy π

Up

Down

Left

Right

Action a_0

+1 for darker image

-1 for lighter image

Reward r_0

Second image

state s_1

…

Continue until final state (see tumor)

Optimization Goal: Find policy π* that maximizes total "discounted" reward $\sum_{t\geq 0} \gamma^t r_t$

TL;DR

-> Use a CNN to map images to actions, optimize CNN with respect to loss function that depends on reward *in a recursive manner*

Movement choices: How'd we do?

First image (of skin) → Policy π → Up / Down / Left / Right → +1 for darker image / -1 for lighter image → Second image → …

Initial state s_0

Policy π

Up

Down

Left

Right

Action a_0

+1 for darker image

-1 for lighter image

Reward r_0

Second image

state s_1

…

Continue until final state (see tumor)

Optimization Goal: Find policy π* that maximizes total "discounted" reward $\sum_{t \geq 0} \gamma^t r_t$

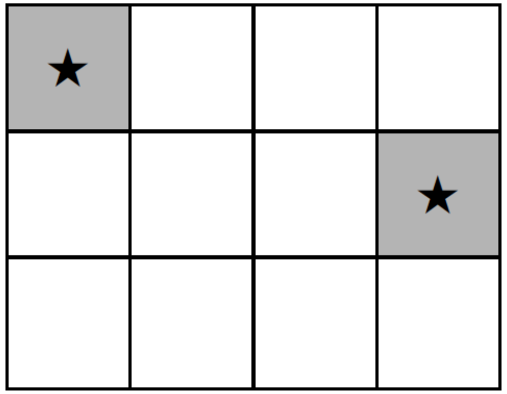
A simple MDP: Grid World

actions = {

1. right →
2. left ←
3. up ↑
4. down ↓

}

states

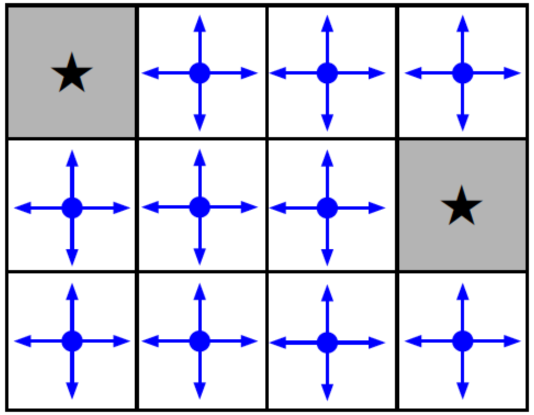


Set a negative "reward" for each transition (e.g. $r = -1$)

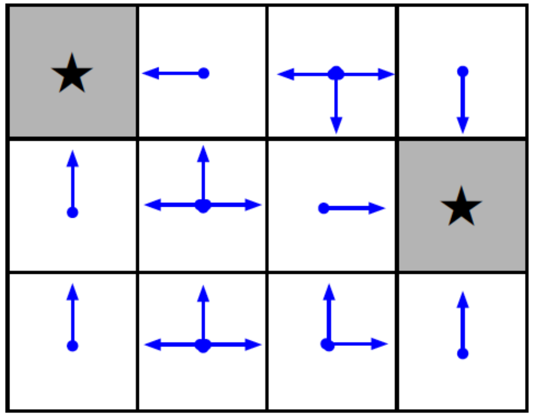
Objective: reach one of terminal states (greyed out) in least number of actions

From Stanford CS231n Lecture 17

A simple MDP: Grid World



Random Policy



Optimal Policy

From Stanford CS231n Lecture 17

Let's jump into the math….

Definition of a Markov process:

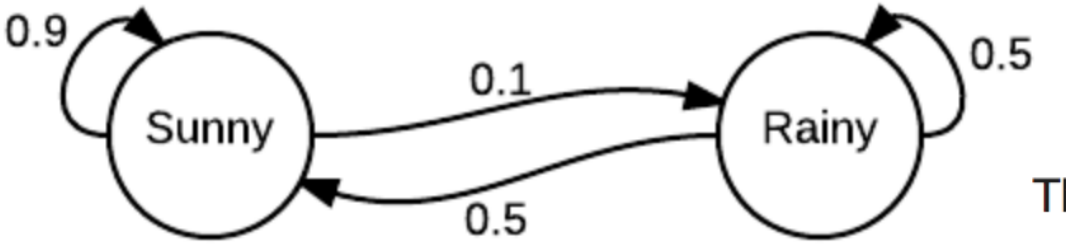
$$\Pr(X_{n+1} = x \mid X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = \Pr(X_{n+1} = x \mid X_n = x_n)$$

Let's jump into the math….

deep imaging

Definition of a Markov process:

$$\Pr(X_{n+1} = x \mid X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = \Pr(X_{n+1} = x \mid X_n = x_n)$$



$$\mathbf{x}^{(0)} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The weather on day 2 can be predicted by:

$$\mathbf{x}^{(1)} = \mathbf{x}^{(0)} P = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 \end{bmatrix}$$

2 states: Sunny and Rainy

Thus, there is a 90% chance that day 2 will also be sunny.

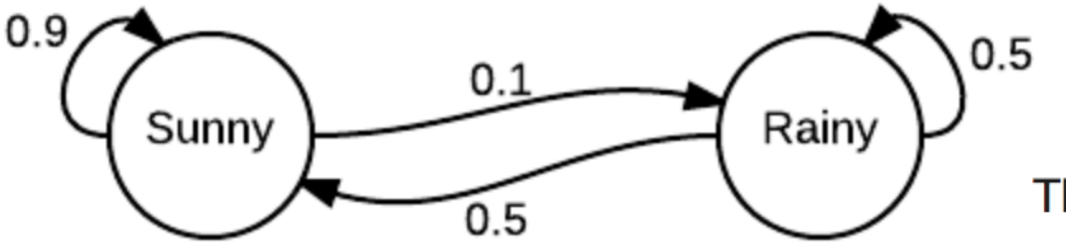
https://en.wikipedia.org/wiki/Markov_chain

Let's jump into the math….

deep imaging

Definition of a Markov process:

$$\Pr(X_{n+1} = x \mid X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = \Pr(X_{n+1} = x \mid X_n = x_n)$$



$$\mathbf{x}^{(0)} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The weather on day 2 can be predicted by:

$$\mathbf{x}^{(1)} = \mathbf{x}^{(0)} P = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 \end{bmatrix}$$

Thus, there is a 90% chance that day 2 will also be sunny.

2 states: Sunny and Rainy

Transition matrix – try to learn this from state to state

https://en.wikipedia.org/wiki/Markov_chain

Assume transition between states follows Markov process

$P(s_{t+1}|s_t, s_{t-1}\ldots s_0) = P(s_{t+1} \mid s_t)$

Andrey Markov

Markov chain

$\mathcal{M} = \{\mathcal{S}, \mathcal{T}\}$

$\mathcal{S}$ – state space — states $s \in \mathcal{S}$ (discrete or continuous)

$\mathcal{T}$ – transition operator — $p(s_{t+1}|s_t)$

why "operator"? — let $\mu_{t,i} = p(s_t = i)$ — $\vec{\mu}_t$ is a vector of probabilities

let $\mathcal{T}_{i,j} = p(s_{t+1} = i|s_t = j)$ — then $\vec{\mu}_{t+1} = \mathcal{T}\vec{\mu}_t$

$$\mathbf{s}_1 \xrightarrow[p(\mathbf{s}_{t+1}|\mathbf{s}_t)]{} \mathbf{s}_2 \xrightarrow[p(\mathbf{s}_{t+1}|\mathbf{s}_t)]{} \mathbf{s}_3$$

Markov property
independent of $\mathbf{s}_{t-1}$

Add in dependence on action: Markov *decision* process

$P(s_{t+1}|s_t) \Rightarrow P(s_{t+1} \mid s_t, a_t) = P(s_{t+1} \mid s_t, a_t, \ldots s_0, a_0)$

Markov decision process $\quad \mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$

$\mathcal{S}$ – state space $\quad$ states $s \in \mathcal{S}$ (discrete or continuous)

$\mathcal{A}$ – action space $\quad$ actions $a \in \mathcal{A}$ (discrete or continuous)

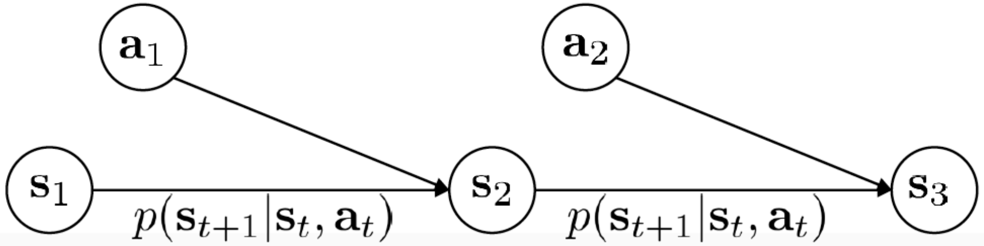
$\mathcal{T}$ – transition operator (now a tensor!)

let $\mu_{t,j} = p(s_t = j)$

let $\xi_{t,k} = p(a_t = k)$

let $\mathcal{T}_{i,j,k} = p(s_{t+1} = i | s_t = j, a_t = k)$

$$\mu_{t,i} = \sum_{j,k} \mathcal{T}_{i,j,k} \mu_{t,j} \xi_{t,k}$$



Andrey Markov

Richard Bellman

Add in dependence on action: Markov *decision* process

$P(s_{t+1}|s_t, a_t)$ can include reward $r(s_t, a_t)$

Markov decision process $\qquad \mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$

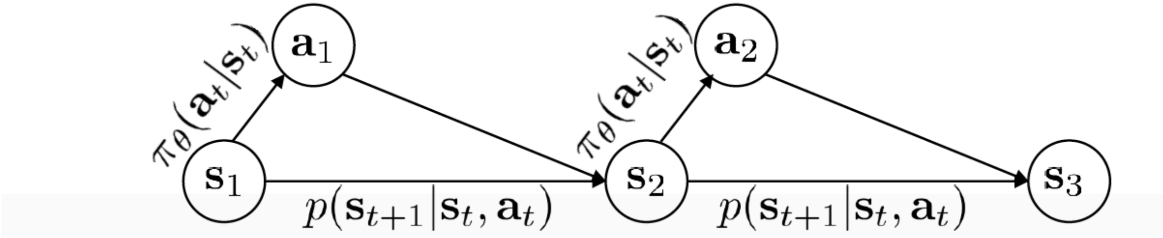
$\mathcal{S}$ – state space $\qquad$ states $s \in \mathcal{S}$ (discrete or continuous)

$\mathcal{A}$ – action space $\qquad$ actions $a \in \mathcal{A}$ (discrete or continuous)

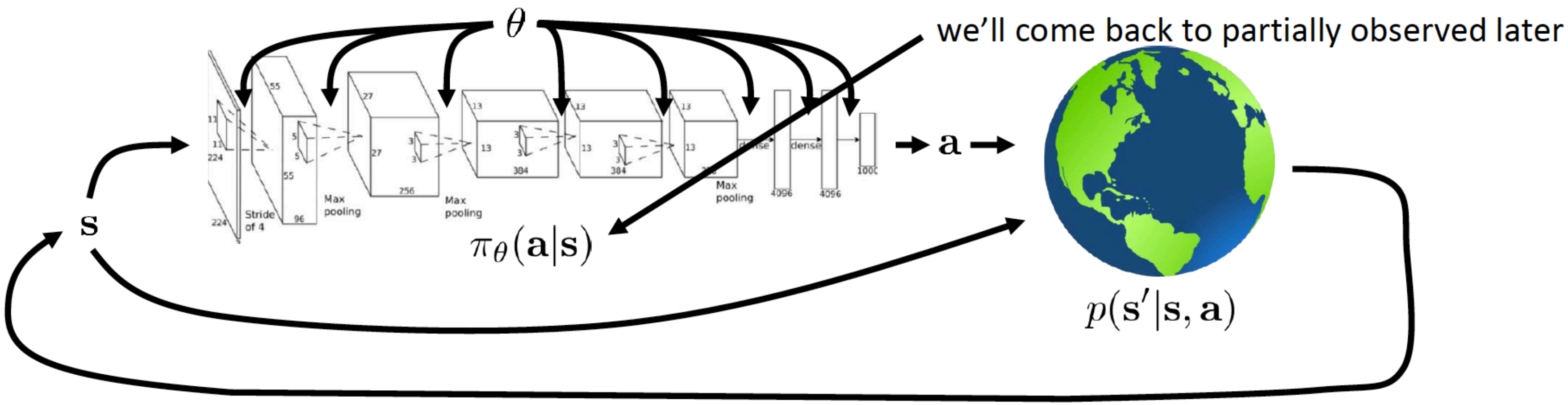
$\mathcal{T}$ – transition operator (now a tensor!)

r – reward function $\qquad r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$

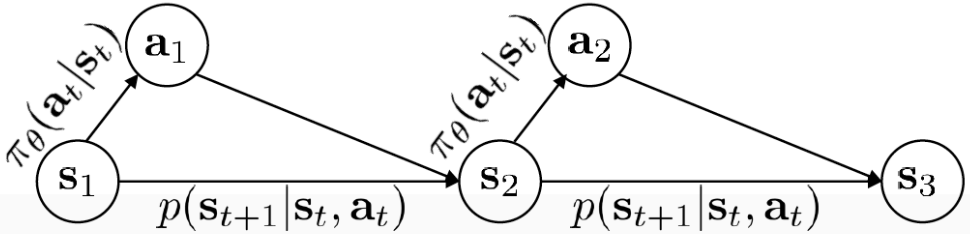
$r(s_t, a_t)$ – reward



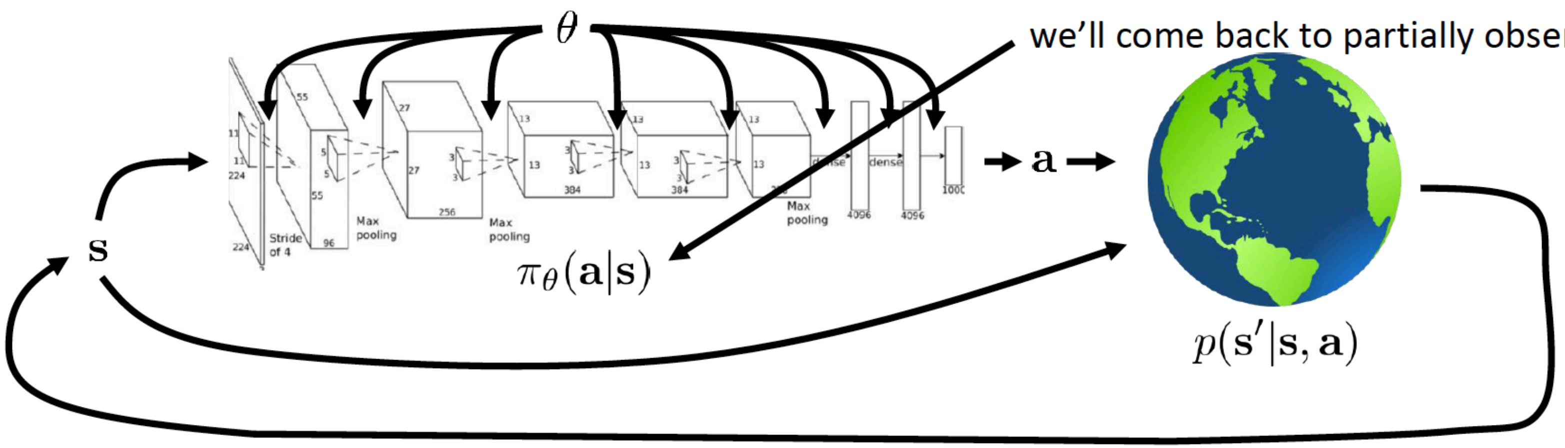
The goal of reinforcement learning



$$\underbrace{p_\theta(\mathbf{s}_1,\mathbf{a}_1,\ldots,\mathbf{s}_T,\mathbf{a}_T)}_{\pi_\theta(\tau)} = p(\mathbf{s}_1)\prod_{t=1}^{T}\underbrace{\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)}_{\text{Markov chain on }(\mathbf{s},\mathbf{a})}$$



The goal of reinforcement learning



$$\underbrace{p_\theta(\mathbf{s}_1,\mathbf{a}_1,\ldots,\mathbf{s}_T,\mathbf{a}_T)}_{\pi_\theta(\tau)} = p(\mathbf{s}_1)\prod_{t=1}^{T}\underbrace{\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)}_{\text{Markov chain on }(\mathbf{s},\mathbf{a})}$$

$$\theta^\star = \arg\max_\theta \sum_{t=1}^{T} E_{(\mathbf{s}_t,\mathbf{a}_t)\sim p_\theta(\mathbf{s}_t,\mathbf{a}_t)}[r(\mathbf{s}_t,\mathbf{a}_t)]$$

The optimal policy **π***

We want to find optimal policy **π*** that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability…)?
Maximize the **expected sum of rewards!**

Formally: $\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi\right]$ with $s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)$

Discount factor: accumulate the rewards
“acquired” up to current state, but they become
less important the longer they were in the past

From Stanford CS231n Lecture 17

The optimal policy **π***

We want to find optimal policy **π*** that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability…)?
Maximize the **expected sum of rewards!**

Formally: $\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi\right]$ with $s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)$

The **Q-value function** at state s and action a, is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^{\pi}(s,a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Don't have access to all policies, so use Q in practice

Bellman equation

The optimal Q-value function Q* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s,a) = \max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Q* satisfies the following **Bellman equation**:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q^*(s',a') | s, a\right]$$

Intuition: if the optimal state-action values for the next time-step Q*(s',a') are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s',a')$

From Stanford CS231n Lecture 17

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning**!

From Stanford CS231n Lecture 17

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q^*(s',a')|s,a\right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)}\left[(y_i - Q(s,a;\theta_i))^2\right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1})|s,a\right]$

From Stanford CS231n Lecture 17

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s',a') | s,a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)} \left[(y_i - Q(s,a;\theta_i))^2 \right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) | s,a \right]$

Iteratively try to make the Q-value close to the target value (y_i) it should have, if Q-function corresponds to optimal Q* (and optimal policy **π***)

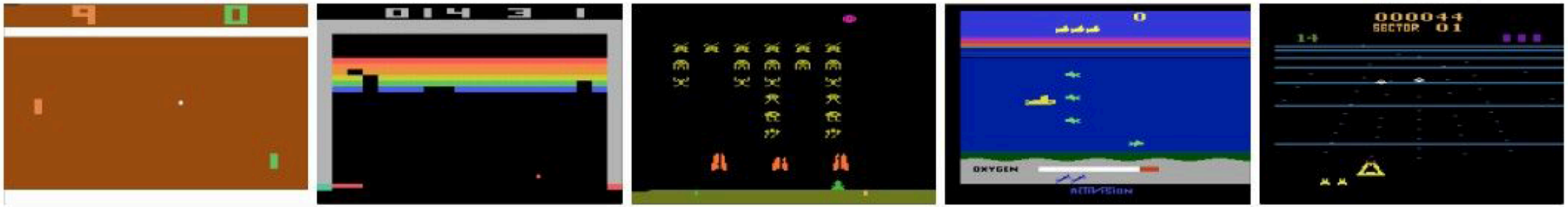
Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) - Q(s,a;\theta_i)) \nabla_{\theta_i} Q(s,a;\theta_i) \right]$$

From Stanford CS231n Lecture 17

Case Study: Playing Atari Games



Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

From Stanford CS231n Lecture 17

Q-network Architecture

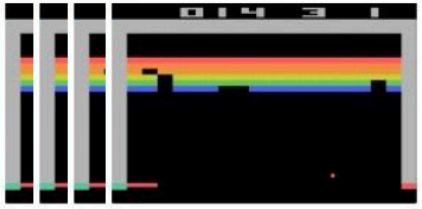
$Q(s, a; \theta)$:
neural network
with weights θ

FC-4 (Q-values)

FC-256

32 4x4 conv, stride 2

16 8x8 conv, stride 4



Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t, a_4)$

Input: state s_t

Current state s_t: 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

From Stanford CS231n Lecture 17

https://www.youtube.com/watch?v=V1eYniJ0Rnk

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

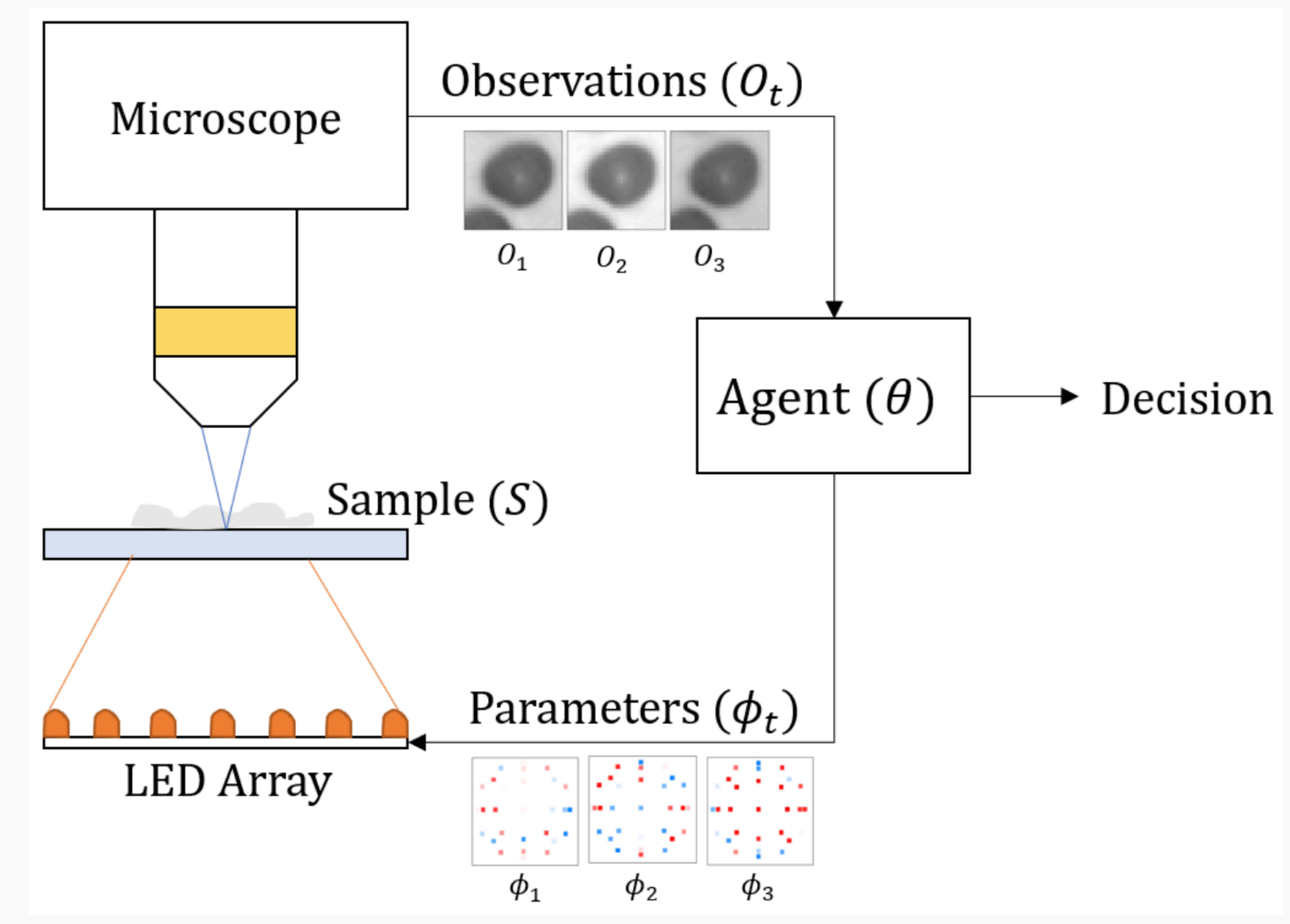
Address these problems using **experience replay**

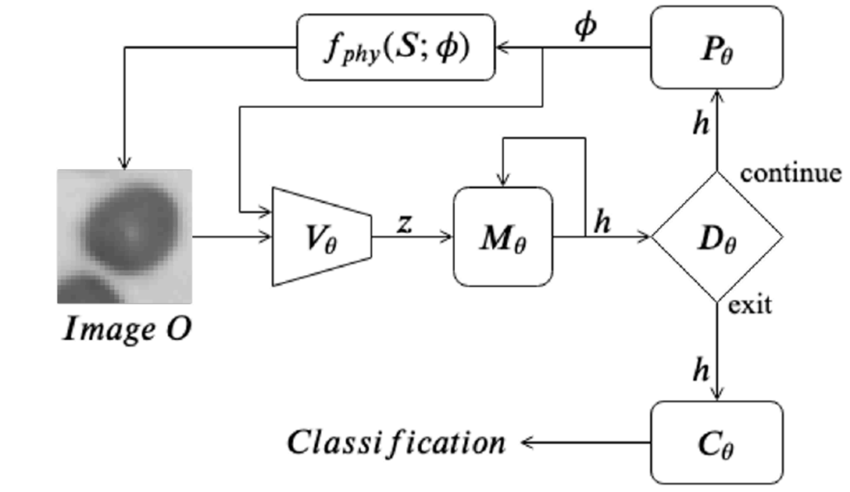
- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

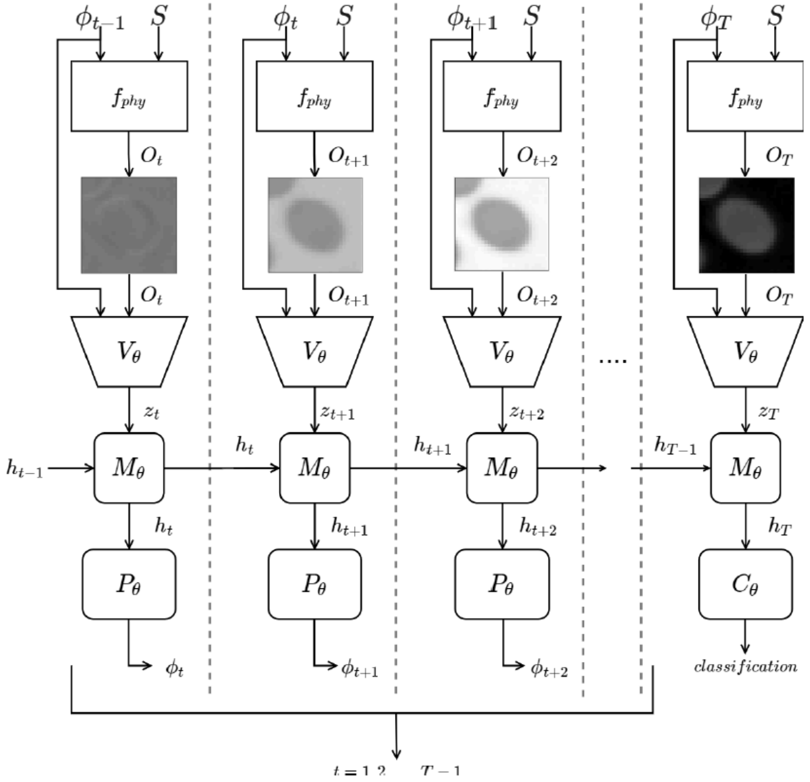
Each transition can also contribute to multiple weight updates => greater data efficiency

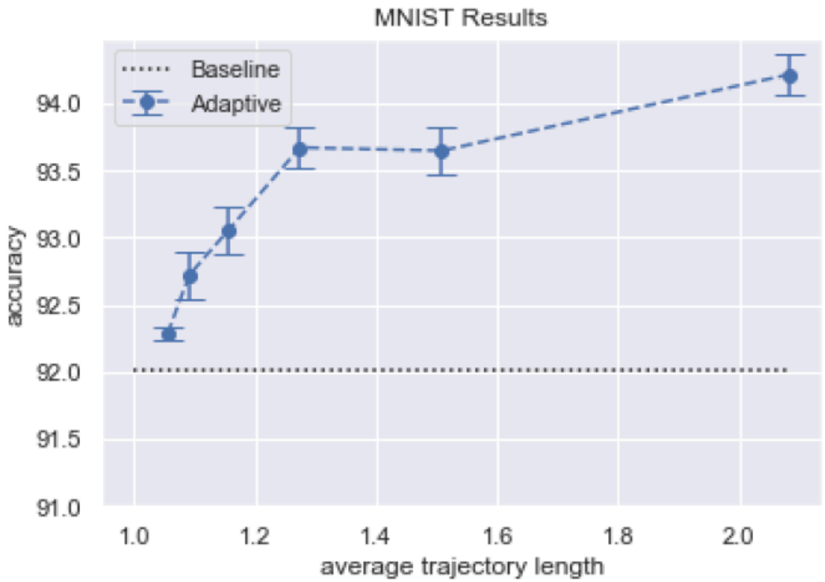
From Stanford CS231n Lecture 17

How can this be applied to optimized imaging?

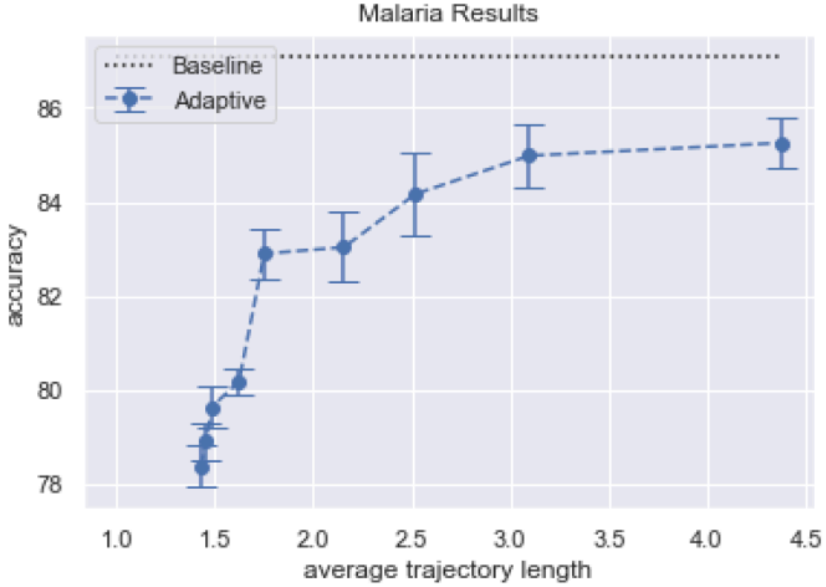








(a) MNIST results



(b) Malaria results

deep imaging

How can this be applied to optimized imaging?