deep imaging

Lecture 21: Reinforcement Learning

Machine Learning and Imaging

BME 590L
Roarke Horstmeyer

Resources for this lecture

Stanford CS231n, Lecture 17

Berkeley CS 294: Deep Reinforcement Learning
http://rail.eecs.berkeley.edu/deeprlcourse-fa17/f17docs/lecture_3_rl_intro.pdf

V. Mnih et al., “Human-level control through deep reinforcement learning,” Nature (2016)

Technical note: Q-Learning
http://www.gatsby.ucl.ac.uk/~Dayan/papers/cjch.pdf

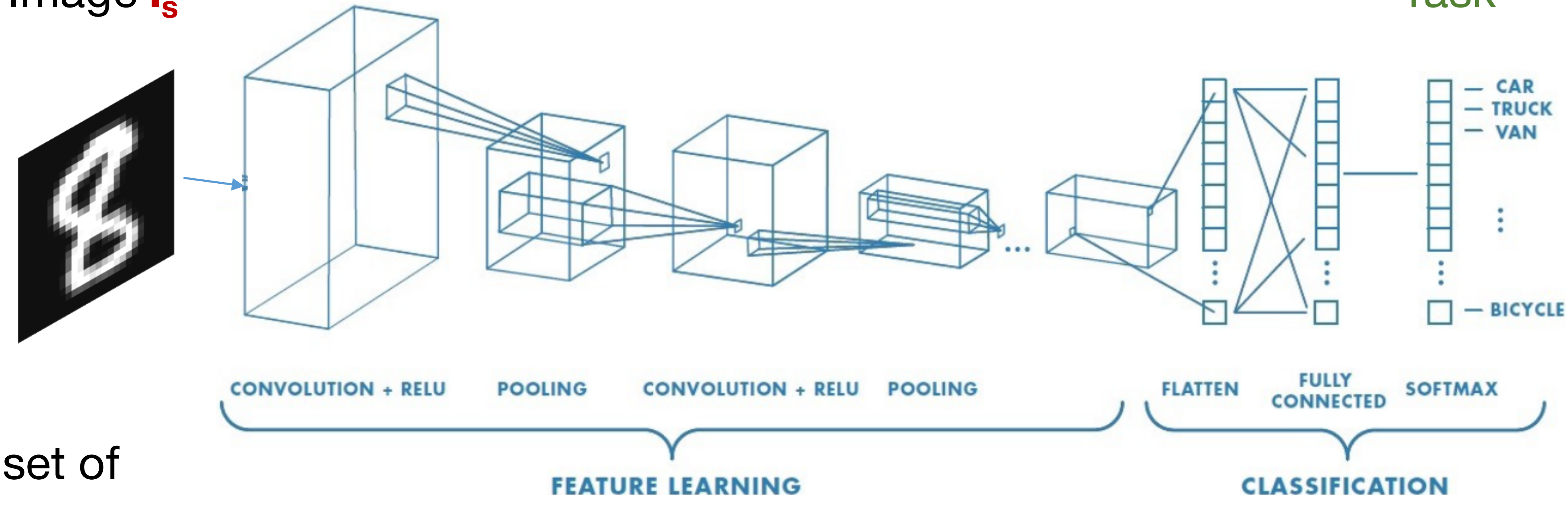
Reinforcement learning - in a nutshell

- So far, we've looked at:
  1) Decisions from fixed images (classification, detection, segmentation)

CNN's

Image $\mathbf{I_s}$

Task



Fixed set of training images

$$\text{Task} = \mathbf{W_n} \ldots \text{ReLU}[\mathbf{W_1}\, \text{ReLU}[\mathbf{W_0}\, \mathbf{I_s}] \ldots]$$

Reinforcement learning - in a nutshell

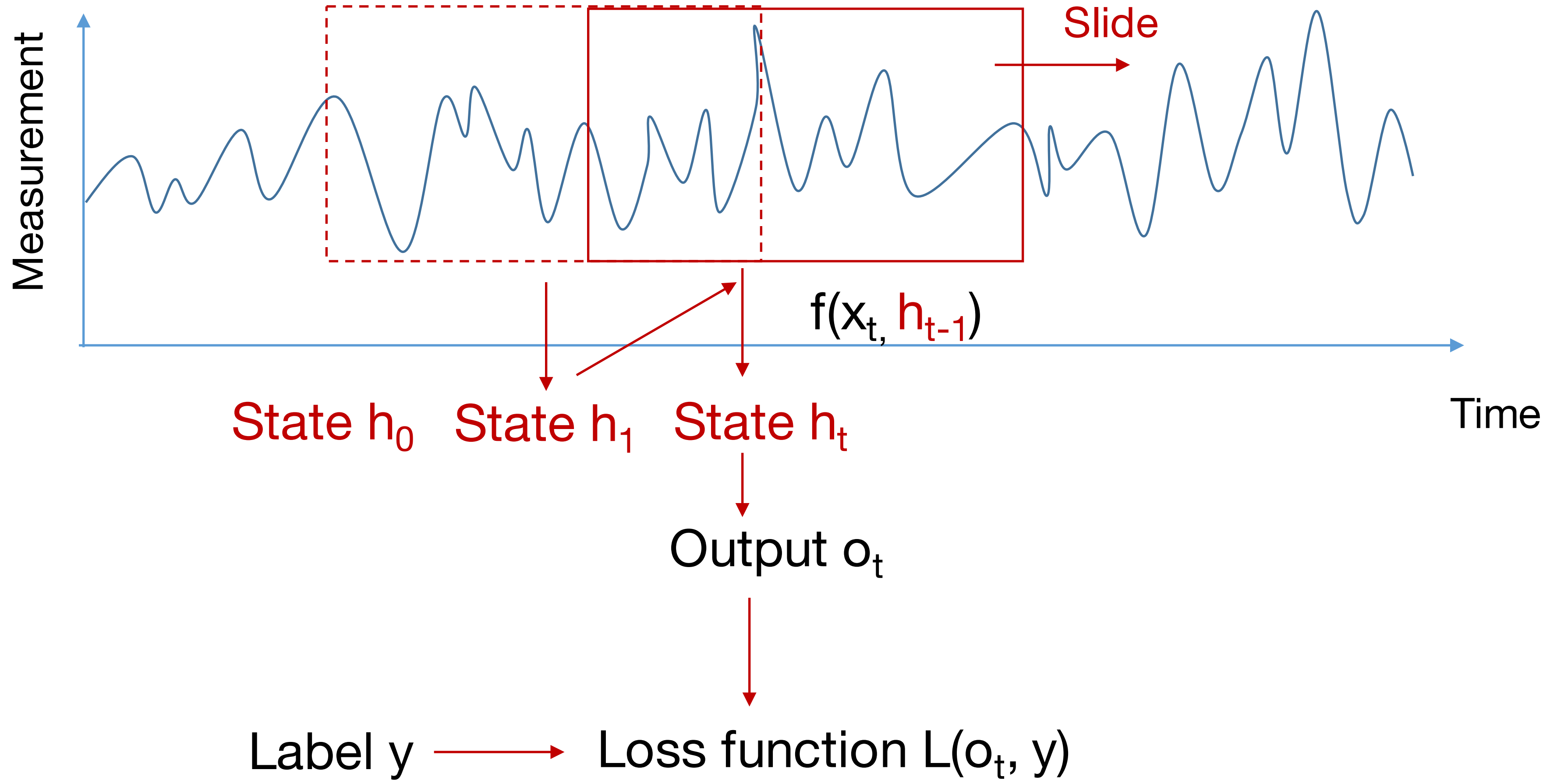
- So far, we've looked at:
  1) Decisions from fixed images (classification, detection, segmentation)

CNN's

2) Decisions from time-sequence data (captioning as classification, etc.)

RNN's

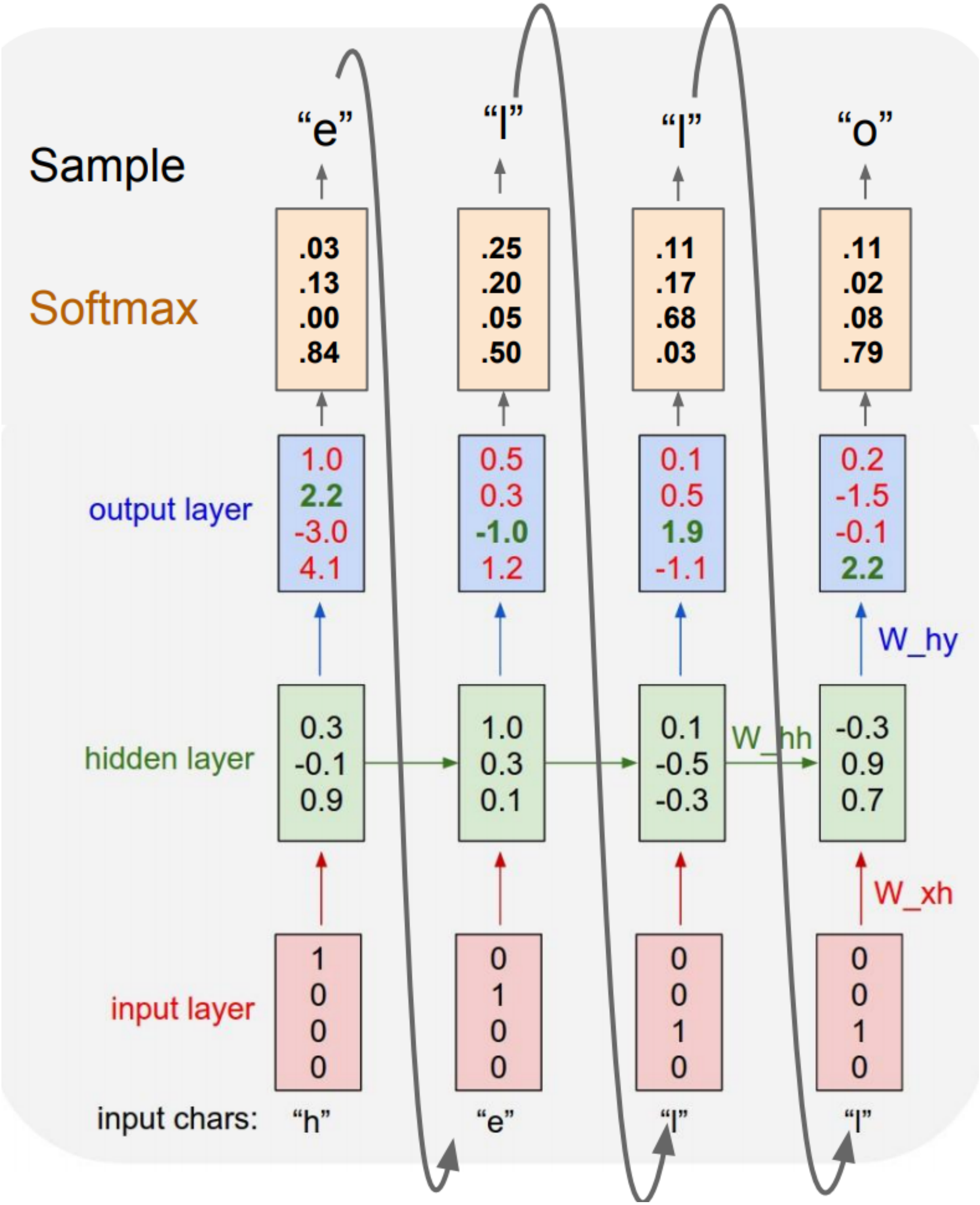
Fixed set of
temporal sequences



State h_0 State h_1 State h_t

Output o_t

Label y → Loss function $L(o_t, y)$



From Stanford CS231n Lecture 10 slides

Reinforcement learning - in a nutshell

- So far, we've looked at:
  1) Decisions from fixed images (classification, detection, segmentation)

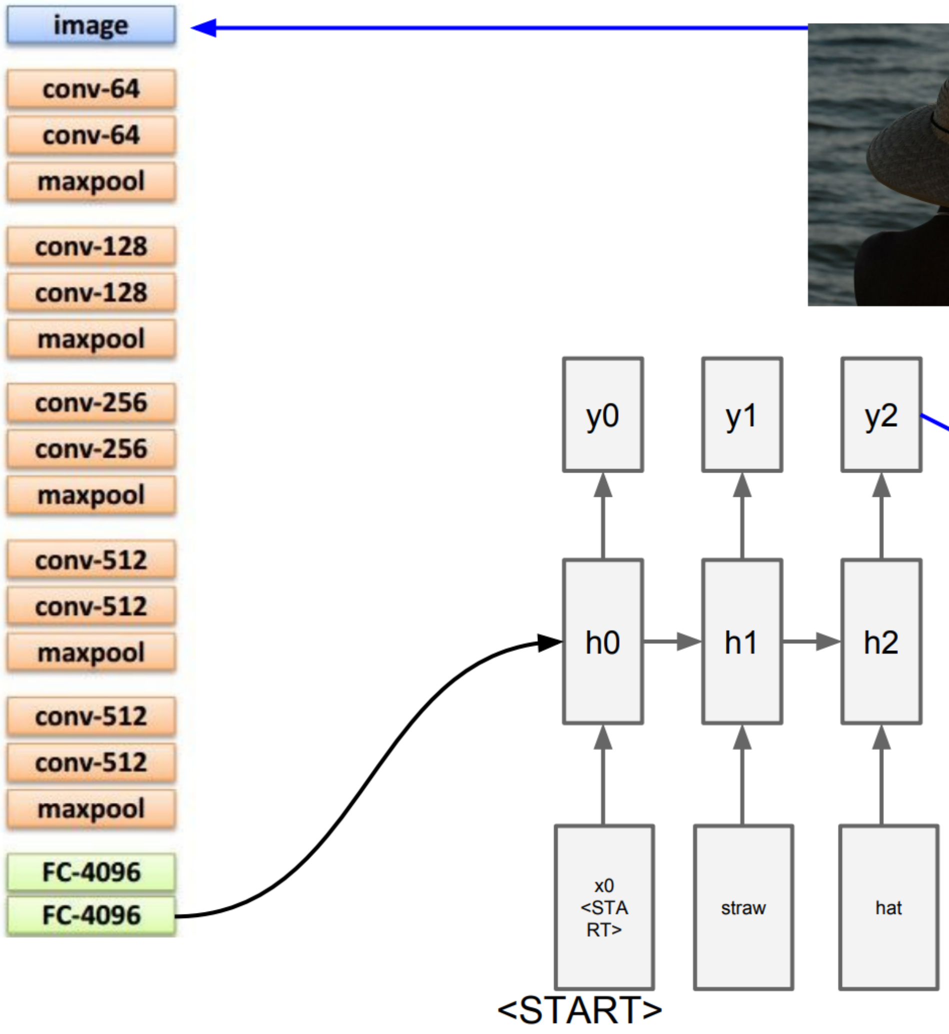
CNN's

2) Decisions from time-sequence data (captioning as classification, etc.)

Decisions from images and time-sequence data (video classification, etc.)

RNN's

Example: Image captioning



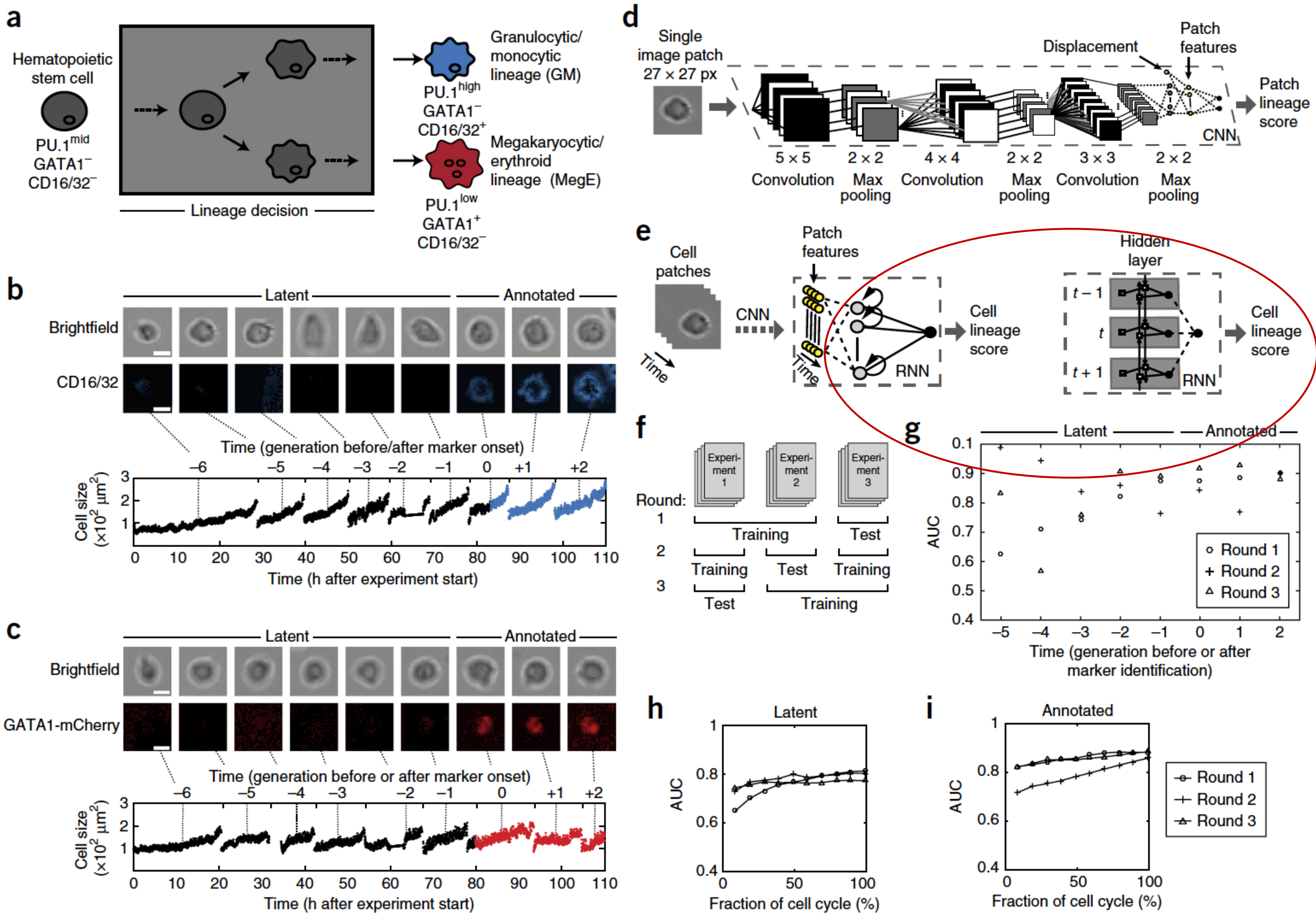
test image

sample <END> token => finish.

From Stanford CS231n Lecture 10 slides

Prospective identification of hematopoietic lineage choice by deep learning

Felix Buggenthin[1,6], Florian Buettner[1,2,6], Philipp S Hoppe[3,4], Max Endele[3], Manuel Kroiss[1,5], Michael Strasser[1], Michael Schwarzfischer[1], Dirk Loeffler[3,4], Konstantinos D Kokkaliaris[3,4], Oliver Hilsenbeck[3,4], Timm Schroeder[3,4], Fabian J Theis[1,5] & Carsten Marr[1]



Reinforcement learning - in a nutshell

- So far, we've looked at:
  1) Decisions from fixed images (classification, detection, segmentation)

CNN's

2) Decisions from time-sequence data (captioning as classification, etc.)

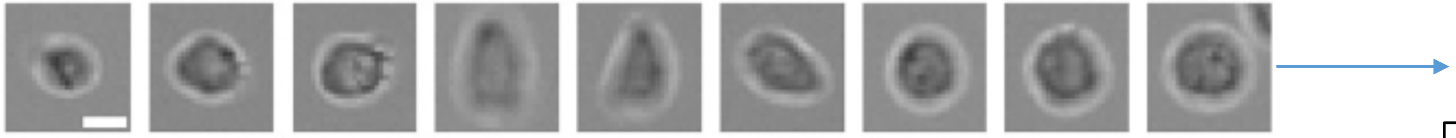
Decisions from images and time-sequence data (video classification, etc.)

RNN's

- Now, we're going to consider decisions for *dynamic data*
  - Most successful application: dynamic image data
    e.g.: video games, images of a Go game, car turning through obstacles

Reinforcement Learning

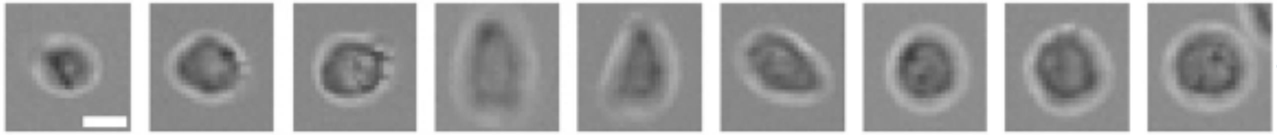
The step from fixed video to dynamic video



Outcome:
Cell type B

Goal: examine *all* data to make final decision

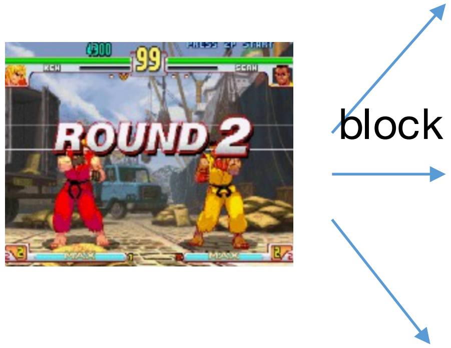
The step from fixed video to dynamic video



Outcome:

Cell type B

Goal: examine *all* data to make final decision

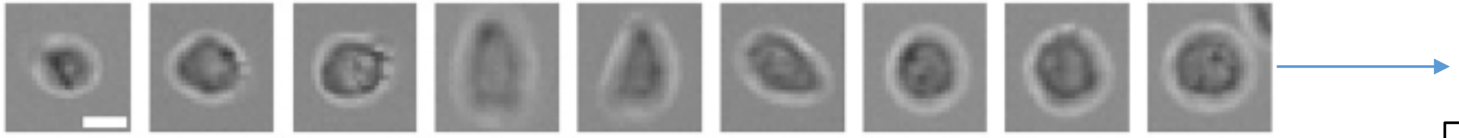


“jump”

block

forward

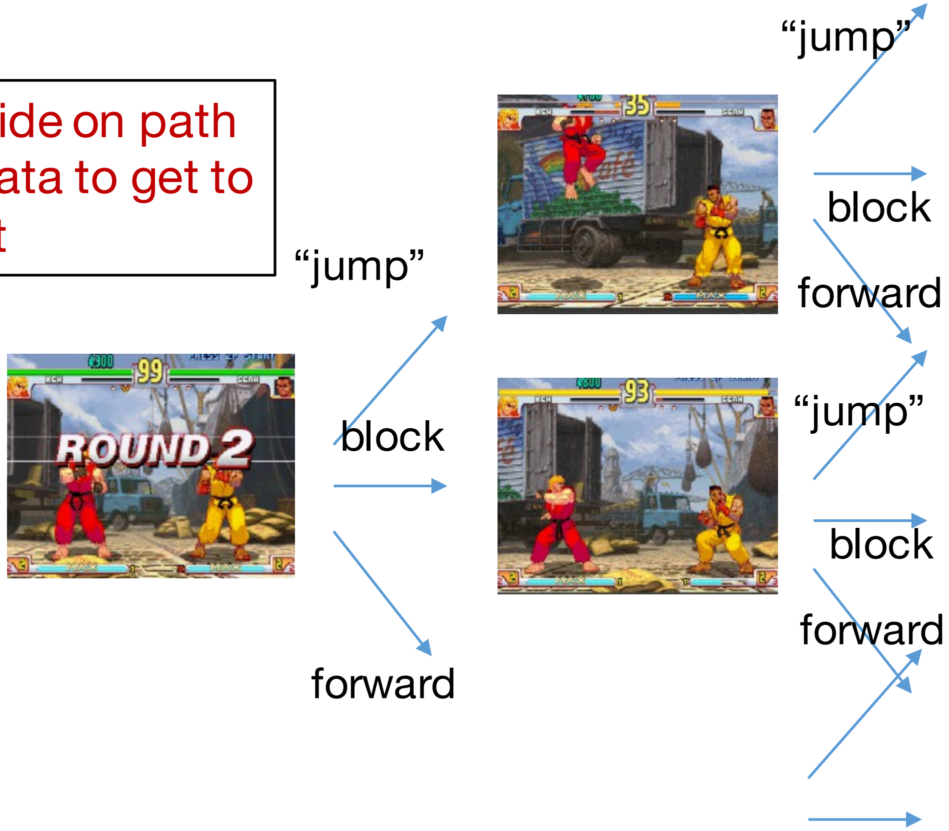
The step from fixed video to dynamic video



Outcome:
Cell type B

Goal: examine *all* data to make final decision

Goal: decide on path through data to get to final result



“jump”

block

forward

“jump”

block

forward

“jump”

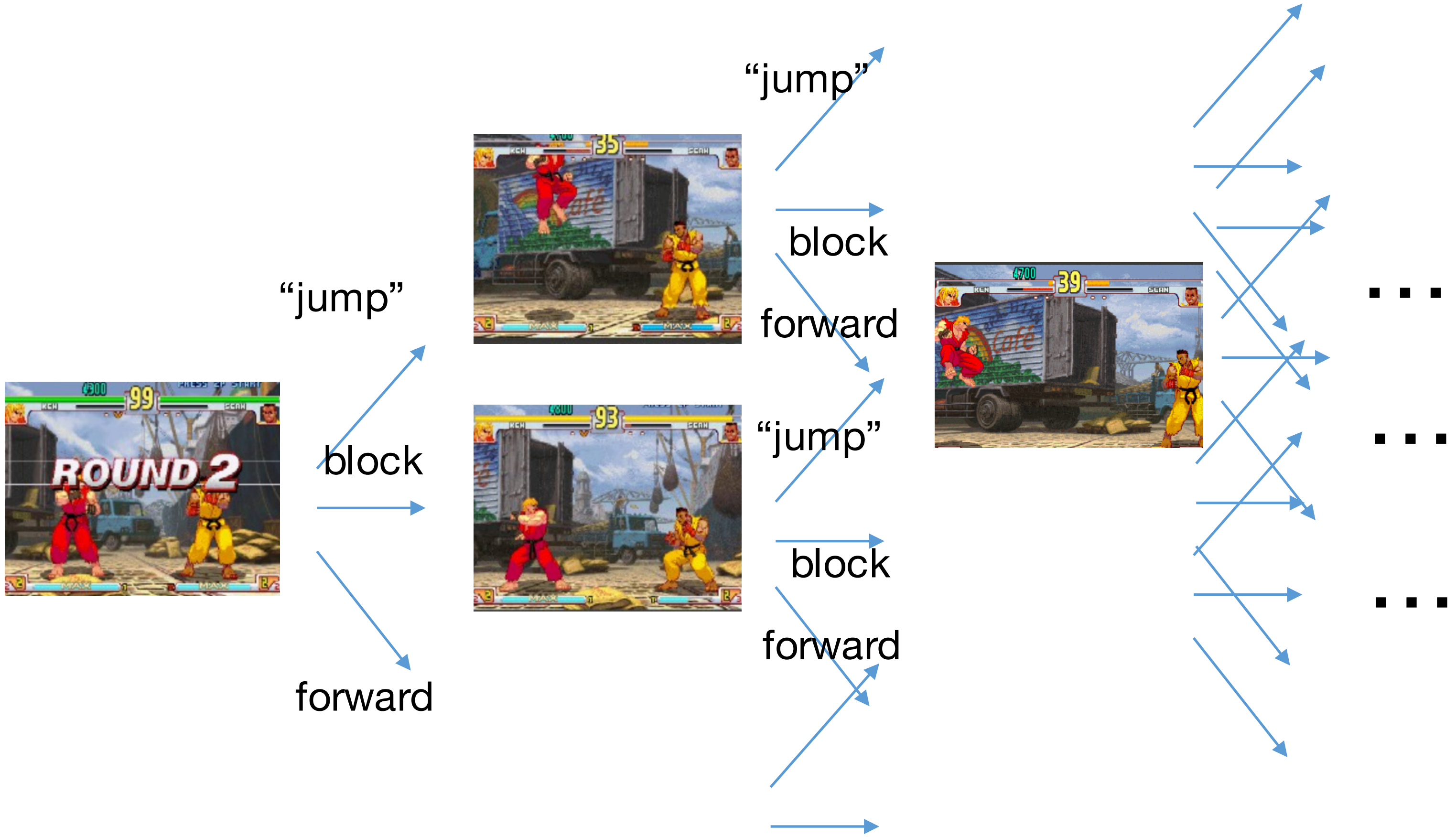
block

forward

The step from fixed video to dynamic video

Outcome:
Cell type B

Goal: examine *all* data to make final decision



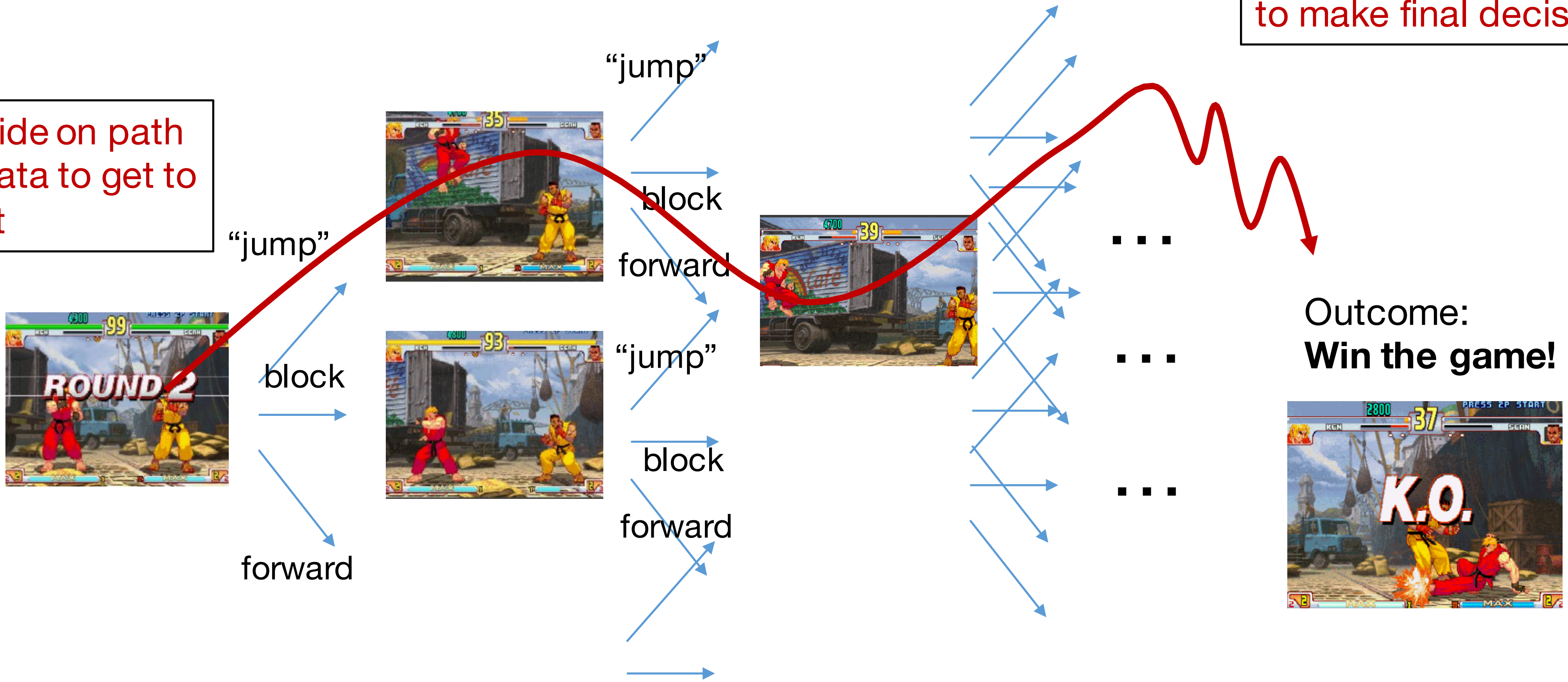
Outcome:
Win the game!

The step from fixed video to dynamic video

Outcome:
Cell type B

Goal: examine *all* data to make final decision

Goal: decide on path through data to get to final result



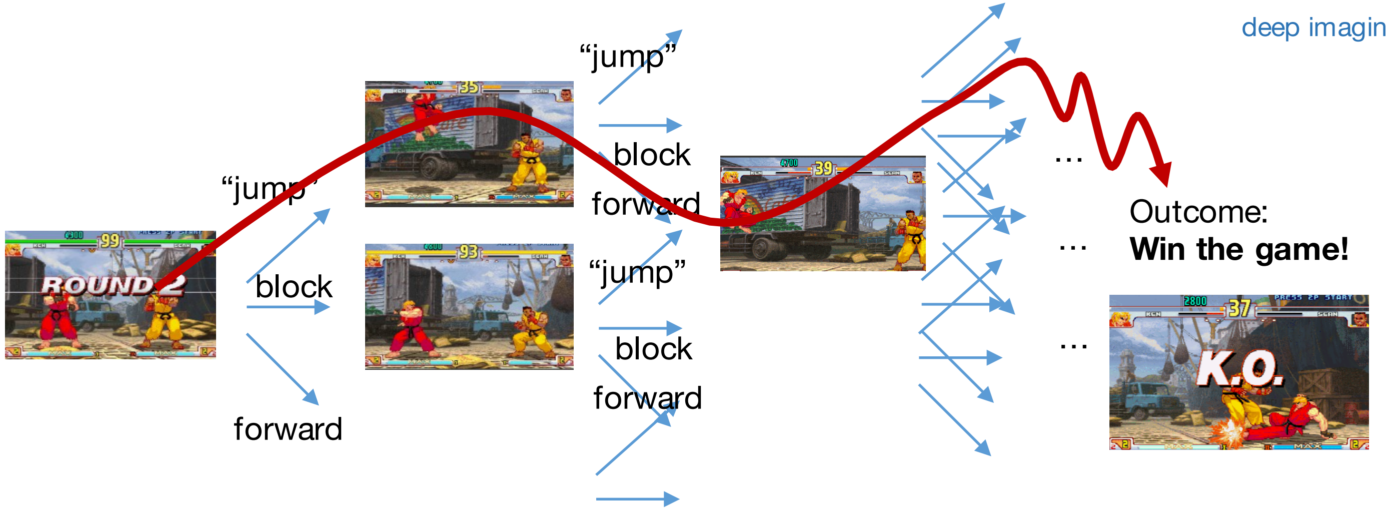
Outcome:
Win the game!

Supervised ML

Outcome:
Cell type B

- Fixed image sequence
- Goal: match to known label (large labeled dataset needed)
- Output: label
- Examines all data

Reinforcement learning



"jump"
block
forward
"jump"
block
forward
"jump"
block
forward
…
…
…

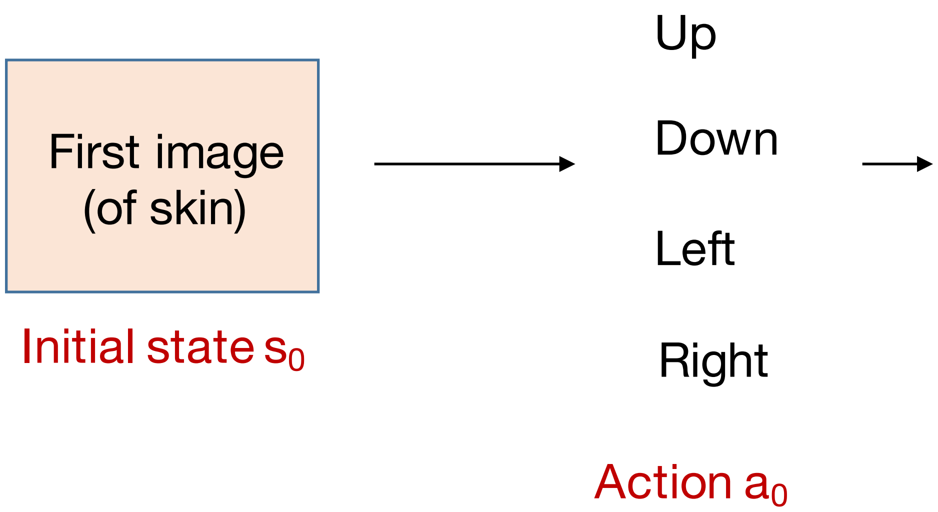
Outcome:
Win the game!

- *Dynamic/active* image sequence
- Goal: get to known desired outcome (no labels needed, really…)
- Output: sequence of actions
- Not possible to examine *all* data

Terms and notation

Example situation: Preparing for surgery using a robotically controlled instrument with an endoscope camera. You want endoscope to guide itself to tumor as quickly as possible

Movement choices:



First image (of skin)

Initial state s_0

Up

Down

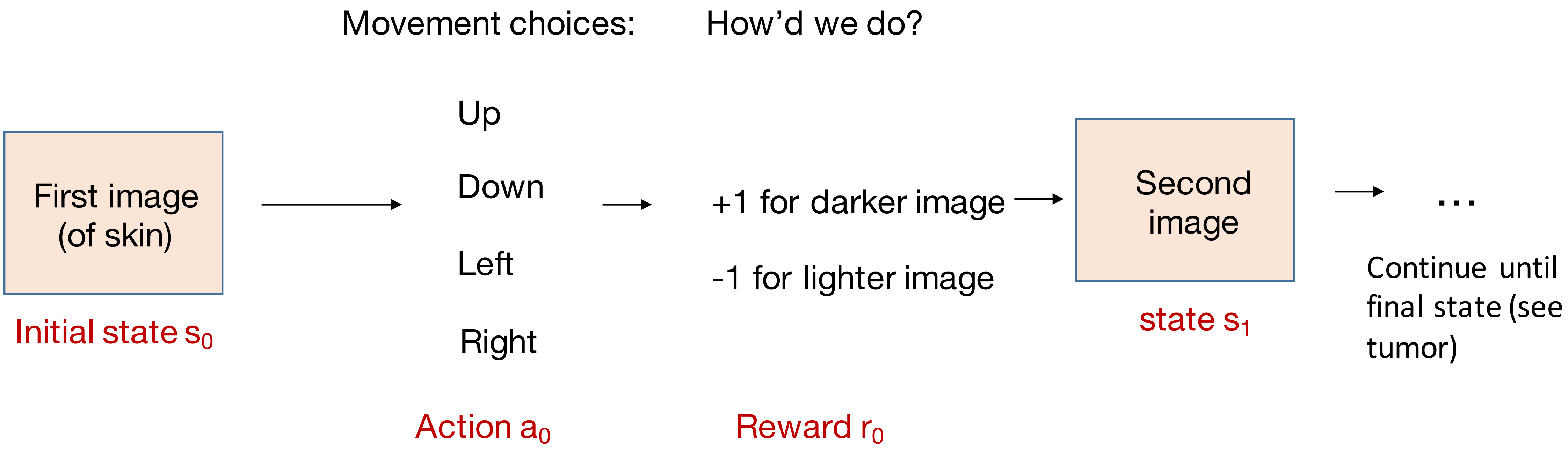
Left

Right

Action a_0

Terms and notation

Example situation: Preparing for surgery using a robotically controlled instrument with an endoscope camera. You want endoscope to guide itself to tumor as quickly as possible



Movement choices:

How'd we do?

First image (of skin)

Initial state s_0

Up

Down

Left

Right

Action a_0

+1 for darker image

-1 for lighter image

Reward r_0

Second image

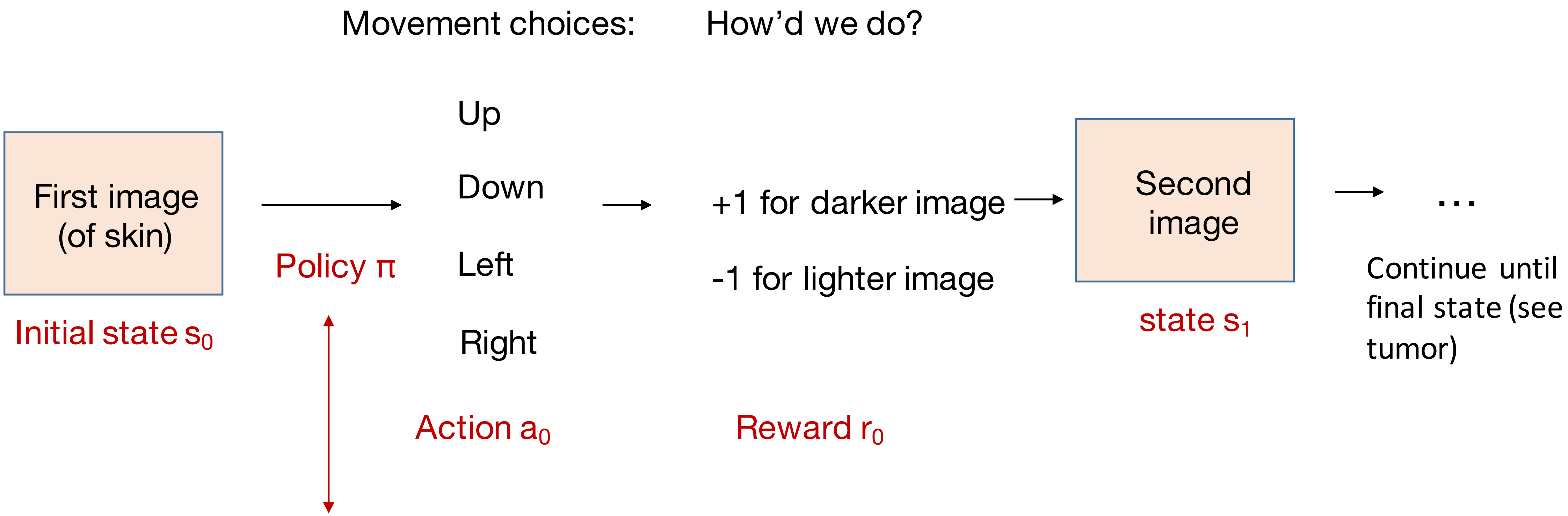
state s_1

…

Continue until final state (see tumor)

Terms and notation

Example situation: Preparing for surgery using a robotically controlled instrument with an endoscope camera. You want endoscope to guide itself to tumor as quickly as possible



Movement choices:

How’d we do?

First image (of skin)

Initial state s_0

Policy π

Up

Down

Left

Right

Action a_0

+1 for darker image

-1 for lighter image

Reward r_0

Second image

state s_1

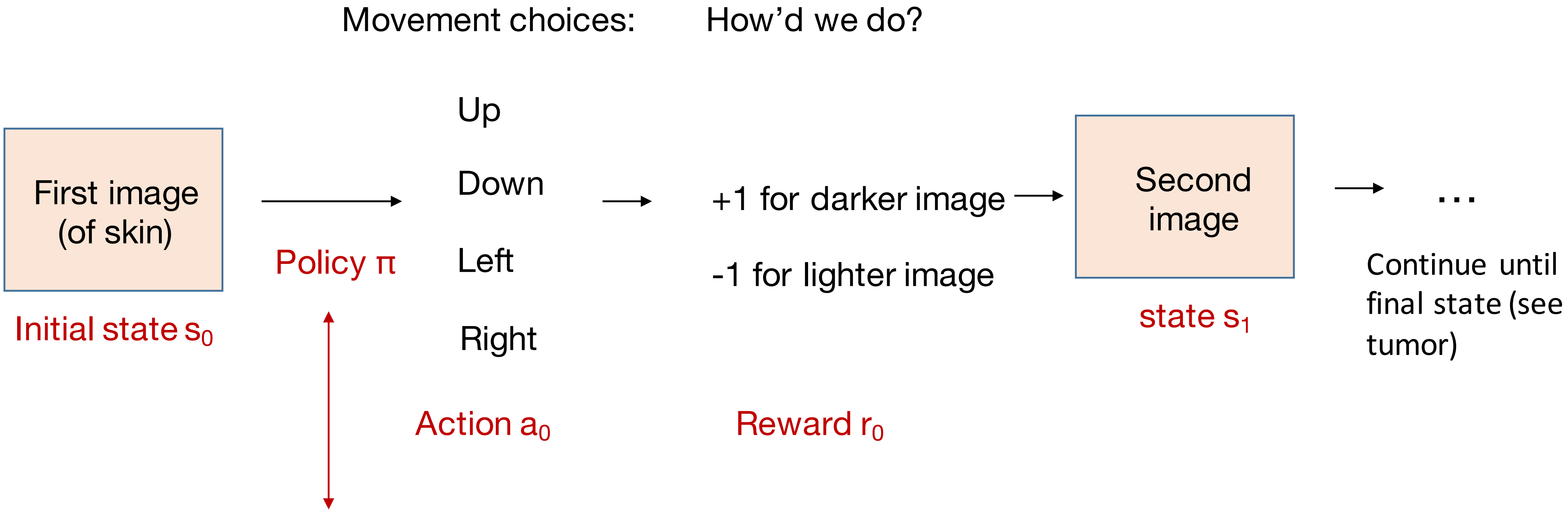
…

Continue until final state (see tumor)

Optimization Goal: Find policy π* that maximizes total “discounted” reward $\sum_{t \geq 0} \gamma^t r_t$

TL;DR

-> Use a CNN to map images to actions, optimize CNN with respect to loss function that depends on reward *in a recursive manner*



Optimization Goal: Find policy π* that maximizes total “discounted” reward $\sum_{t\geq 0}\gamma^t r_t$

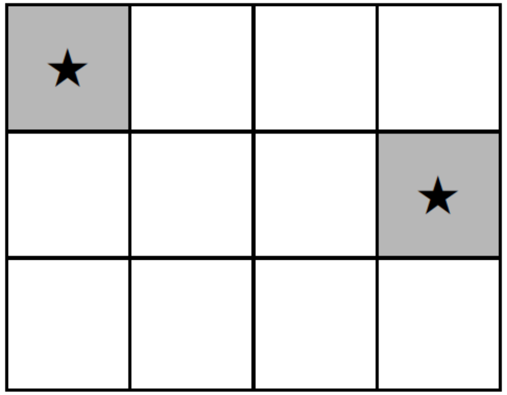
A simple MDP: Grid World

actions = {

1. right →
2. left ←
3. up ↑
4. down ↓

}

states



Set a negative "reward"
for each transition
(e.g. $r = -1$)

Objective: reach one of terminal states (greyed out) in least number of actions

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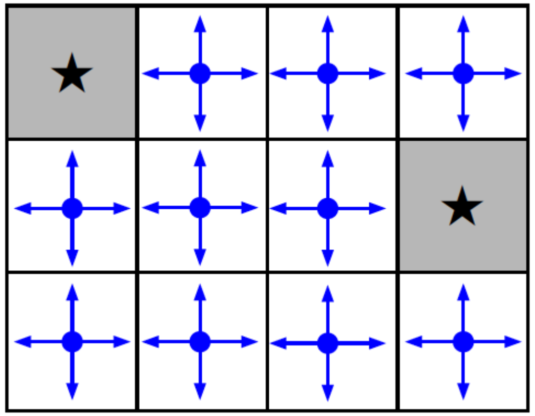
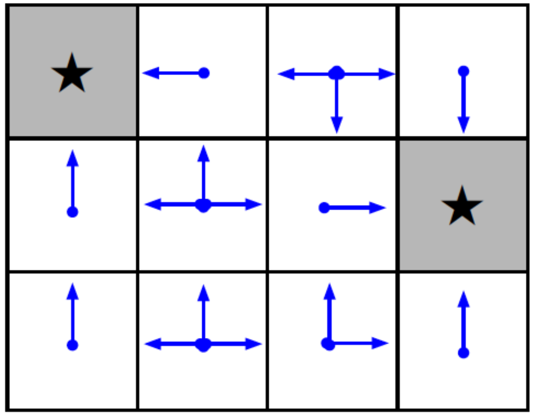
A simple MDP: Grid World



Random Policy



Optimal Policy

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Let’s jump into the math….

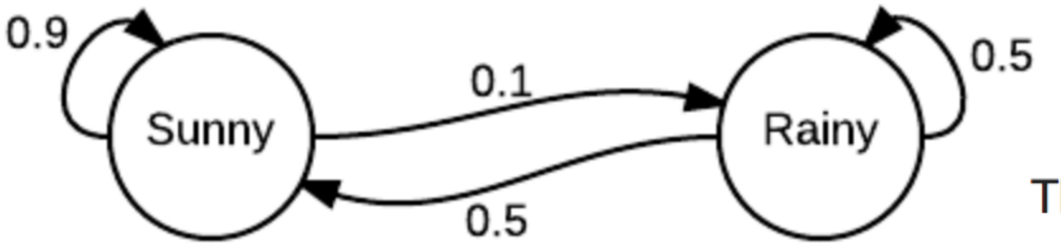
Definition of a Markov process:

$$\Pr(X_{n+1}=x\mid X_1=x_1,X_2=x_2,\ldots,X_n=x_n)=\Pr(X_{n+1}=x\mid X_n=x_n)$$

Let’s jump into the math….

Definition of a Markov process:

$$\Pr(X_{n+1} = x \mid X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = \Pr(X_{n+1} = x \mid X_n = x_n)$$



2 states: Sunny and Rainy

$$\mathbf{x}^{(0)} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The weather on day 2 can be predicted by:

$$\mathbf{x}^{(1)} = \mathbf{x}^{(0)} P = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 \end{bmatrix}$$

Thus, there is a 90% chance that day 2 will also be sunny.

https://en.wikipedia.org/wiki/Markov_chain

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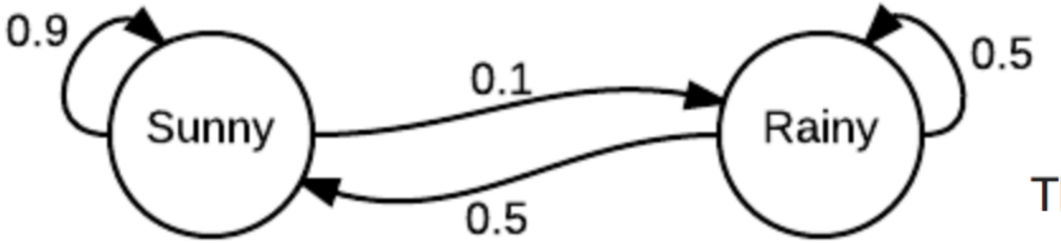

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Thus, there is a 90% chance that day 2 will also be sunny.

2 states: Sunny and Rainy

Transition matrix – try to learn this from state to state

https://en.wikipedia.org/wiki/Markov_chain

Assume transition between states follows Markov process

$P(s_{t+1}|s_t,\ s_{t-1}\ldots s_0) = P(s_{t+1}\ |\ s_t)$

Andrey Markov

Markov chain

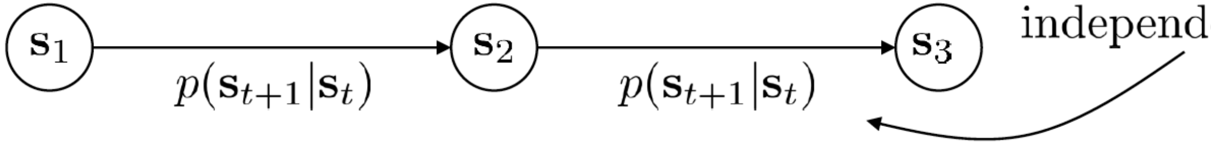
$\mathcal{M} = \{\mathcal{S}, \mathcal{T}\}$

$\mathcal{S}$ – state space — states $s \in \mathcal{S}$ (discrete or continuous)

$\mathcal{T}$ – transition operator — $p(s_{t+1}|s_t)$

why "operator"? — let $\mu_{t,i} = p(s_t = i)$ — $\vec{\mu}_t$ is a vector of probabilities

let $\mathcal{T}_{i,j} = p(s_{t+1} = i|s_t = j)$ — then $\vec{\mu}_{t+1} = \mathcal{T}\vec{\mu}_t$


$\mathbf{s}_1$
$\mathbf{s}_2$
$\mathbf{s}_3$
$p(\mathbf{s}_{t+1}|\mathbf{s}_t)$
$p(\mathbf{s}_{t+1}|\mathbf{s}_t)$

Markov property
independent of $\mathbf{s}_{t-1}$

Add in dependence on action: Markov *decision* process

$P(s_{t+1}|s_t) \Rightarrow P(s_{t+1} \mid s_t, a_t) = P(s_{t+1} \mid s_t, a_t, \ldots s_0, a_0)$

Markov decision process $\quad \mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$

$\mathcal{S}$ – state space $\quad$ states $s \in \mathcal{S}$ (discrete or continuous)

$\mathcal{A}$ – action space $\quad$ actions $a \in \mathcal{A}$ (discrete or continuous)

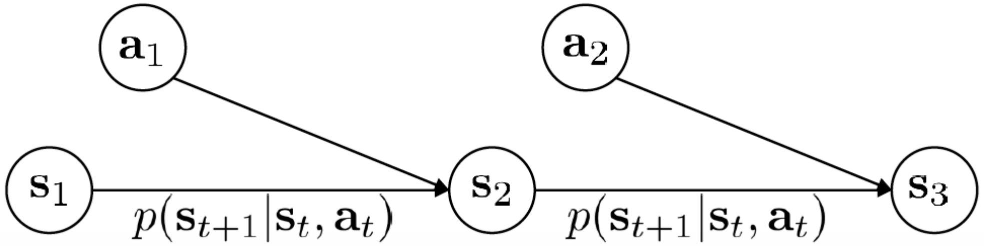
$\mathcal{T}$ – transition operator (now a tensor!)

let $\mu_{t,j} = p(s_t = j)$

let $\xi_{t,k} = p(a_t = k)$

let $\mathcal{T}_{i,j,k} = p(s_{t+1} = i | s_t = j, a_t = k)$

$$\mu_{t,i} = \sum_{j,k} \mathcal{T}_{i,j,k} \mu_{t,j} \xi_{t,k}$$


$\mathbf{a}_1$
$\mathbf{a}_2$
$\mathbf{s}_1$
$\mathbf{s}_2$
$\mathbf{s}_3$
$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$
$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$

Andrey Markov

Richard Bellman

Add in dependence on action: Markov *decision* process

P(s_{t+1}|s_t, a_t) can include reward r(s_t, a_t)

Markov decision process $\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$

$\mathcal{S}$ – state space states $s \in \mathcal{S}$ (discrete or continuous)

$\mathcal{A}$ – action space actions $a \in \mathcal{A}$ (discrete or continuous)

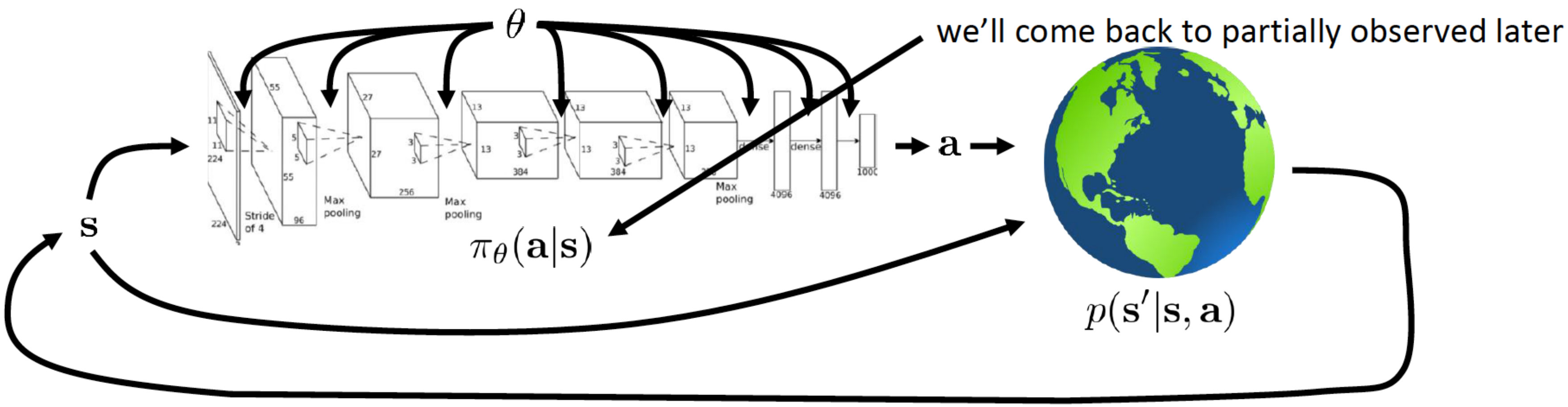
$\mathcal{T}$ – transition operator (now a tensor!)

r – reward function $r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$

$r(s_t, a_t)$ – reward

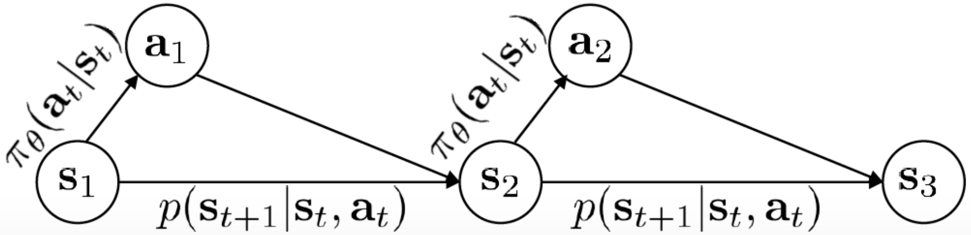
Berkeley CS 294: Deep Reinforcement Learning

The goal of reinforcement learning



we’ll come back to partially observed later

$$\underbrace{p_\theta(\mathbf{s}_1,\mathbf{a}_1,\ldots,\mathbf{s}_T,\mathbf{a}_T)}_{\pi_\theta(\tau)} = p(\mathbf{s}_1)\prod_{t=1}^{T}\underbrace{\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)}_{\text{Markov chain on }(\mathbf{s},\mathbf{a})}$$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$\mathbf{a}_2$
$\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
$\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
$\mathbf{s}_1$
$\mathbf{s}_2$
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$p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)$
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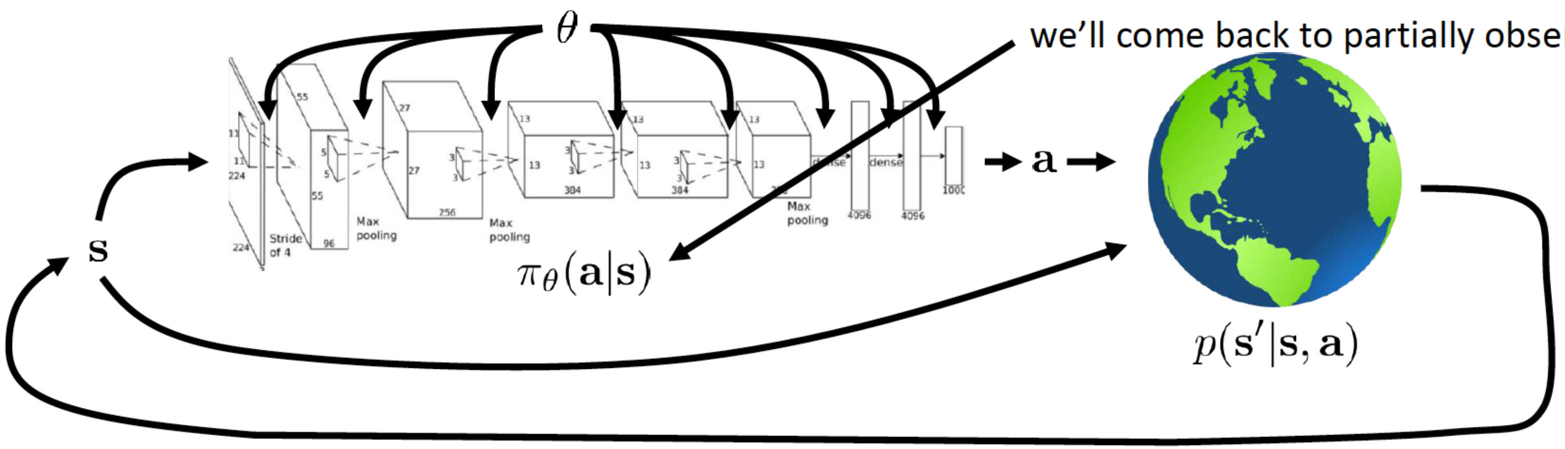
The goal of reinforcement learning



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$$\underbrace{p_\theta(\mathbf{s}_1,\mathbf{a}_1,\ldots,\mathbf{s}_T,\mathbf{a}_T)}_{\pi_\theta(\tau)} = p(\mathbf{s}_1)\prod_{t=1}^{T}\underbrace{\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)}_{\text{Markov chain on } (\mathbf{s},\mathbf{a})}$$

$$\theta^\star = \arg\max_\theta \sum_{t=1}^{T} E_{(\mathbf{s}_t,\mathbf{a}_t)\sim p_\theta(\mathbf{s}_t,\mathbf{a}_t)}[r(\mathbf{s}_t,\mathbf{a}_t)]$$

The optimal policy **π***

We want to find optimal policy **π*** that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability…)?
Maximize the **expected sum of rewards!**

Formally: $\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \underset{\uparrow}{\gamma^t} r_t | \pi\right]$ with $s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)$

Discount factor: accumulate the rewards "acquired" up to current state, but they become less important the longer they were in the past

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The optimal policy **π***

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Maximize the **expected sum of rewards!**

Formally: $\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi\right]$ with $s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)$

The **Q-value function** at state s and action a, is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^\pi(s,a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Don’t have access to all policies, so use Q in practice

Bellman equation

The optimal Q-value function Q* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s,a) = \max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Q* satisfies the following **Bellman equation**:

$$Q^*(s,a) = \mathbb{E}_{s'\sim\mathcal{E}}\left[r + \gamma \max_{a'} Q^*(s',a') | s, a\right]$$

Intuition: if the optimal state-action values for the next time-step Q*(s',a') are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s',a')$

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Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning**!

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Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q^*(s',a')|s,a\right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)}\left[(y_i - Q(s,a;\theta_i))^2\right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1})|s,a\right]$

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Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s',a') | s,a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)} \left[(y_i - Q(s,a;\theta_i))^2 \right]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) | s,a \right]$

Iteratively try to make the Q-value close to the target value (y_i) it should have, if Q-function corresponds to optimal Q* (and optimal policy **π***)

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) - Q(s,a;\theta_i)) \nabla_{\theta_i} Q(s,a;\theta_i) \right]$$

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[Mnih et al. NIPS Workshop 2013; Nature 2015]

Case Study: Playing Atari Games

Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

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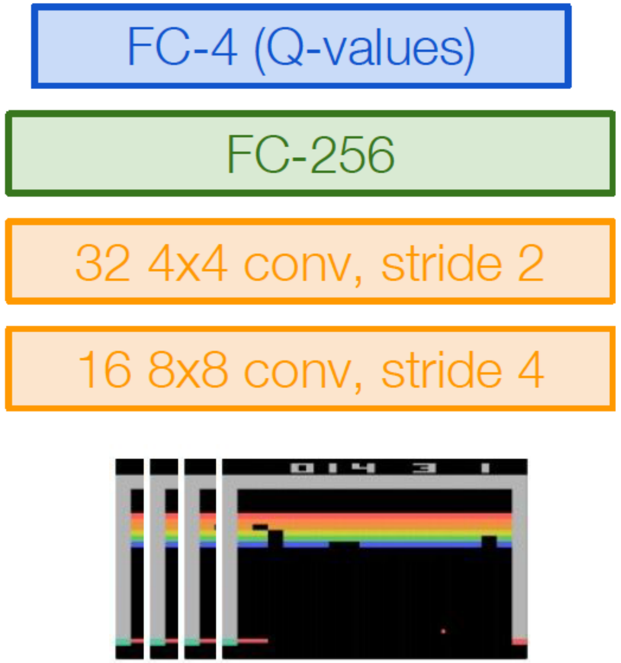
Q-network Architecture

$Q(s, a; \theta)$: neural network with weights θ



FC-4 (Q-values)

FC-256

32 4x4 conv, stride 2

16 8x8 conv, stride 4

Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t,a_4)$

Input: state s_t

Current state s_t: 84x84x4 stack of last 4 frames

(after RGB->grayscale conversion, downsampling, and cropping)

From Stanford CS231n Lecture 17

https://www.youtube.com/watch?v=V1eYniJ0Rnk

Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

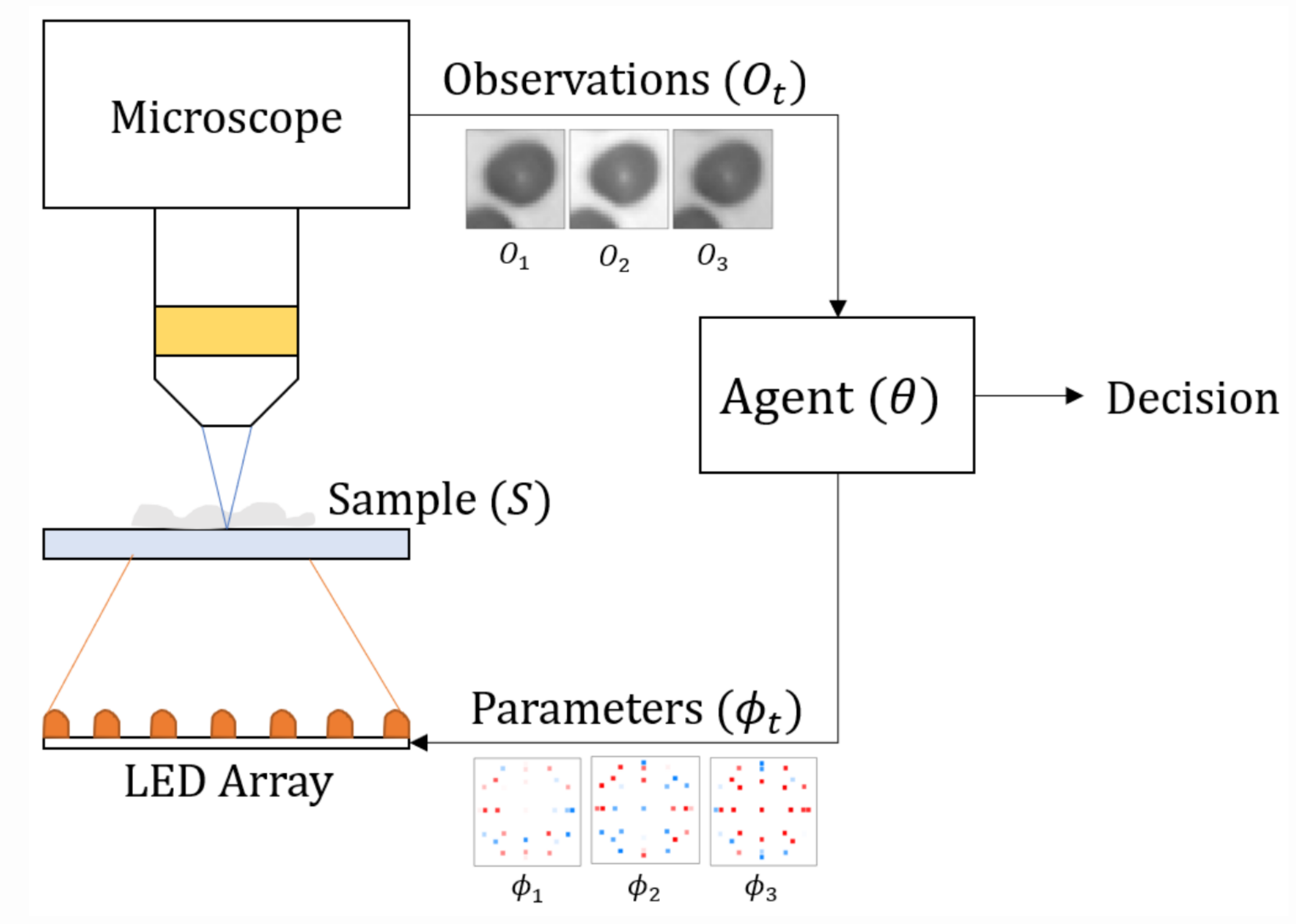
Address these problems using **experience replay**

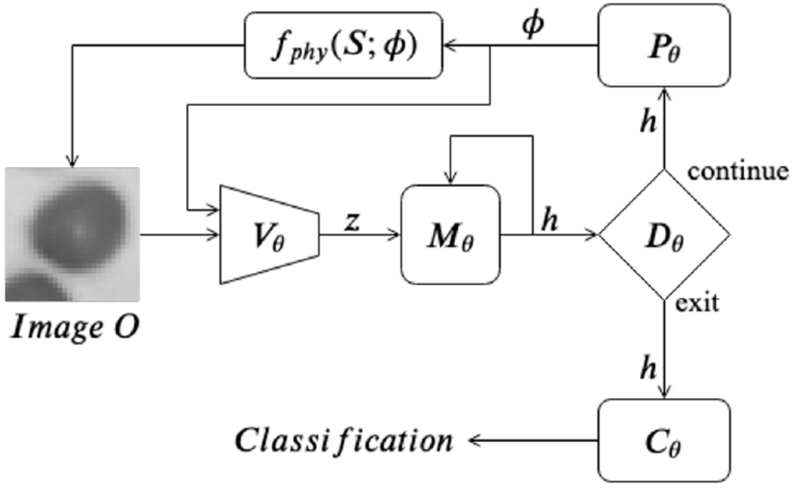
- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

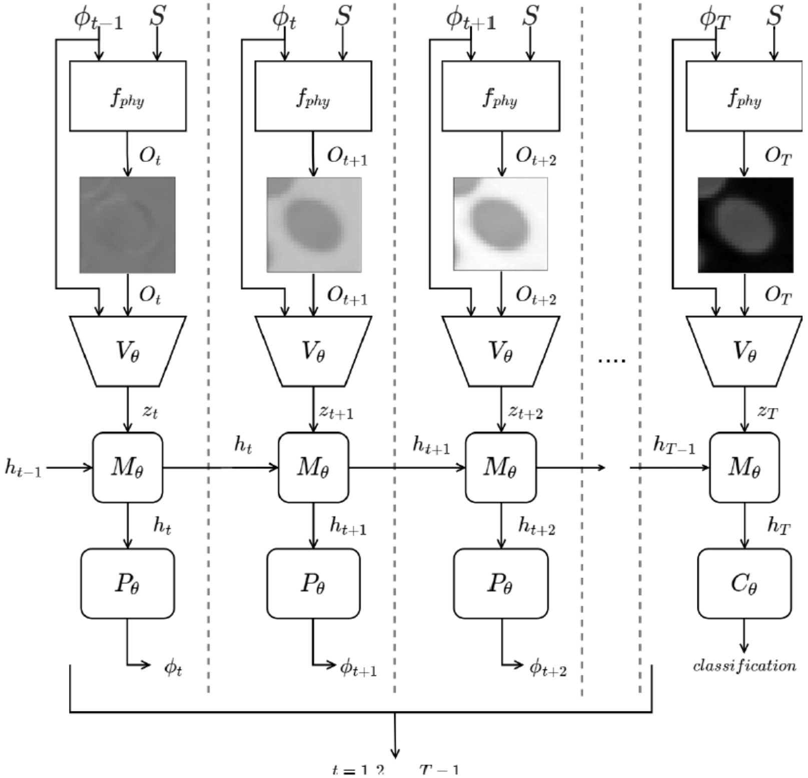
Each transition can also contribute to multiple weight updates => greater data efficiency

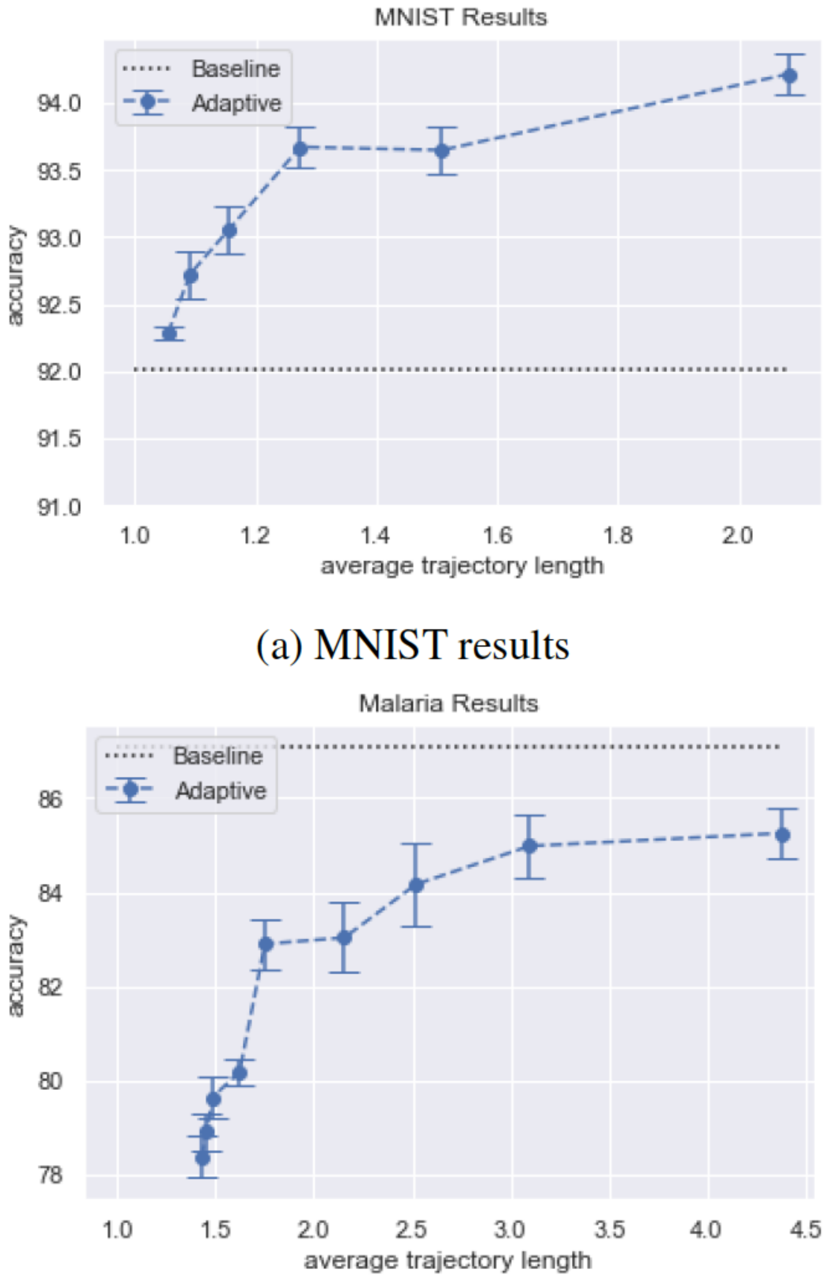
From Stanford CS231n Lecture 17

How can this be applied to optimized imaging?





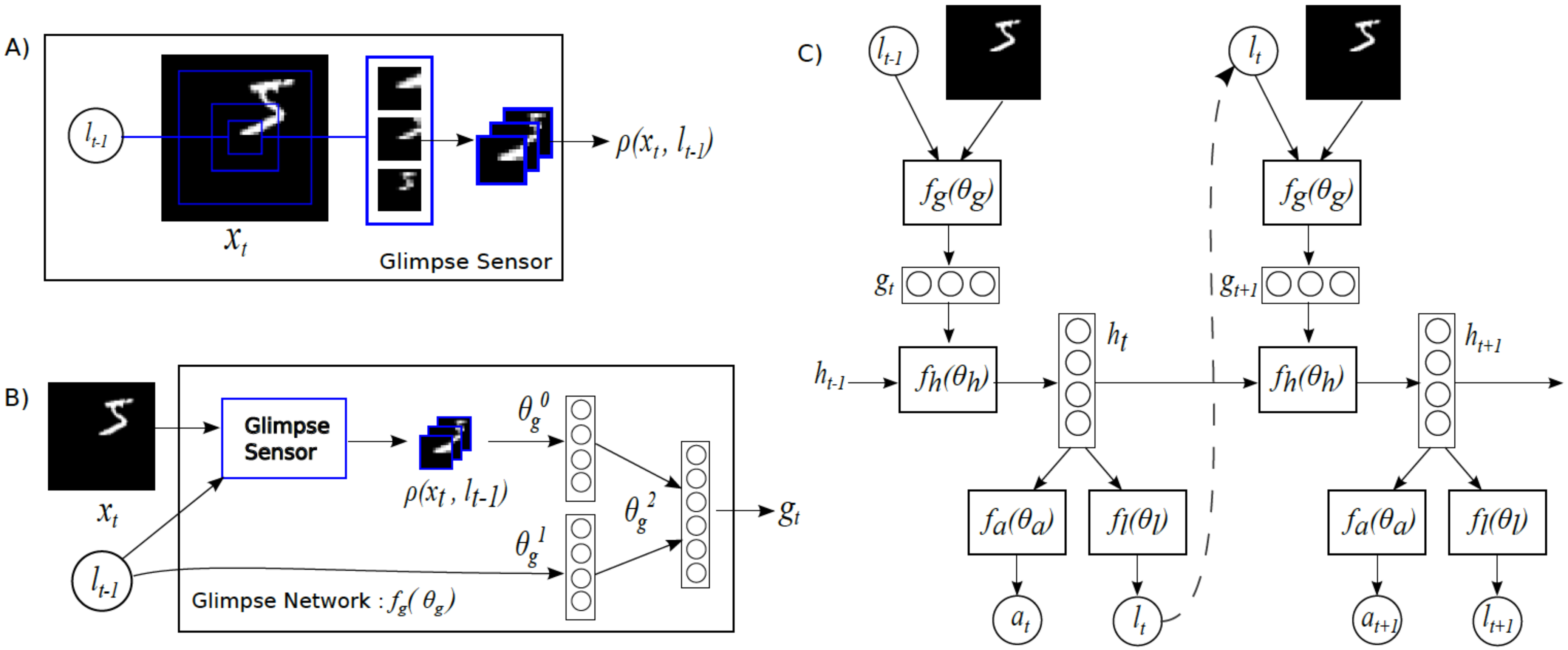




(a) MNIST results

(b) Malaria results

How can this be applied to optimized imaging?


A)
l_{t-1}
x_t
$\rho(x_t, l_{t-1})$
Glimpse Sensor
B)
x_t
l_{t-1}
Glimpse Sensor
$\rho(x_t, l_{t-1})$
θ_g^0
θ_g^1
θ_g^2
g_t
Glimpse Network : $f_g(\theta_g)$
C)
l_{t-1}
$f_g(\theta_g)$
g_t
h_{t-1}
$f_h(\theta_h)$
h_t
$f_a(\theta_a)$
$f_l(\theta_l)$
a_t
l_t
l_t
$f_g(\theta_g)$
g_{t+1}
$f_h(\theta_h)$
h_{t+1}
$f_a(\theta_a)$
$f_l(\theta_l)$
a_{t+1}
l_{t+1}