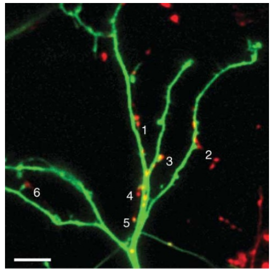
Lecture 17: Wave optics and Fourier optics

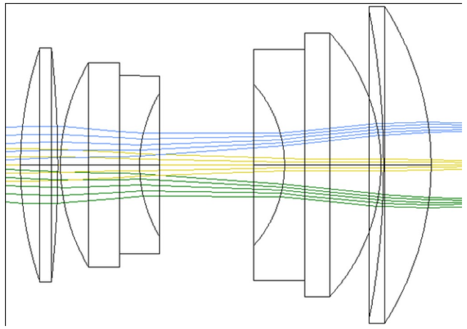
Machine Learning and Imaging

BME 548L
Roarke Horstmeyer

What is light and how can we model it?

- Interpretation #1: Radiation (*Incoherent*)
- Model: Rays

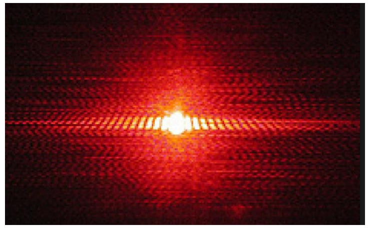



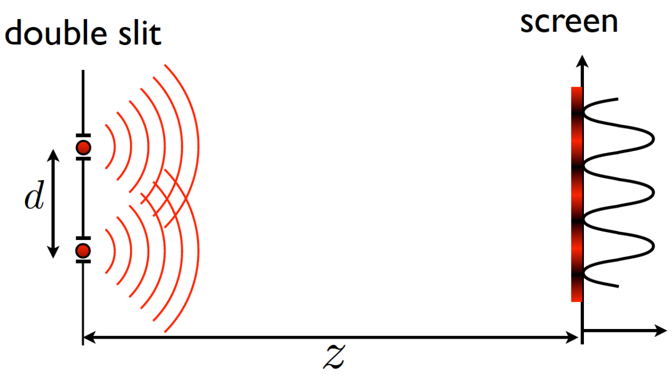


- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

- Interpretation #2: Electromagnetic wave (*Coherent*)
- Model: Waves





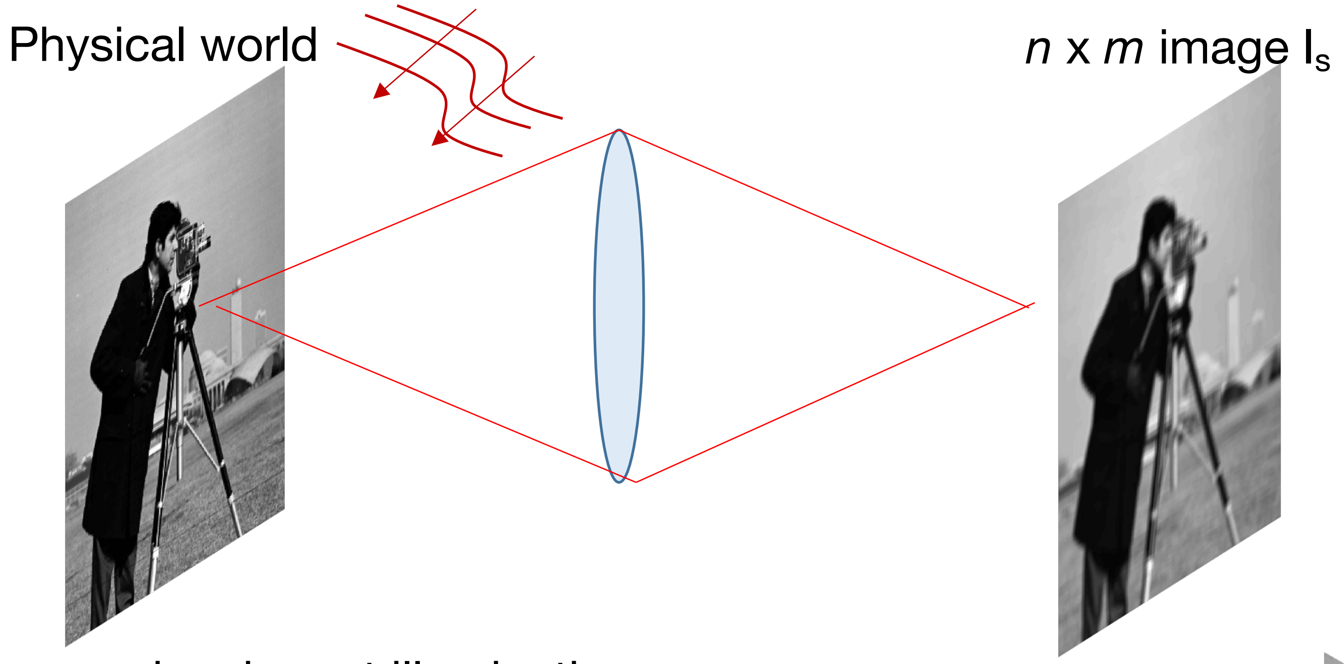
- Complex field
- Models Interference

$$E_{tot} = E_1 + E_2$$

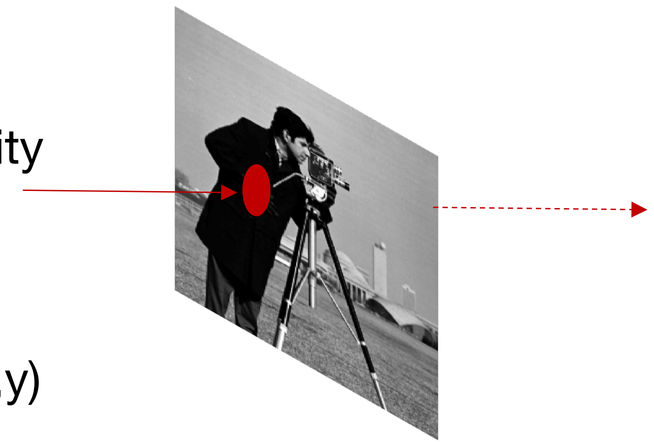
- Interpretation #3: Particle
- Model: Photons

Simple mathematical model of incoherent image formation

deep imaging

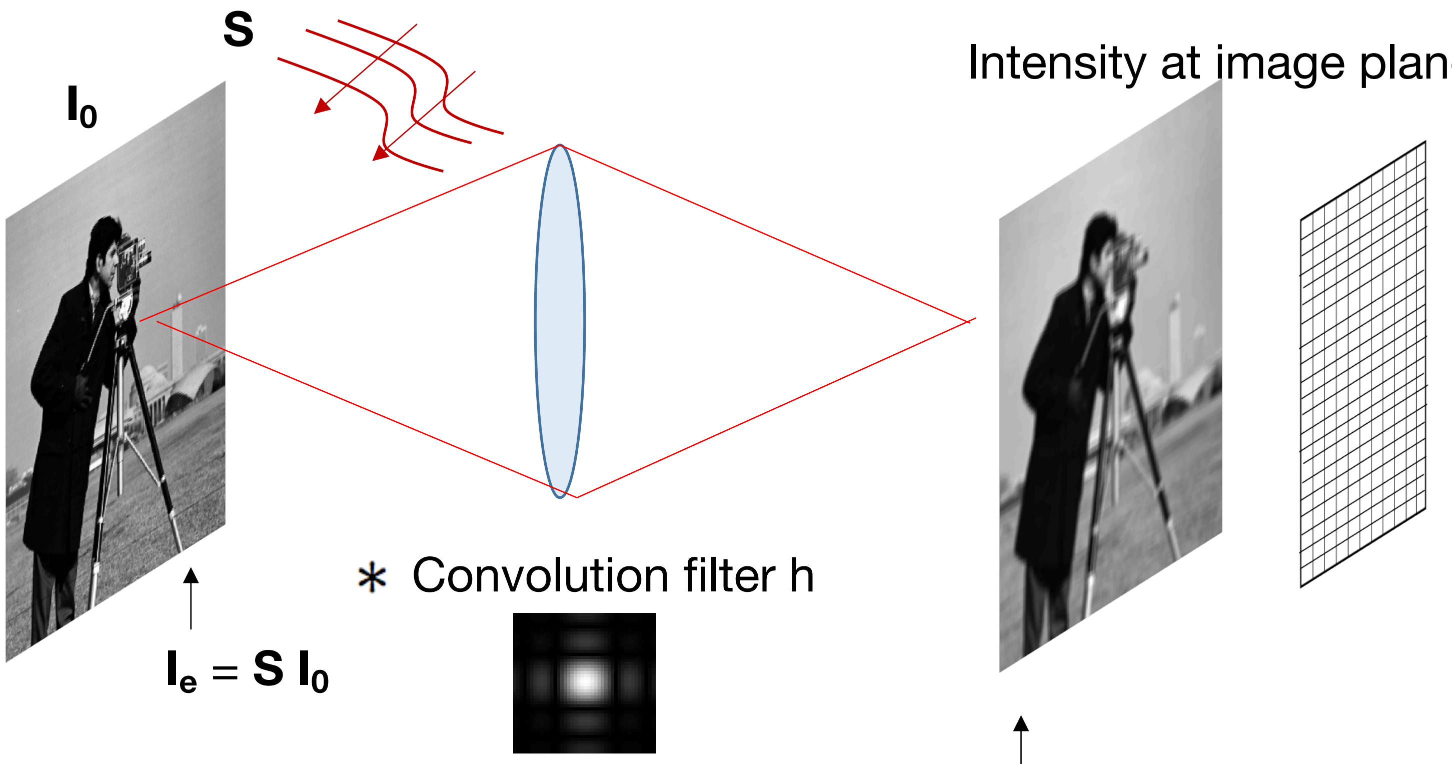


- Assume incoherent illumination
- Assume thin 2D object
- Object is real, non-negative map of absorption/reflectivity



Object absorption: $\mathbf{I_0}(x,y)$
Illumination pattern: $\mathbf{s}(x,y)$
Light exiting object surface: $\mathbf{I_e}(x,y) = \mathbf{I_0}(x,y) \circ \mathbf{s}(x,y)$

Simple mathematical model of incoherent image formation



$$\boxed{\mathbf{I_p} = \mathbf{H}\ \mathbf{I_e} = \mathbf{H}\ \mathbf{S}\ \mathbf{I_0}}$$

Summary: simple physical layers for incoherent imaging

- Deal with sample/image intensities I, real and non-negative

- Effect of illumination is element-wise multiplication $\boxed{I_e(x,y) = \mathbf{S}\, I_0(x,y)}$

λ

- Imaging systems blur the object via point-spread function matrix **H**

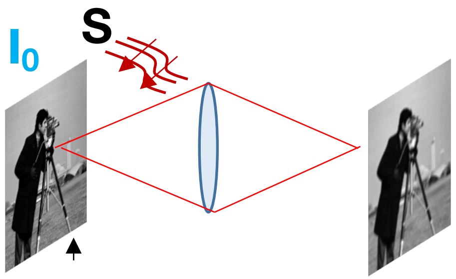
$$\boxed{I_b(x,y) = \mathbf{H}\, I_0(x,y)}$$

- Discrete pixels down-sample the object via $\boxed{I_d(x,y) = \mathbf{D}\, I_0(x,y)}$

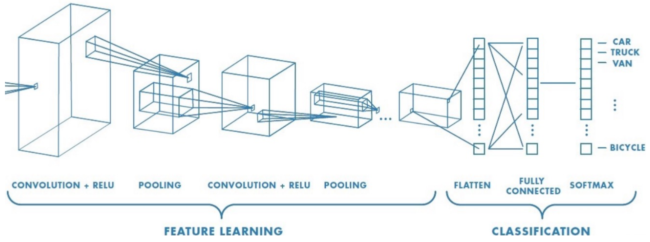
- Add noise into measurement $\boxed{I_N(x,y) = \mathbf{D}\, I_0(x,y) + \mathbf{N}}$

- Different colors add linearly $\boxed{I_s(x, y) = \Sigma\, I_0(x, y, \lambda)}$

Example #1: Optimized illumination pattern (one color)



$[I_e(x, y), y]$



Training data:
$[I_0(x, y), y]$
I_0:100 x 100
Label y: 1x2

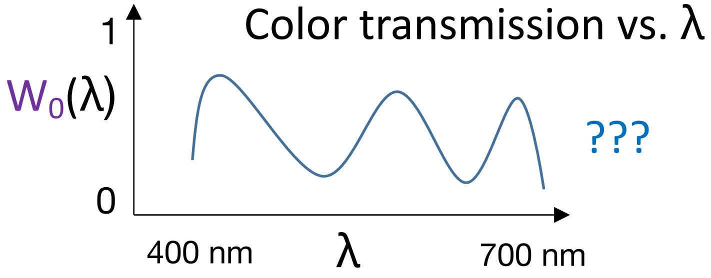
$\boxed{I_e(x,y) = \mathbf{S}\, I_0(x,y)}$

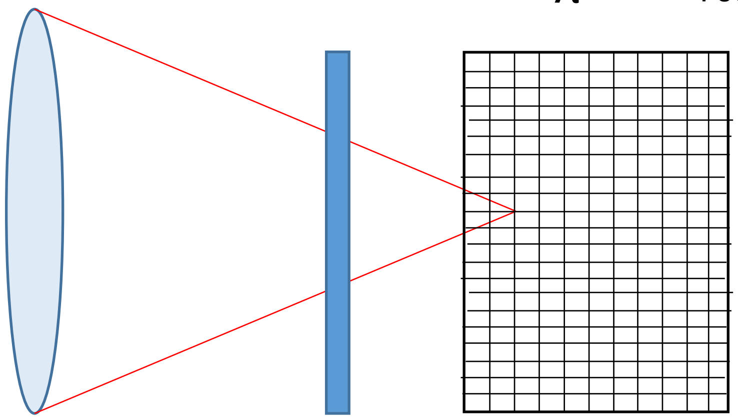
Physical Layer

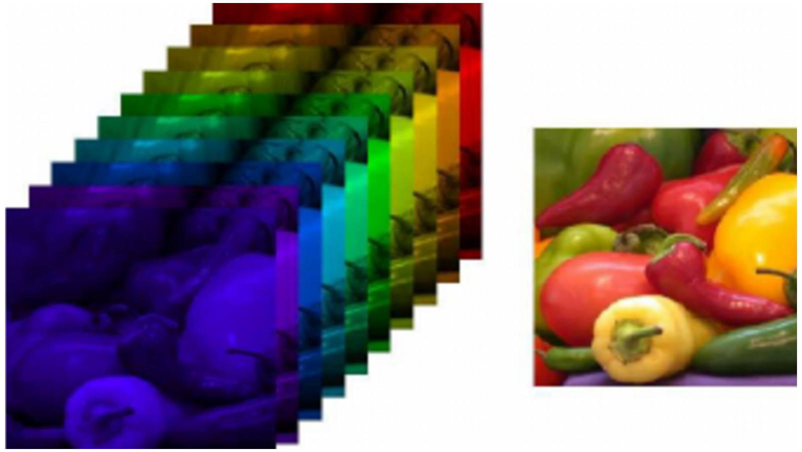
Task = $\mathbf{W_n} \ldots \mathrm{ReLU}[\mathbf{W_1}\, \mathrm{ReLU}[\mathbf{W_0}\, I_e] \ldots]$

Example #2: Optimized color filter for a grayscale camera

Design optimal color filter for classification:







Monochromatic camera sensor

Training data:
[$I_0(x, y, \lambda)$, y]
I_0:100 x 100 pix. x 30
Label y: 1x3 - pepper, broccoli, green beans

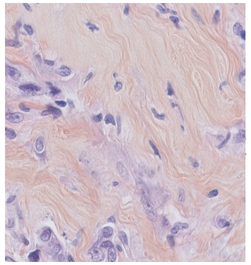
$$I_s(x, y) = \sum_{\lambda} W_0(\lambda)\, I_0(x, y, \lambda)$$

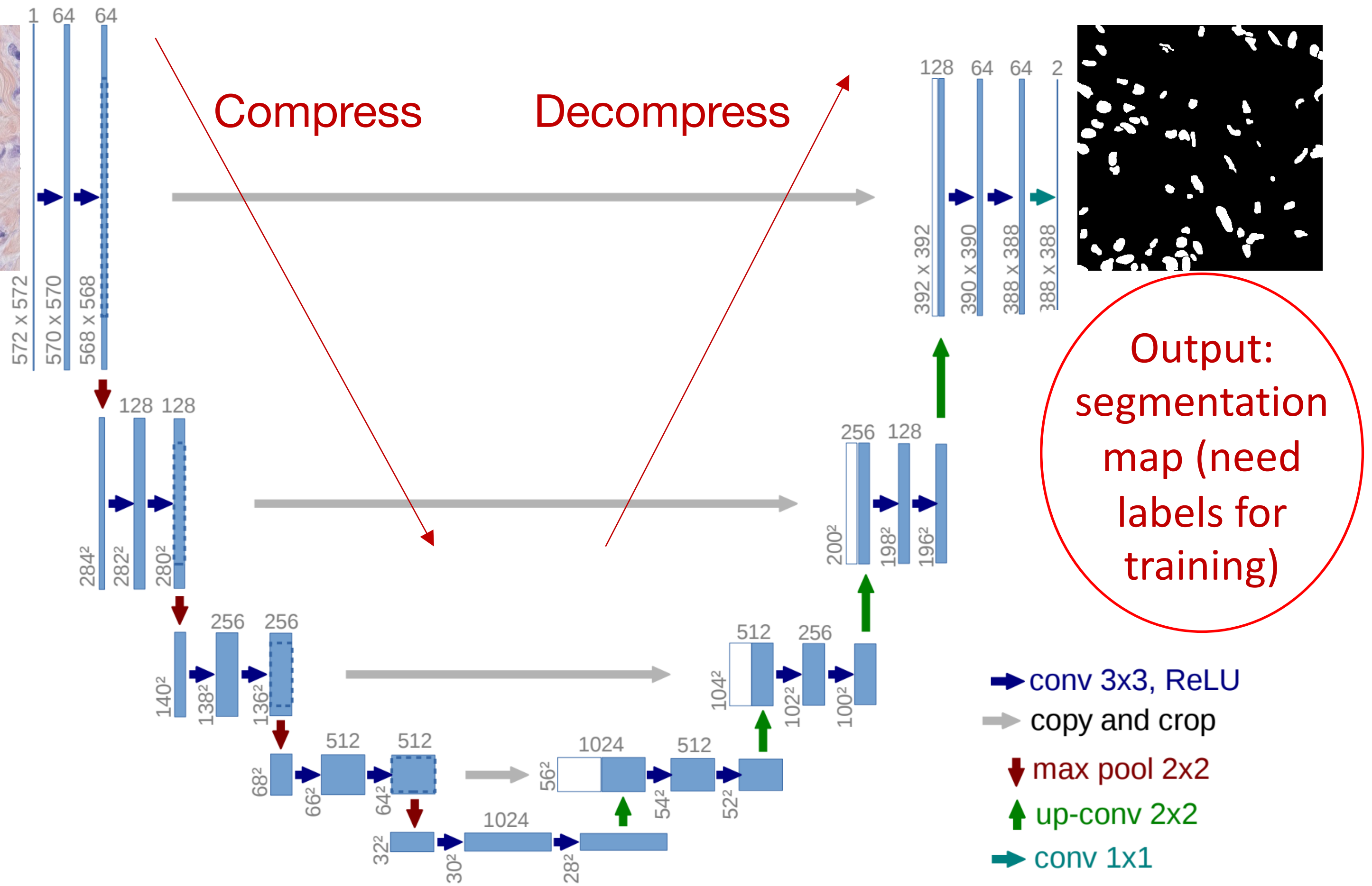
Physical Layer

Example 3: learned illumination pattern for improved segmentation

U-Net Architecture

Input image





U-Net: Convolutional Networks for Biomedical Image Segmentation

Olaf Ronneberger, Philipp Fischer, and Thomas Brox

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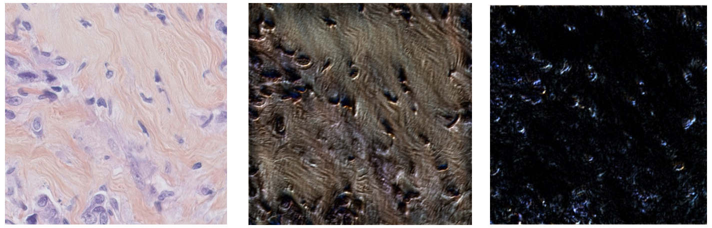
University of Freiburg, Germany
ronneber@informatik.uni-freiburg.de,
WWW home page: http://lmb.informatik.uni-freiburg.de/

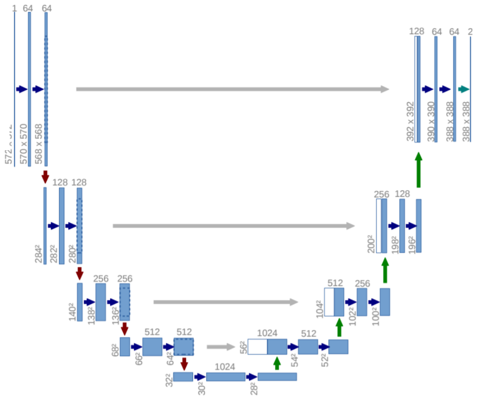
Example 3: learned illumination pattern for improved segmentation

Variably illuminated images from different LEDs

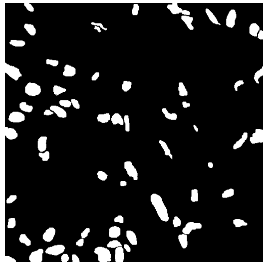


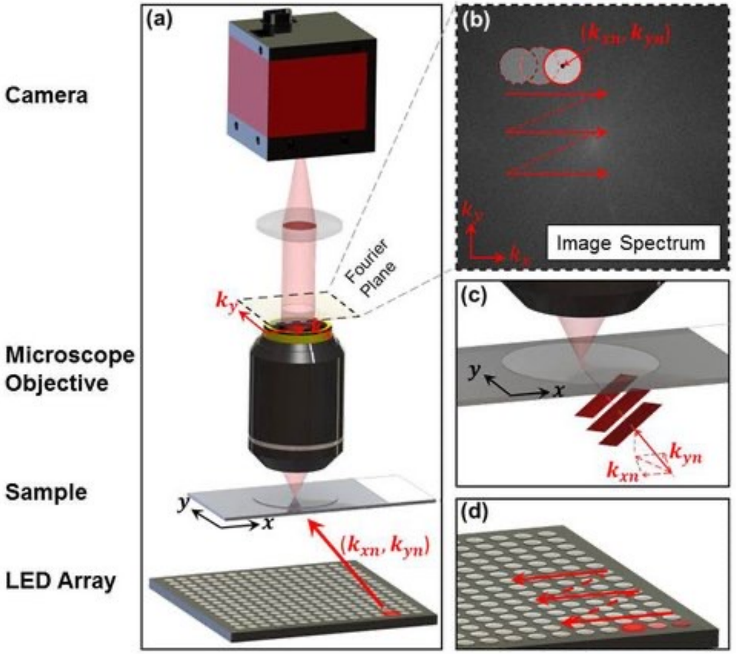
…

U-Net CNN

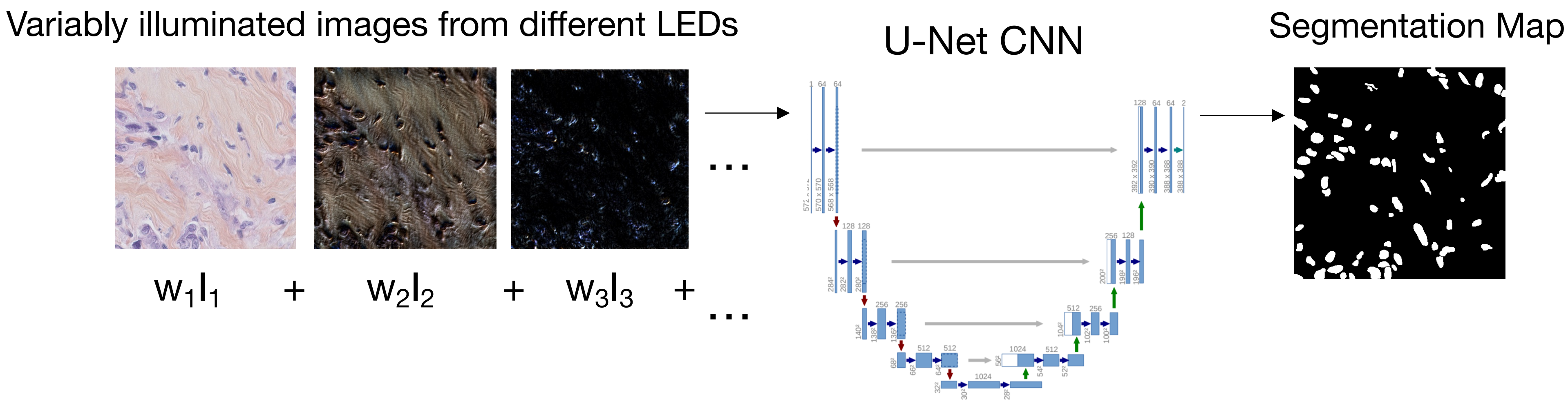


Segmentation Map





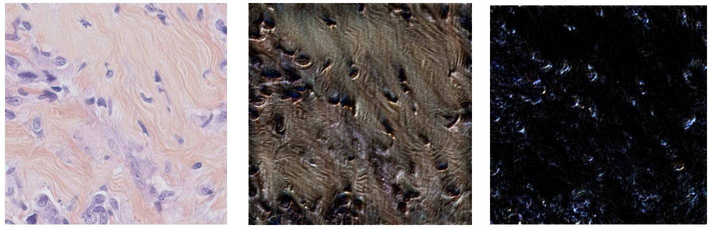
Example 3: learned illumination pattern for improved segmentation



*If we allow w's here to be trainable weights, then we can find ideal brightnesses for different LEDs to illuminate a sample of interest!

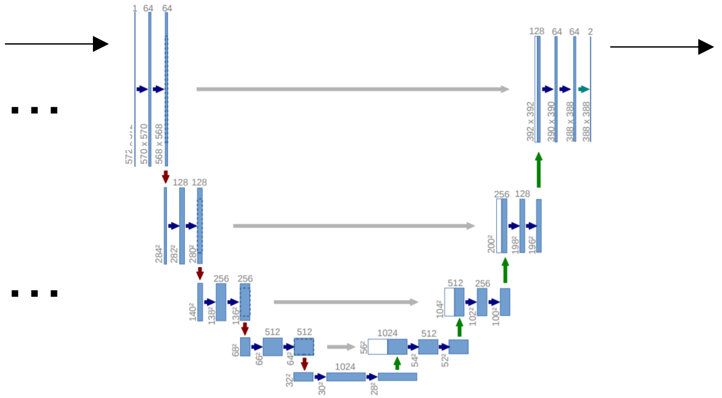
Example 3: learned illumination pattern for improved segmentation

Variably illuminated images



$w_1 I_1$ + $w_2 I_2$ + $w_3 I_3$ + …

U-Net CNN



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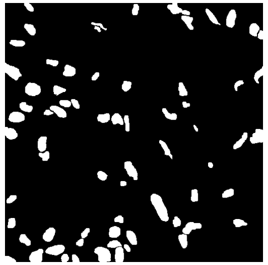
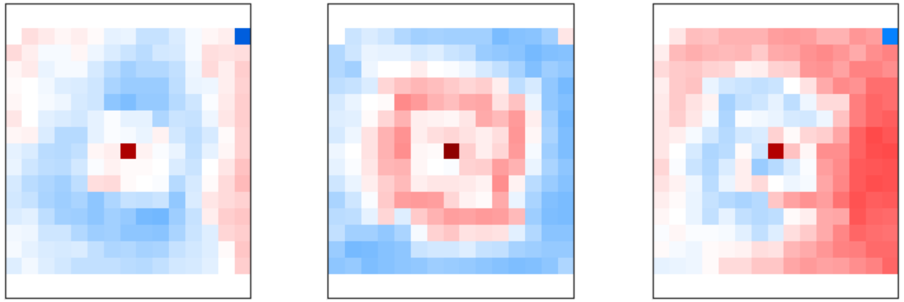
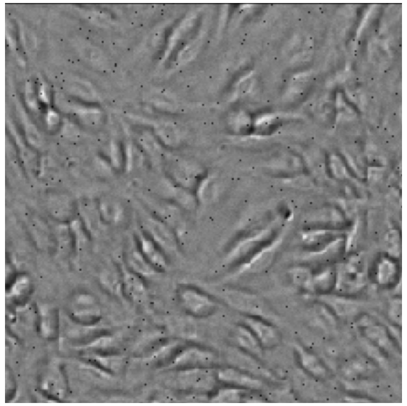


Optimized illumination for nuclei segmentation

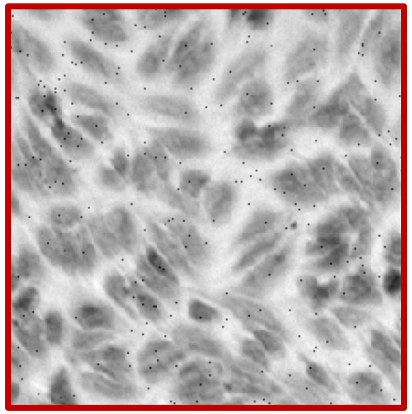
Mean Red channel **Mean Green channel** **Mean Blue channel**



Standard illumination



Learned illumination



+5-10% accuracy

See C. Cooke et al., "Physics-enhanced machine learning for virtual fluorescence microscopy," ICCV (2021)

Multiple Patterns for Fluorescence image inference

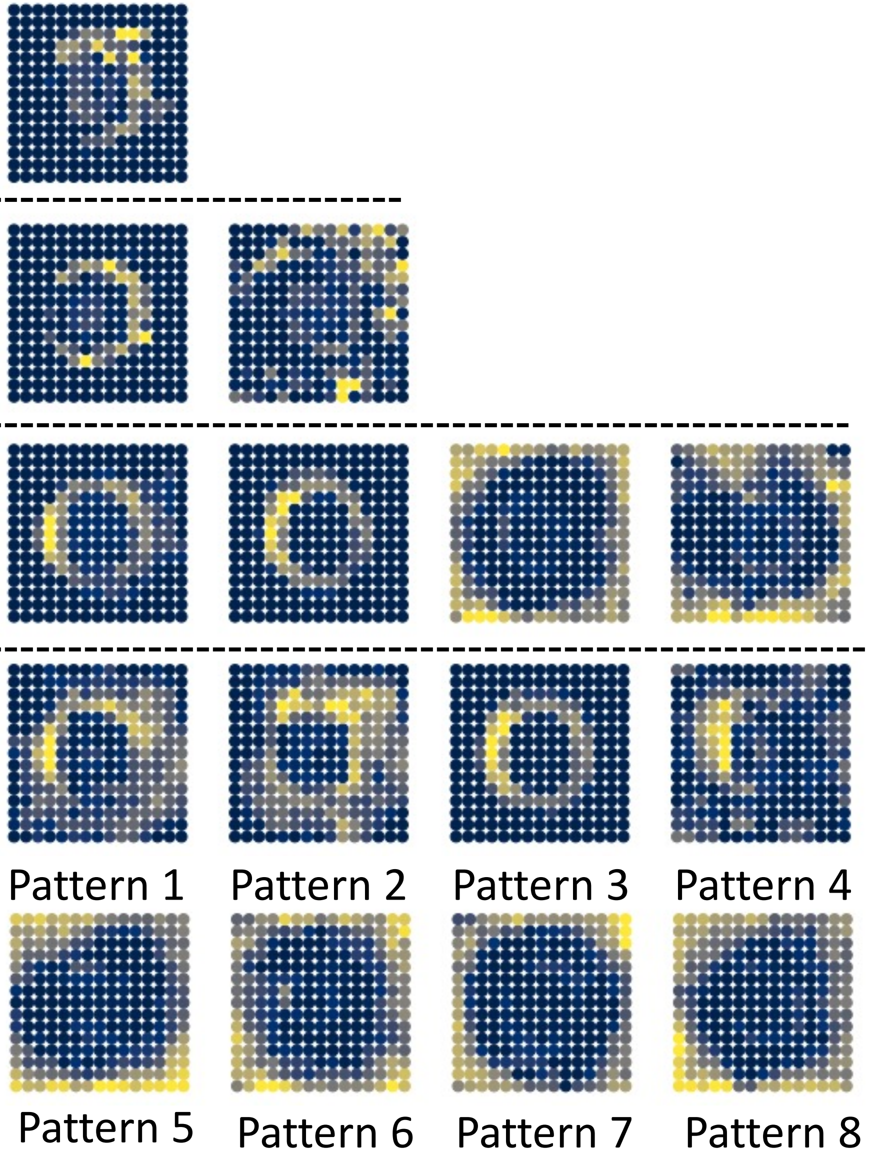
deep imaging

1x Pattern Optimization

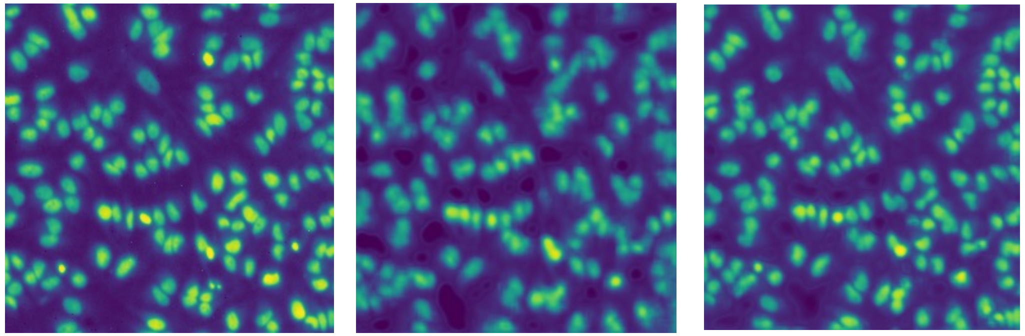
2x Pattern Optimization

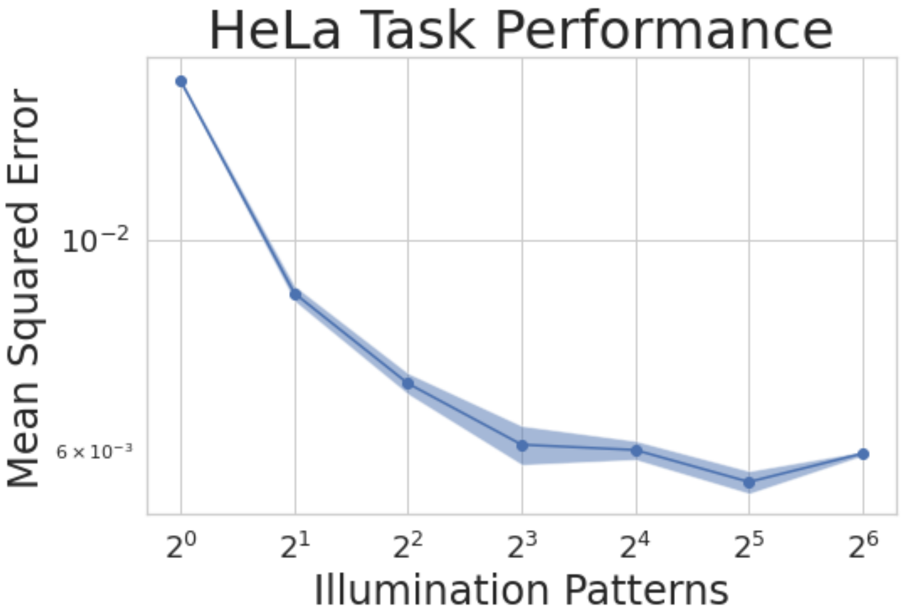
4x Pattern Optimization

8x Pattern Optimization



Ground Truth | 1 Pattern | 4 Patterns





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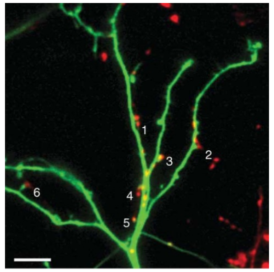

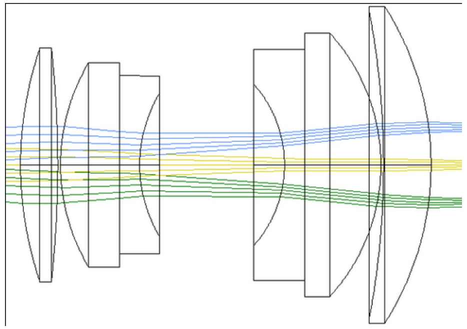

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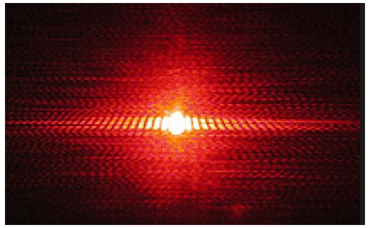

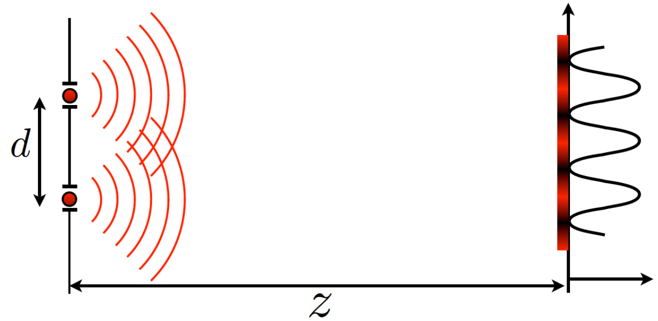

- Complex field
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$E_{tot} = E_1 + E_2$

This class: Modeling coherent radiation as a wave

Let's take a step back: how does light propagate?

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Maxwell's equations
without any charge

$$\nabla \times \vec{\mathcal{E}} = -\mu \frac{\partial \vec{\mathcal{H}}}{\partial t}$$

$$\nabla \times \vec{\mathcal{H}} = \epsilon \frac{\partial \vec{\mathcal{E}}}{\partial t}$$

$$\nabla \cdot \epsilon \vec{\mathcal{E}} = 0$$

$$\nabla \cdot \mu \vec{\mathcal{H}} = 0.$$

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$$\nabla \cdot \epsilon \vec{\mathcal{E}} = 0$$

$$\nabla \cdot \mu \vec{\mathcal{H}} = 0.$$

1. Take the curl of both sides of first equation
2. Substitute 2nd and 3rd equation
3. Arrive at the wave equation:

$$\nabla^2 \vec{\mathcal{E}} - \frac{n^2}{c^2} \frac{\partial^2 \vec{\mathcal{E}}}{\partial t^2} = 0 \qquad n = \left(\frac{\epsilon}{\epsilon_0}\right)^{1/2} \qquad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

Let’s take a step back: how does light propagate?

Considering light that isn’t pulsed over time, we can use the following solution:

$$u(P,t) = A(P)\cos[2\pi\nu t + \phi(P)]$$

$$u(P,t) = Re\{U(P)\exp(-j2\pi\nu t)\},$$

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With this particular solution, we get the following important time-independent equation:

Helmholtz Equation

$$\boxed{(\nabla^2 + k^2)U = 0.}$$

$$k = 2\pi n \frac{\nu}{c} = \frac{2\pi}{\lambda},$$

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Helmholtz Equation

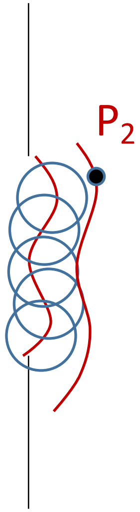
$$\boxed{(\nabla^2 + k^2)U = 0.} \qquad k = 2\pi n\frac{\nu}{c} = \frac{2\pi}{\lambda},$$

This is an important equation in physics. We won't go into the details, but it leads to the Huygen-Fresnel principle:

$$U(P_2) = \frac{1}{j\lambda}\iint_{\Sigma} U(P_1)\frac{\exp(jkr_{21})}{r_{21}}\cos\theta\, ds$$

Plane-to-plane light propagation via the ”paraxial approximation”

The Huygens-Fresnel Equation $$U(P_2) = \frac{1}{j\lambda} \iint_{\Sigma} U(P_1) \frac{\exp(jkr_{21})}{r_{21}} \cos\theta \, ds$$



Aperture

Plane-to-plane light propagation via the ”paraxial approximation”

The Huygens-Fresnel Equation

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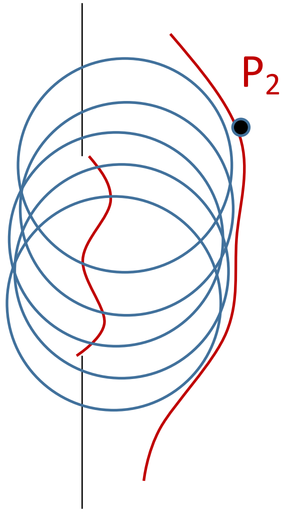


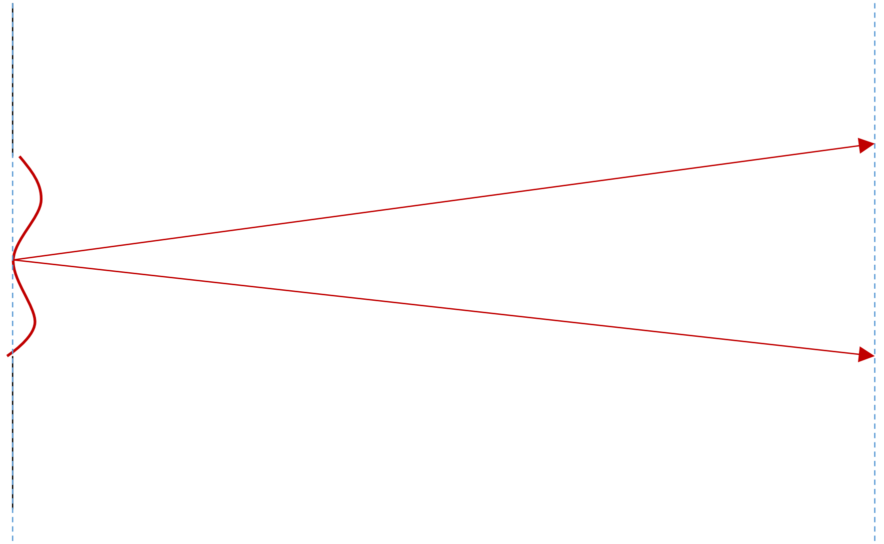
Generally connects two points in 3D:

$$U(P_1) = U(x_1, y_1, z_1)$$

$$U(P_2) = U(x_2, y_2, z_2)$$

Plane-to-plane light propagation via the "paraxial approximation"

We are usually concerned about propagation between two planes (almost always in an optical system):



$$U(P_1) = U(x_1, y_1, z_1 = z_{p1}) \qquad U(P_2) = U(x_2, y_2, z_2 = z_{p2})$$

$$U(P) = E(x, y, z)e^{ikz}$$

Plane-to-plane light propagation via the ”paraxial approximation”

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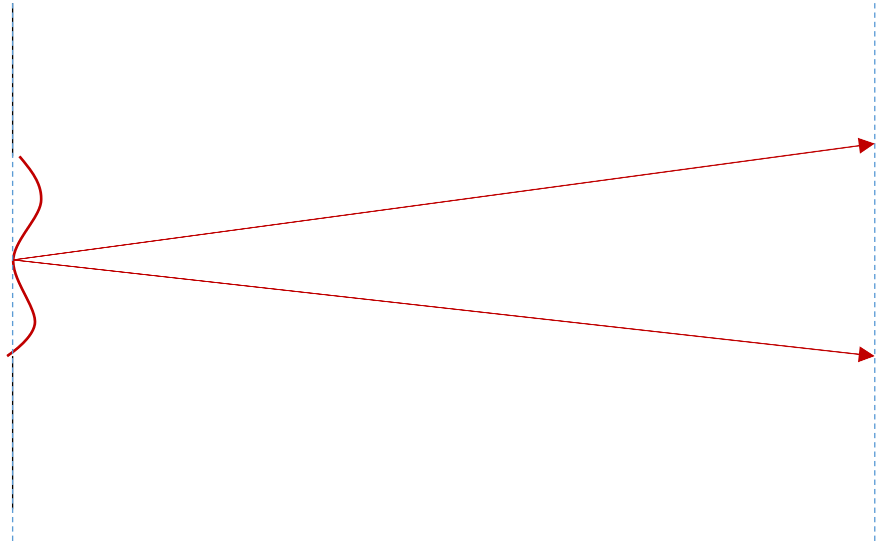

Paraxial approximation:

$$\boxed{\nabla_\perp^2 U + 2ik\frac{dU}{dz} = 0}$$

$$\nabla_\perp^2 \stackrel{\text{def}}{=} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$U(P_1) = U(x_1, y_1, z_1 = z_{p1}) \qquad U(P_2) = U(x_2, y_2, z_2 = z_{p2})$$

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Plane-to-plane light propagation via the "paraxial approximation"

deep imaging

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Paraxial approximation:

$$\nabla_{\perp}^{2} U + 2ik\frac{dU}{dz} = 0$$

Substitute in $U(P) = E(x, y, z)e^{ikz}$ and crank the wheel,

$$\nabla_{\perp}^{2} E + 2ik\frac{dE}{dz} + 2k^{2} E = 0$$

Paraxial Helmholtz Equation. This has an exact integral solution:

Plane-to-plane light propagation via the ”paraxial approximation”

deep imaging

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Paraxial Helmholtz Equation. This has an exact integral solution:

$$\boxed{E(x, y, z) = \frac{e^{ikz}}{i\lambda z}\iint_{-\infty}^{+\infty} E(x', y', 0)e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]}dx'dy'}$$

Fresnel diffraction integral

This is how light propagates from one plane to the next. It’s a convolution!

Fresnel light propagation as a convolution

$$E(x, y, z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{+\infty} E(x', y', 0) e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]} dx' dy'$$

$$h(x, y, z) = \frac{e^{ikz}}{i\lambda z} e^{i\frac{k}{2z}(x^2+y^2)}$$

$$E(x, y, z) = E(x, y, 0) * h(x, y, z)$$

Fresnel light propagation as a convolution

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{+\infty} E(x',y',0) e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]} dx' dy'$$

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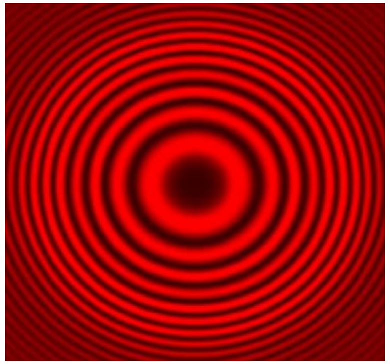

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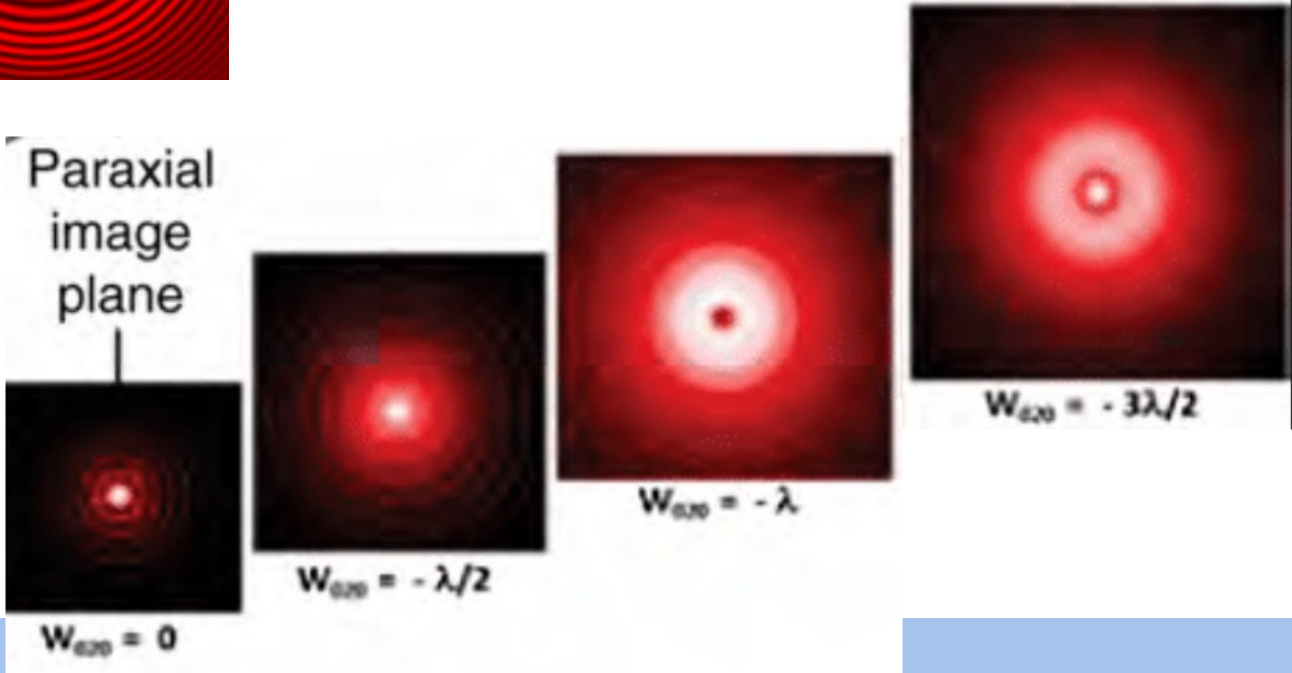

From the Fresnel approximation to the Fraunhofer approximation

Fresnel Approximation:

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{+\infty} E(x',y',0) e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]} dx' dy'$$

Lets assume that the second plane is “pretty far away” from the first plane. Then,

$$\boxed{z > \frac{2D^2}{\lambda}}$$

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From the Fresnel approximation to the Fraunhofer approximation

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$$\boxed{z > \frac{2D^2}{\lambda}}$$

1. Expand the squaring

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint E(x',y',0) e^{\frac{ik}{2z}(x^2+y^2)} e^{\frac{ik}{2z}(x'^2+y'^2)} e^{\frac{ik}{2z}(xx'+yy')} dx'dy'$$

From the Fresnel approximation to the Fraunhofer approximation

Fresnel Approximation:

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{+\infty} E(x',y',0) e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]} dx'dy'$$

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1. Expand the squaring

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint E(x',y',0) e^{\frac{ik}{2z}(x^2+y^2)} e^{\frac{ik}{2z}(x'^2+y'^2)} e^{\frac{ik}{2z}(xx'+yy')} dx'dy'$$

2. Front term comes out, assume second term goes away, then,

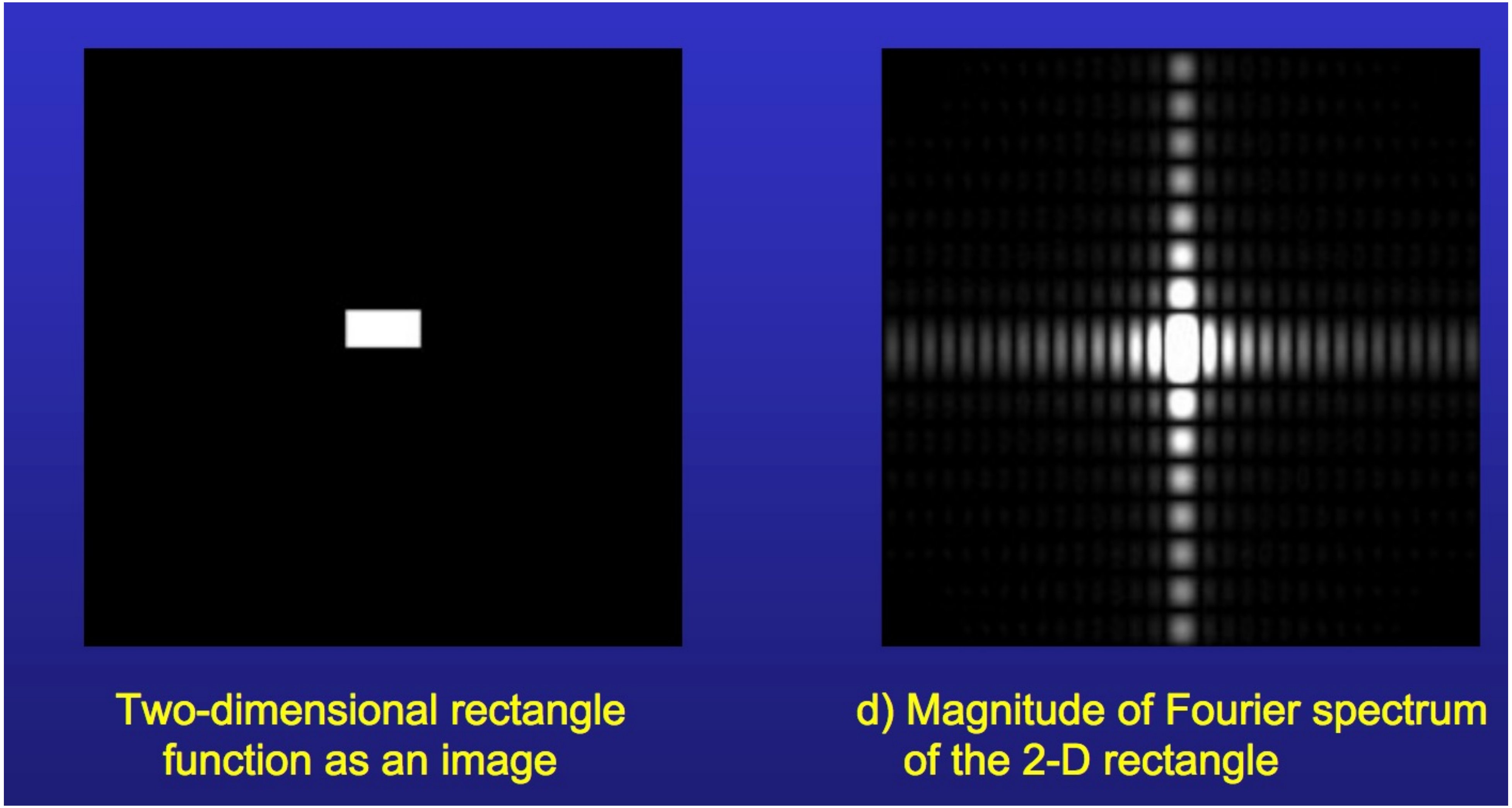
$$\boxed{E(x,y,z) = C \iint E(x',y',0) e^{\frac{ik}{2z}(xx'+yy')} dx'dy'}$$

$$C = \frac{e^{ikz}}{i\lambda z} e^{\frac{ik}{2z}(x^2+y^2)}$$

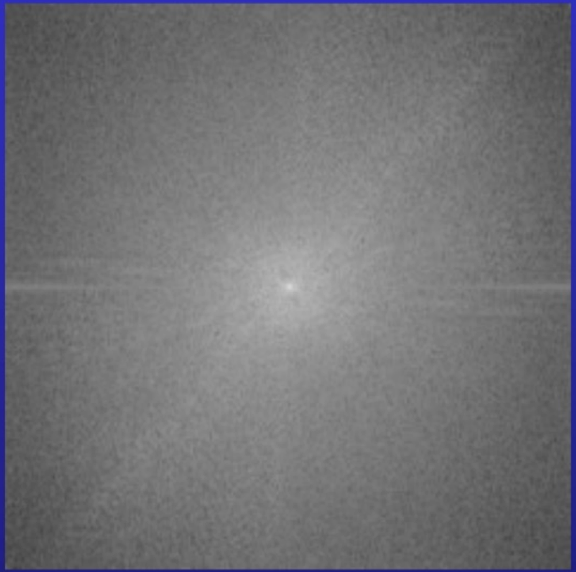
Fraunhofer diffraction is a Fourier transform!!!!!!!

This is the aperture

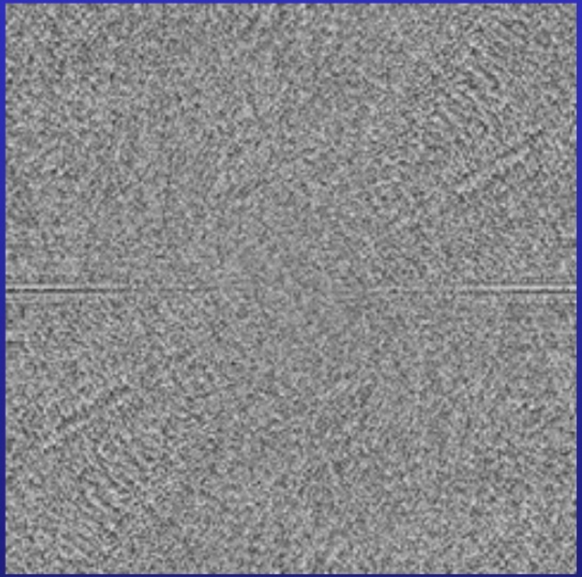
This is what you see far away



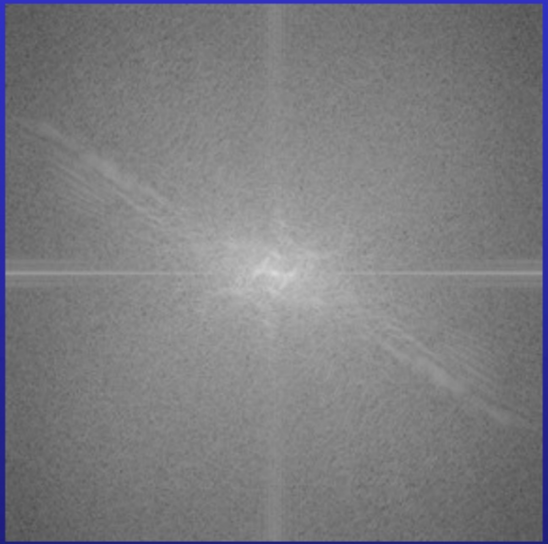

Cheetah
Zebra



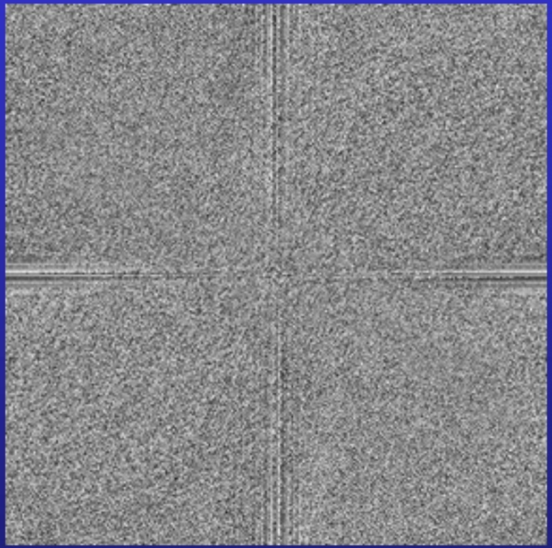
magnitude of cheetah



phase of cheetah

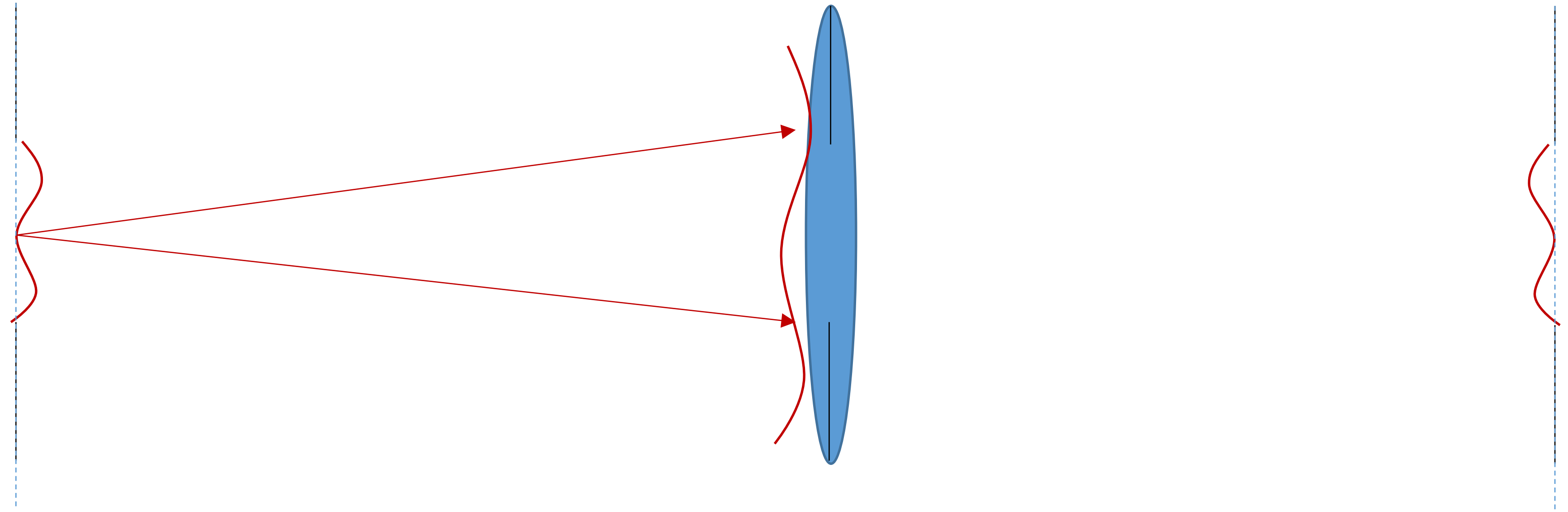


magnitude of zebra



phase of zebra

Model of a microscope (or camera) using Fourier transforms:



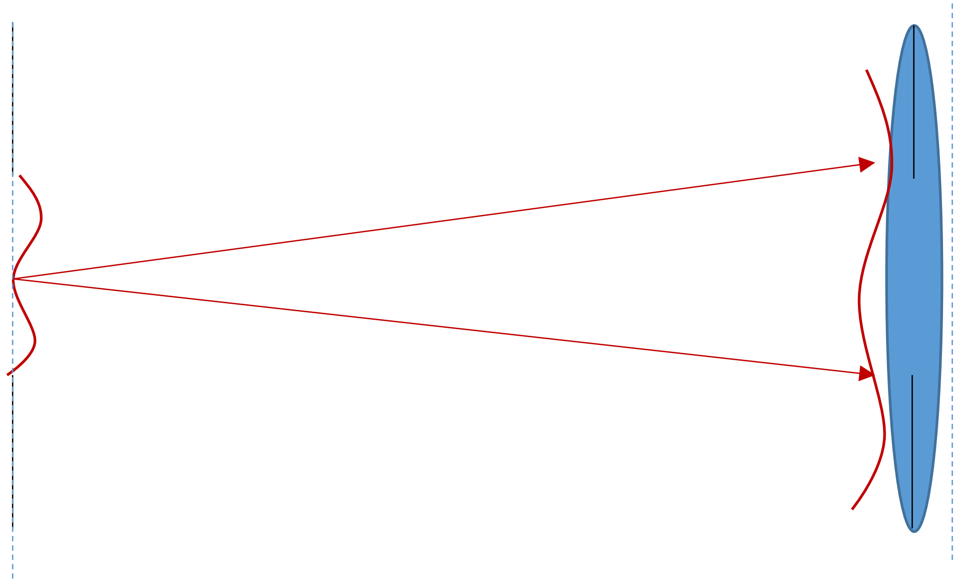
$$E_s(x_s, y_s, 0)$$

$$E_a(x_a, y_a, w)$$

2D Fourier Transform

Model of a microscope (or camera) using Fourier transforms:

$$E'_a(x_a, y_a, w)$$



$$E_s(x_s, y_s, 0) \qquad E_a(x_a, y_a, w)$$

2D Fourier Transform

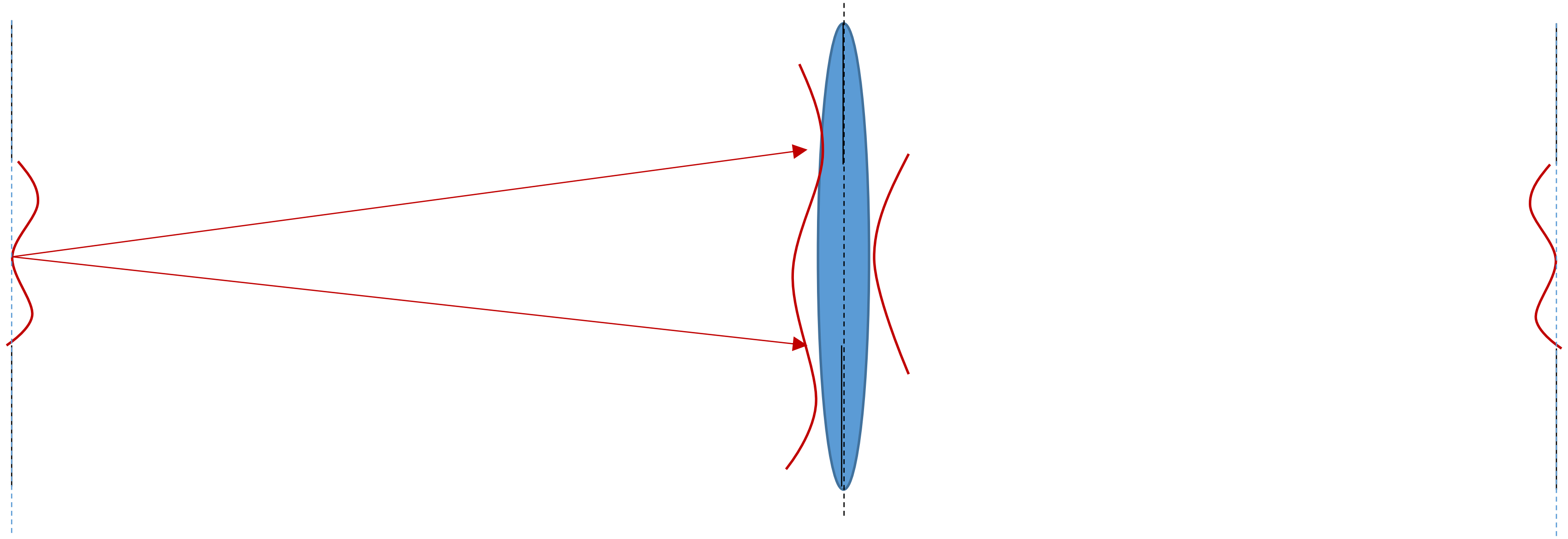
Effect of the lens is to block light.

Use *thin object approximation* to determine distribution of light on the immediate other side of the lens stop:

$$E'_a(x_d, y_d) = E_a(x_d, y_d)A(x_d, y_d)$$

Model of a microscope (or camera) using Fourier transforms:

Last piece of the puzzle: what happens from lens to sensor?



$$E_s(x_s, y_s, 0)$$

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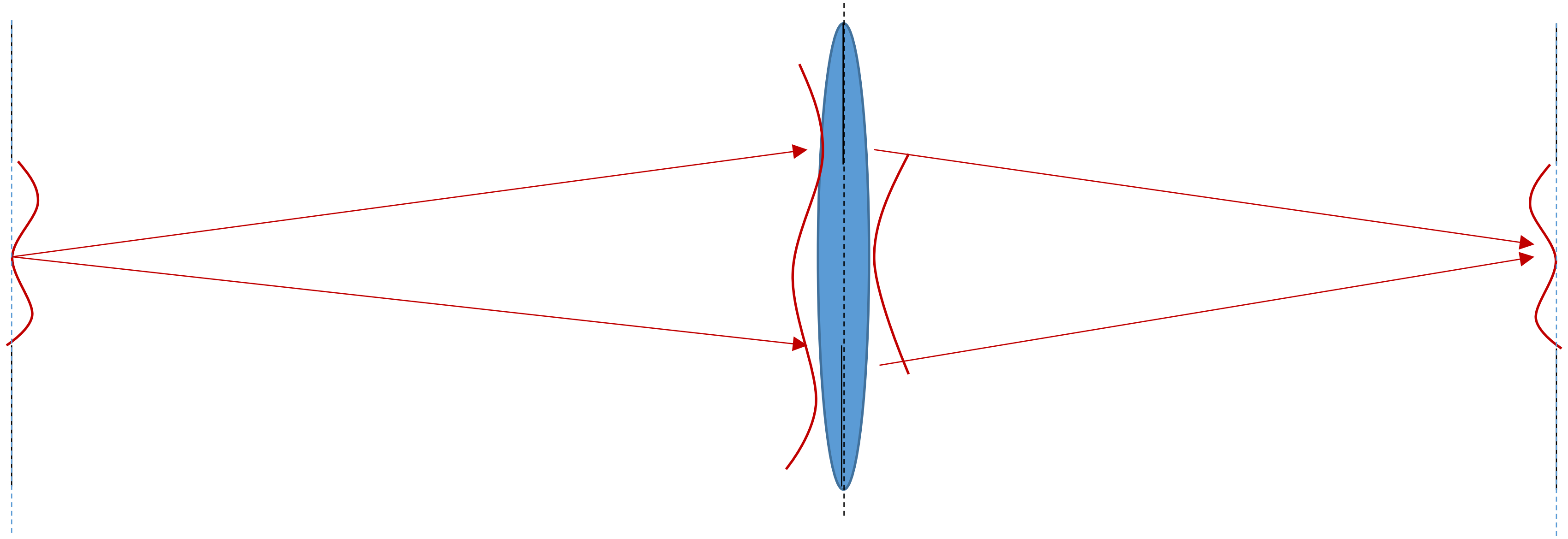
2D Fourier Transform

Model of a microscope (or camera) using Fourier transforms:

Last piece of the puzzle: what happens from lens to sensor?

inverse Fourier transform!



$$E_s(x_s, y_s, 0)$$

$$E'_a(x_d, y_d) = E_a(x_d, y_d)A(x_d, y_d)$$

2D Fourier Transform

2D inverse Fourier Transform

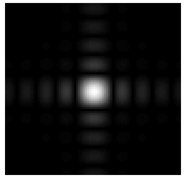
This process should sound familiar….

Input image

$U_1(x,y)$

Convolution filter h

$*$



$=$

Output image

$U_2(x,y)$

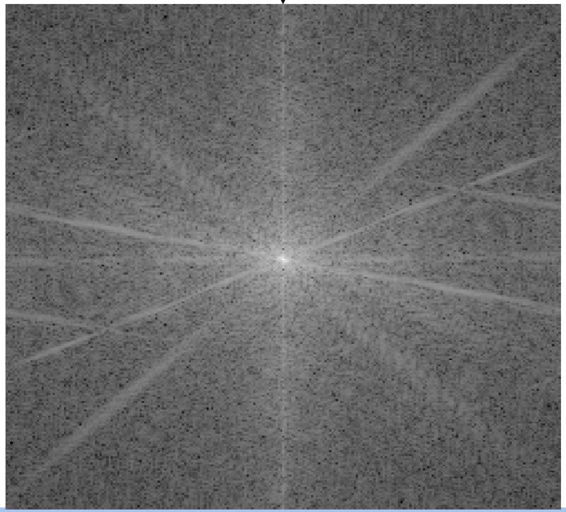
$F\,[U_1]$

$F\,[h]$

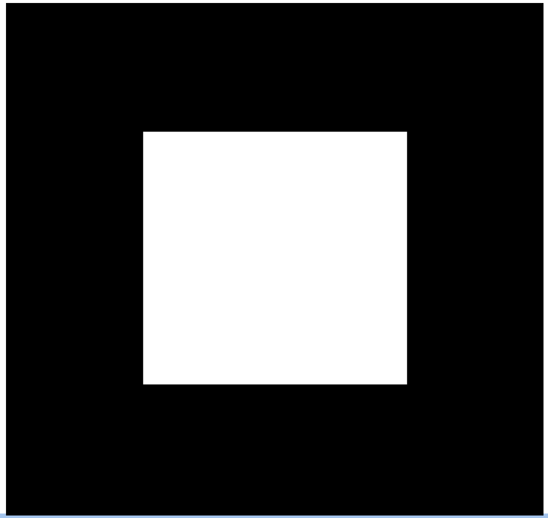
$F^{-1}[H\hat{U}_1]$

Input spectrum

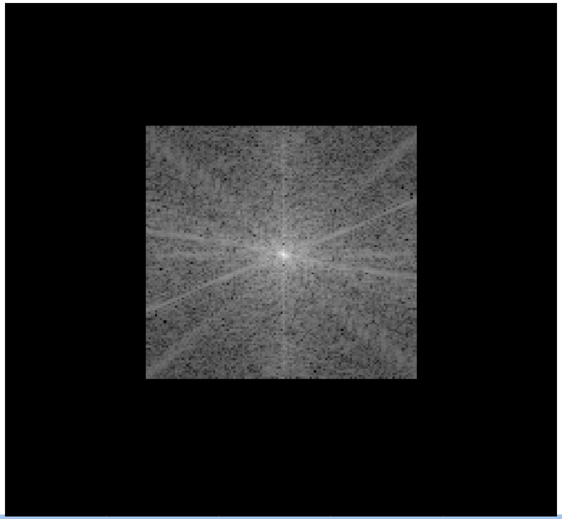
$\hat{U}_1(f_x,f_y)$



$\cdot$

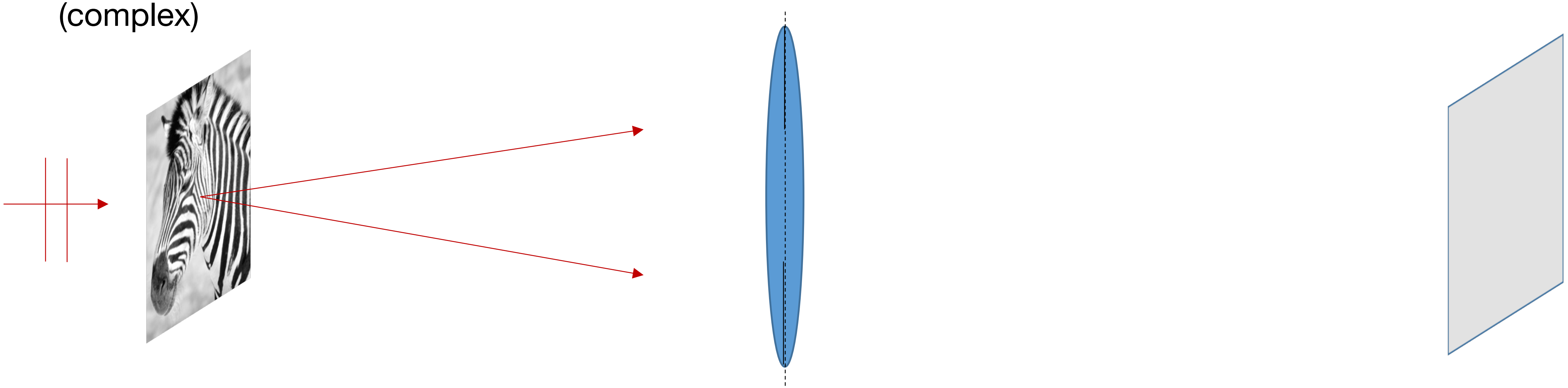


$=$

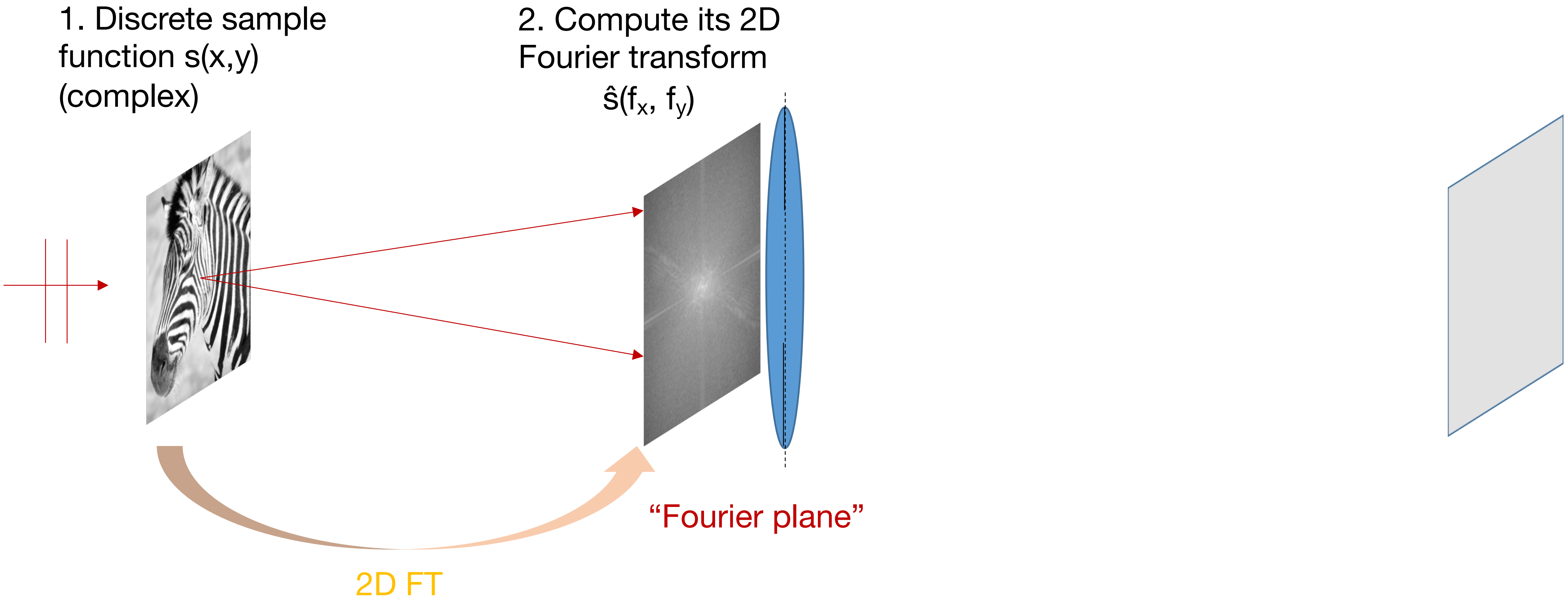


Model of image formation for wave optics (coherent light):

1. Discrete sample function s(x,y) (complex)



Model of image formation for wave optics (coherent light):



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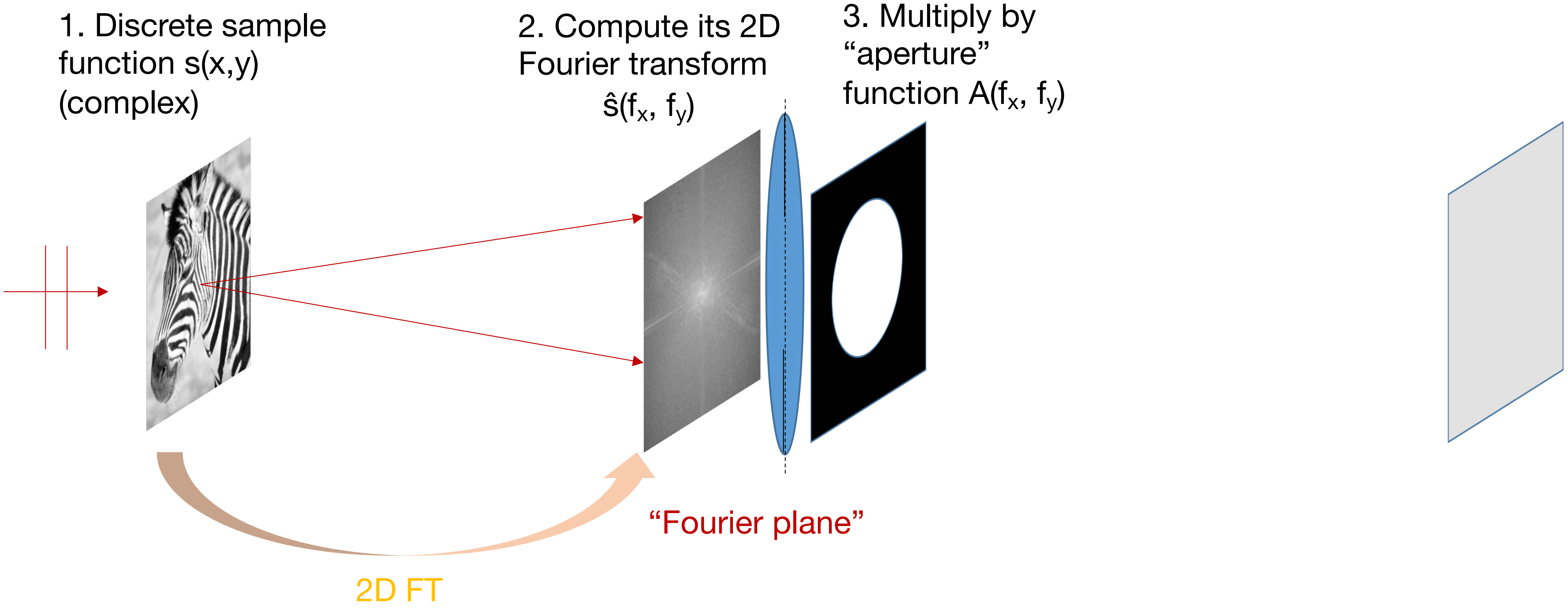

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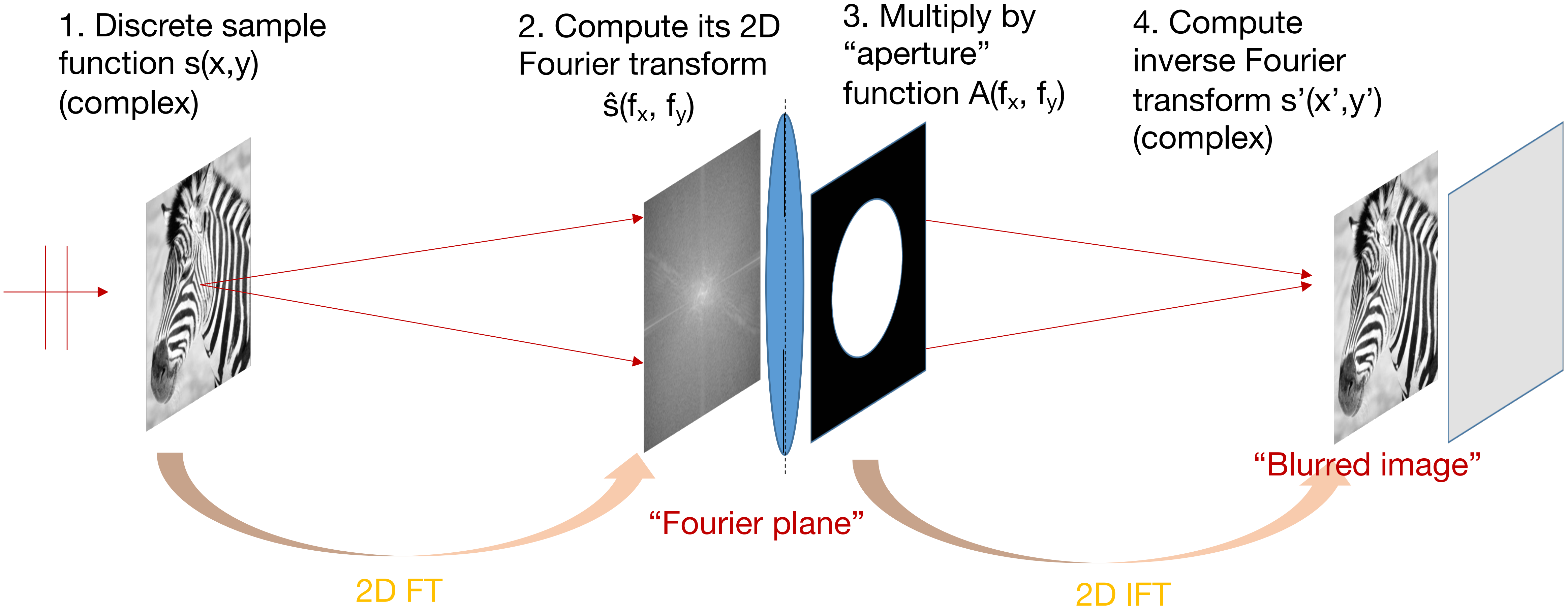

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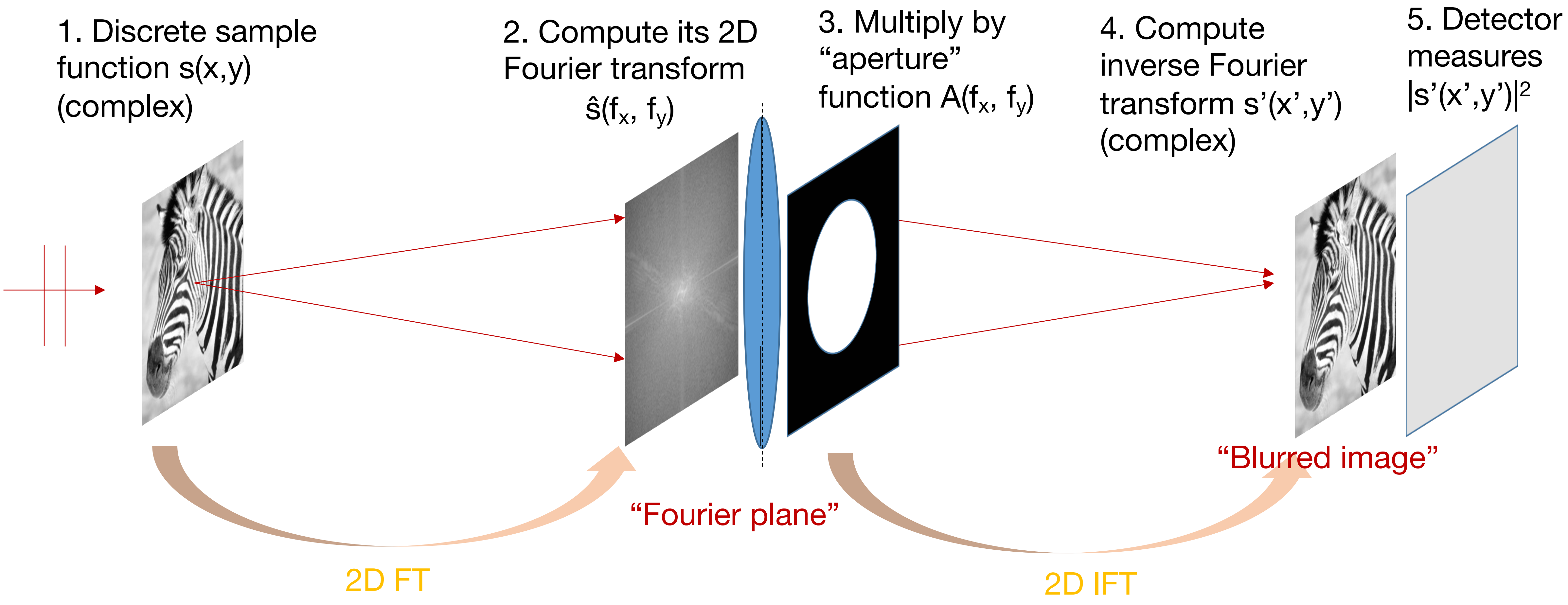


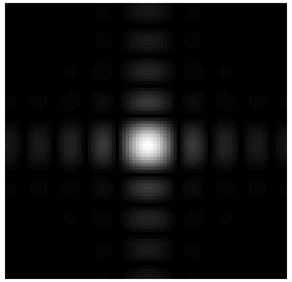
Can also model this using the Convolution Theorem

Aperture function (lens shape)

$A(f_x, f_y)$

Camera blur function (IFT of lens shape)

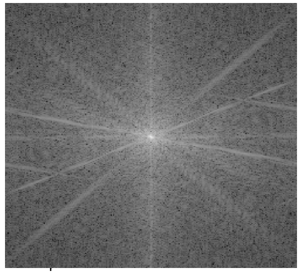
$h(x, y)$



2D IFT

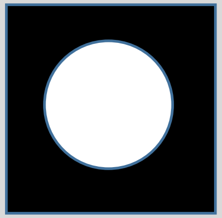
Two modeling choices for the camera:

Spatial Frequency Domain

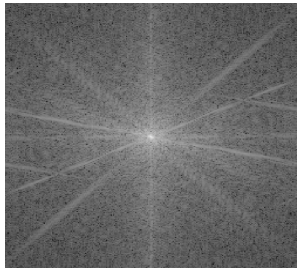


$\hat{s}_1(f_{xi},f_{yi})$

Multiply



$A\,(f_x,f_y)$

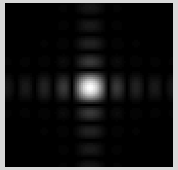


$\hat{s}_2(f_{xo},f_{yo})$

Spatial Domain

$s_1(x_i,y_i)$

Convolve = blur



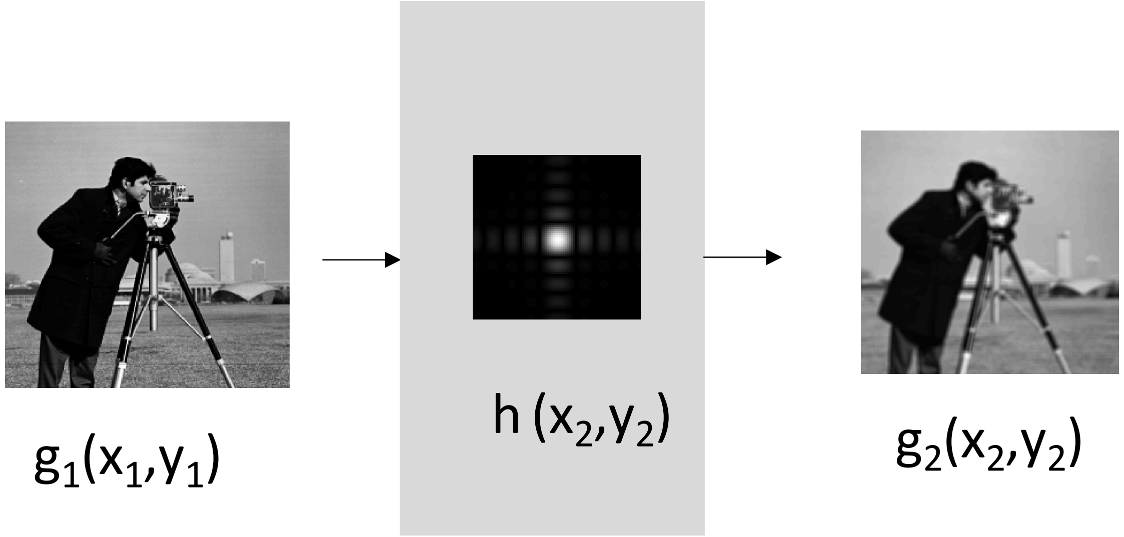
$h\,(x,y)$

$s_2(x_o,y_o)$

Linear systems and the black box

The optical black box system and the point-spread function:

Light $g_1(x_i,y_i)$ entering “black box” optical system modified by system point-spread function

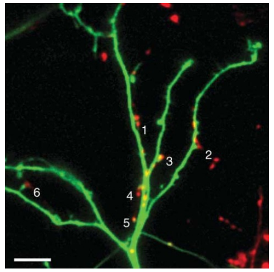


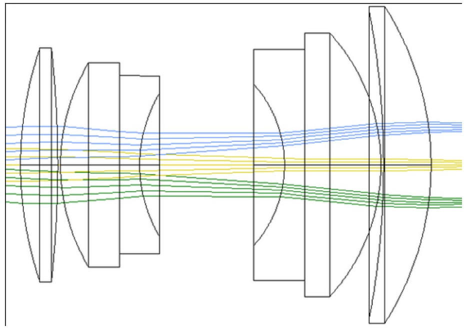
$$\boxed{g_2(x_2,y_2) = \iint_{-\infty}^{\infty} g_1(x_1,y_1)h(x_2-x_1,y_2-y_1)dx_1dy_1}$$

Assume shift invariance:
This is the system point-spread function

Summary of two models for image formation

- **Interpretation #1: Radiation (*Incoherent*)**
- Model: Rays





- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

Mathematical model of for incoherent image formation

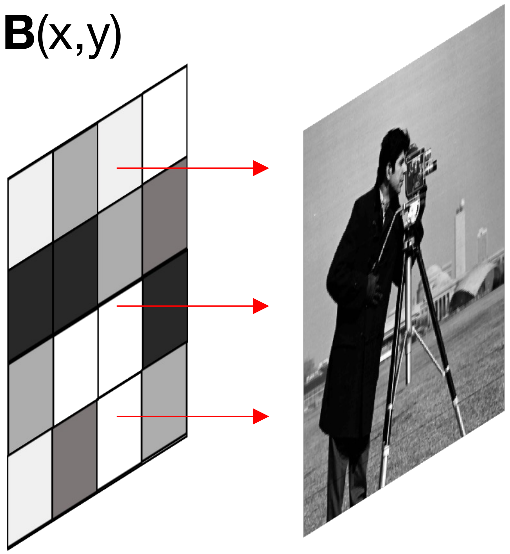
- All quantities are real, and non-negative

Object absorption:

$\mathbf{S_0}(x,y)$

Illumination brightness:

$\mathbf{B}(x,y)$



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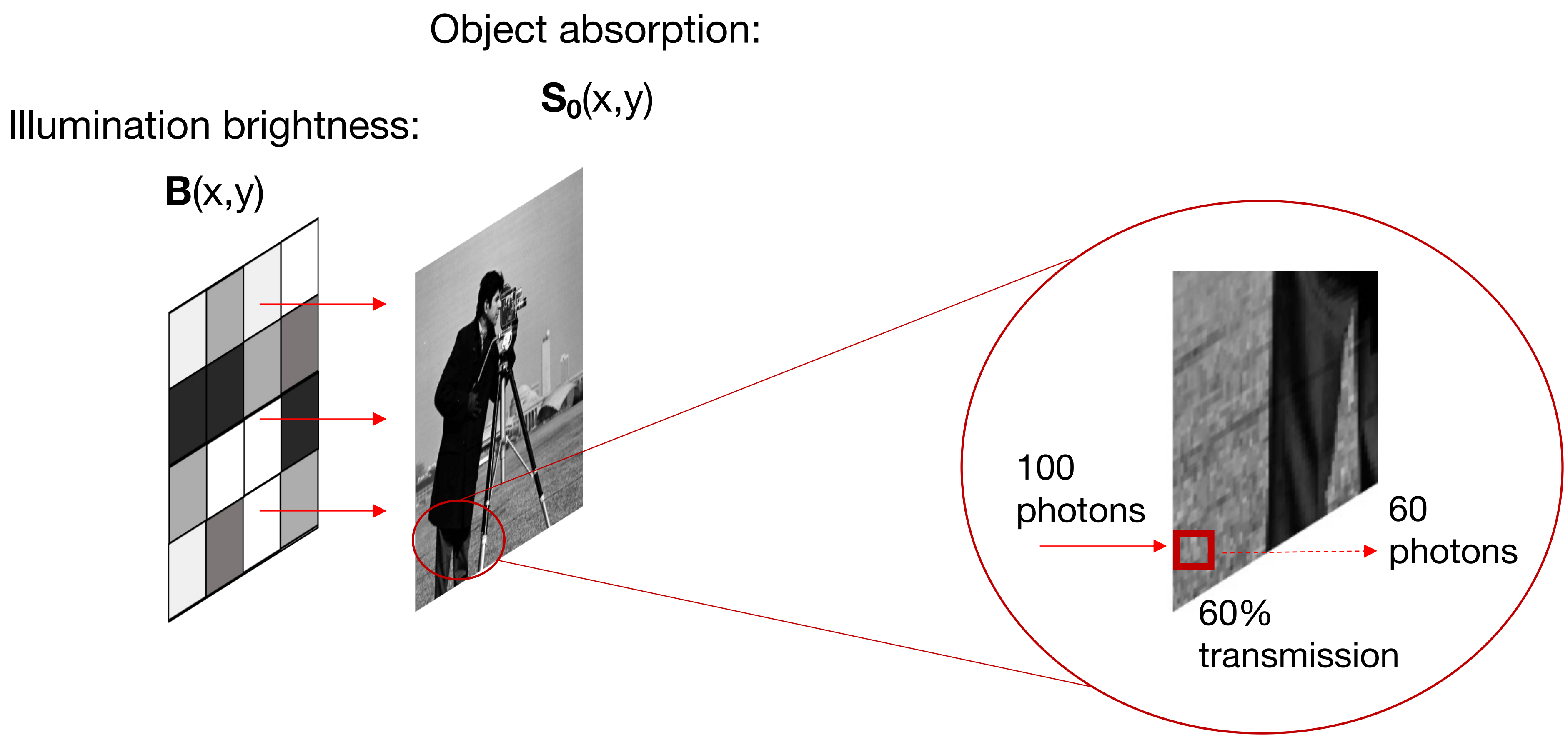

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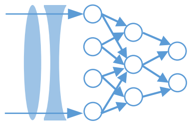

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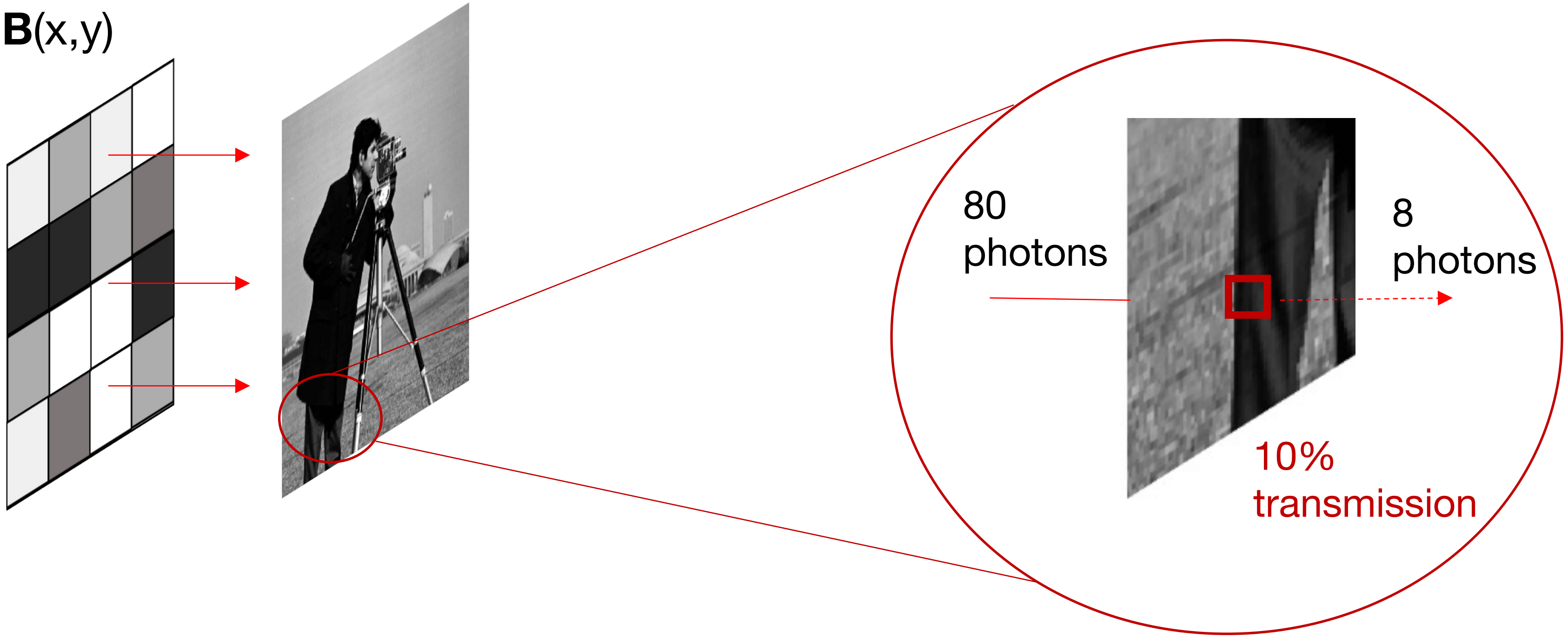

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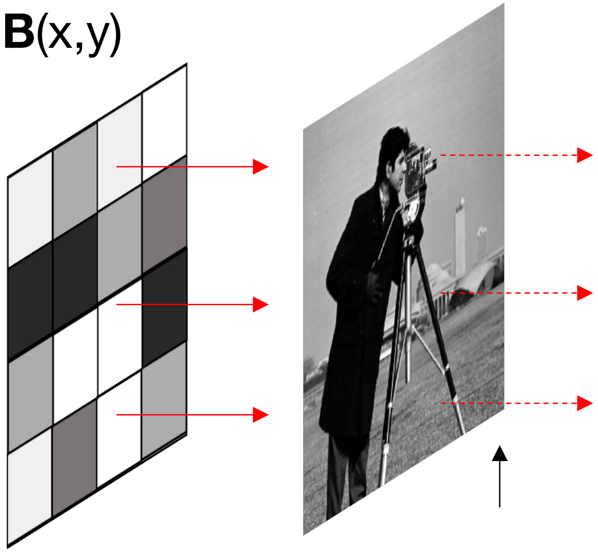

$\mathbf{B\ S_0}$

multiplication

Summary of two models for image formation

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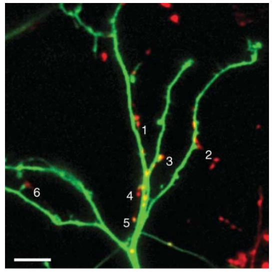
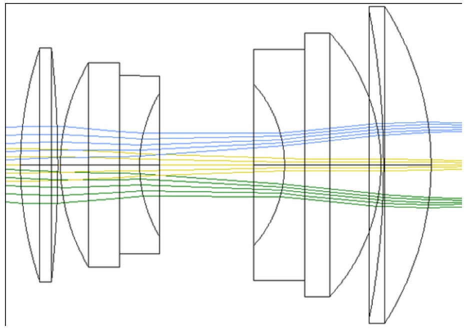


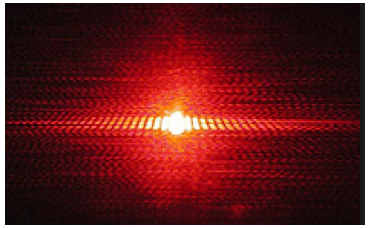


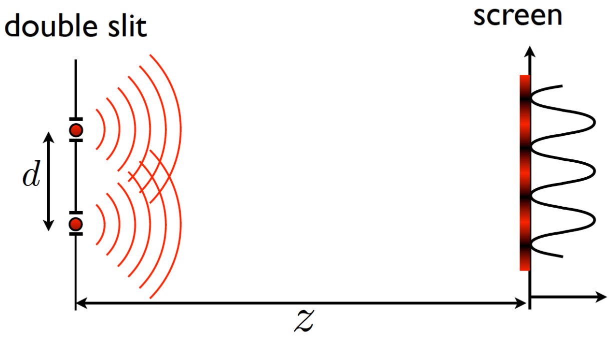
- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

$$\mathbf{I_s = B\ S_0}$$

- **Interpretation #2: Electromagnetic wave (*Coherent*)**
- Model: Waves





- Complex field
- Models Interference

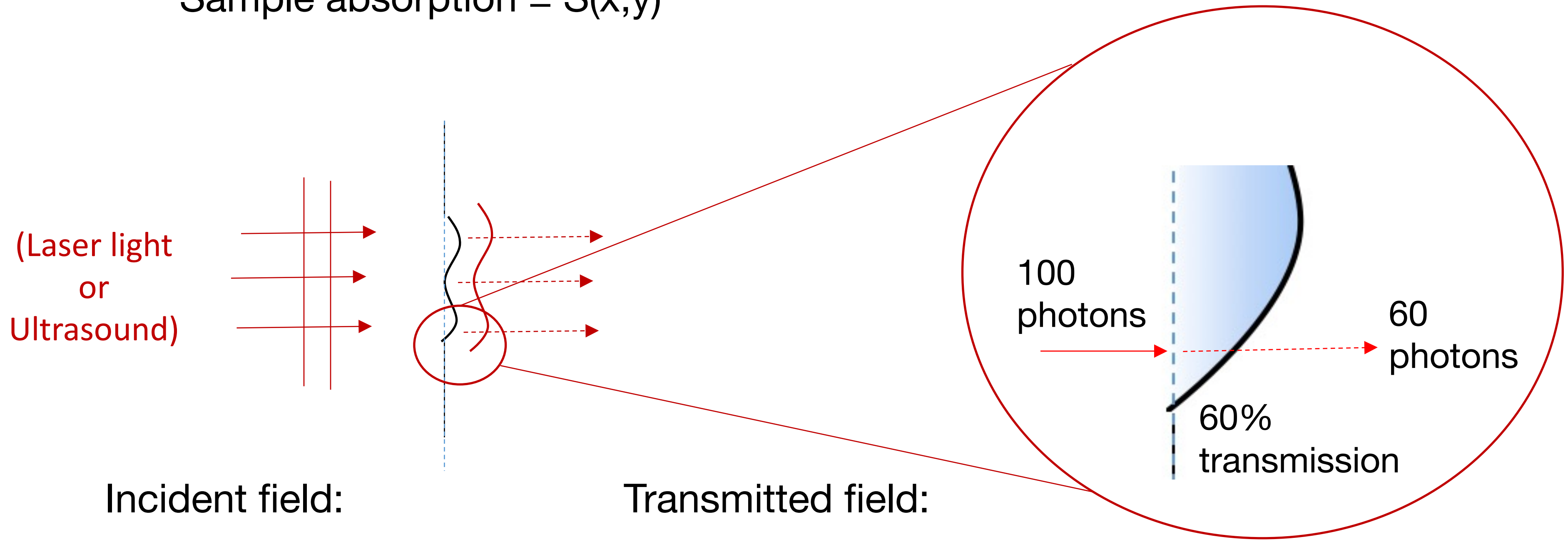
$$E_{tot} = E_1 + E_2$$

Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = S(x,y)

Point #1: Amplitudes behave just like before



$A_t(x,y) = A_i(x,y)\ S(x,y)$

Incident field:

$C(x,y) = A_i(x,y)$

Transmitted field:

$U(x,y) = A_t(x,y) = ?$

Mathematical model of for coherent image formation

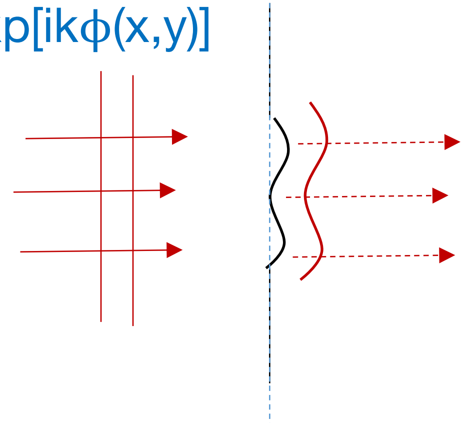
- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = S(x,y)

Sample phase delay = exp[ikϕ(x,y)]

New: complex phase delay
- Needed to represent wave
- Represents wave delay across space

(Laser light or Ultrasound)



Incident field:

$C(x,y) = A_i(x,y)\ \exp[ik\phi_i(x,y)]$

Transmitted field:

$U(x,y) = A_i(x,y)\ S(x,y)\ \exp[ik\phi_t(x,y)]$

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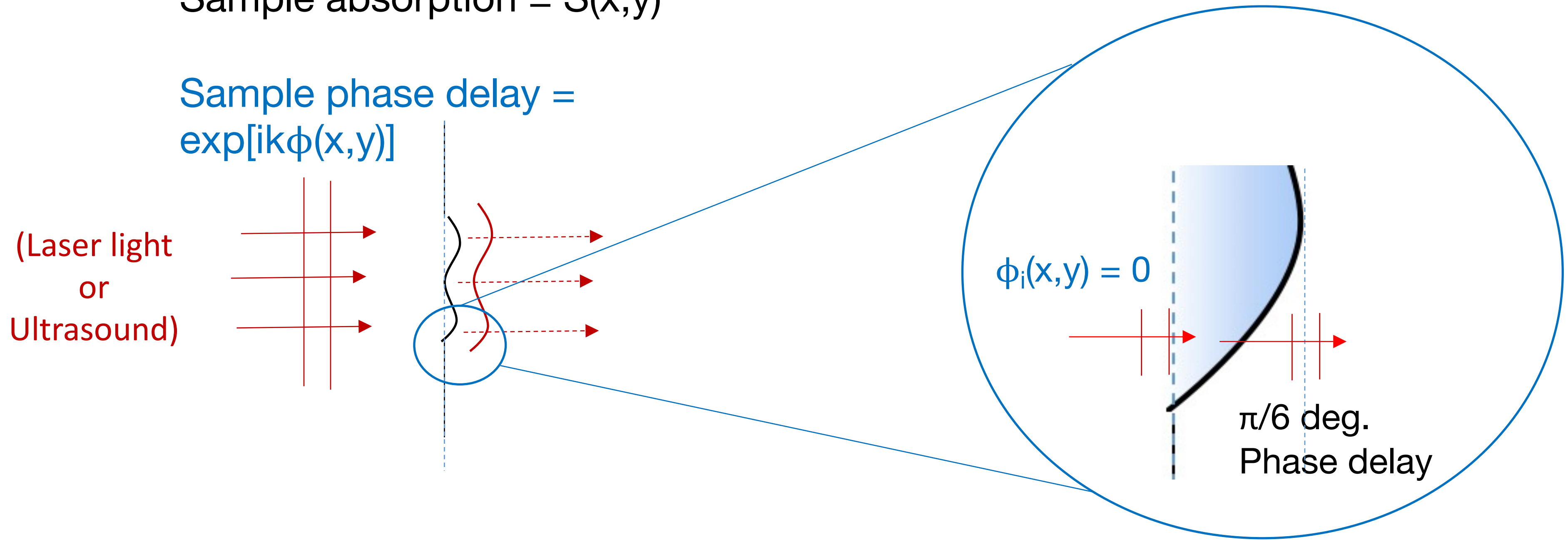

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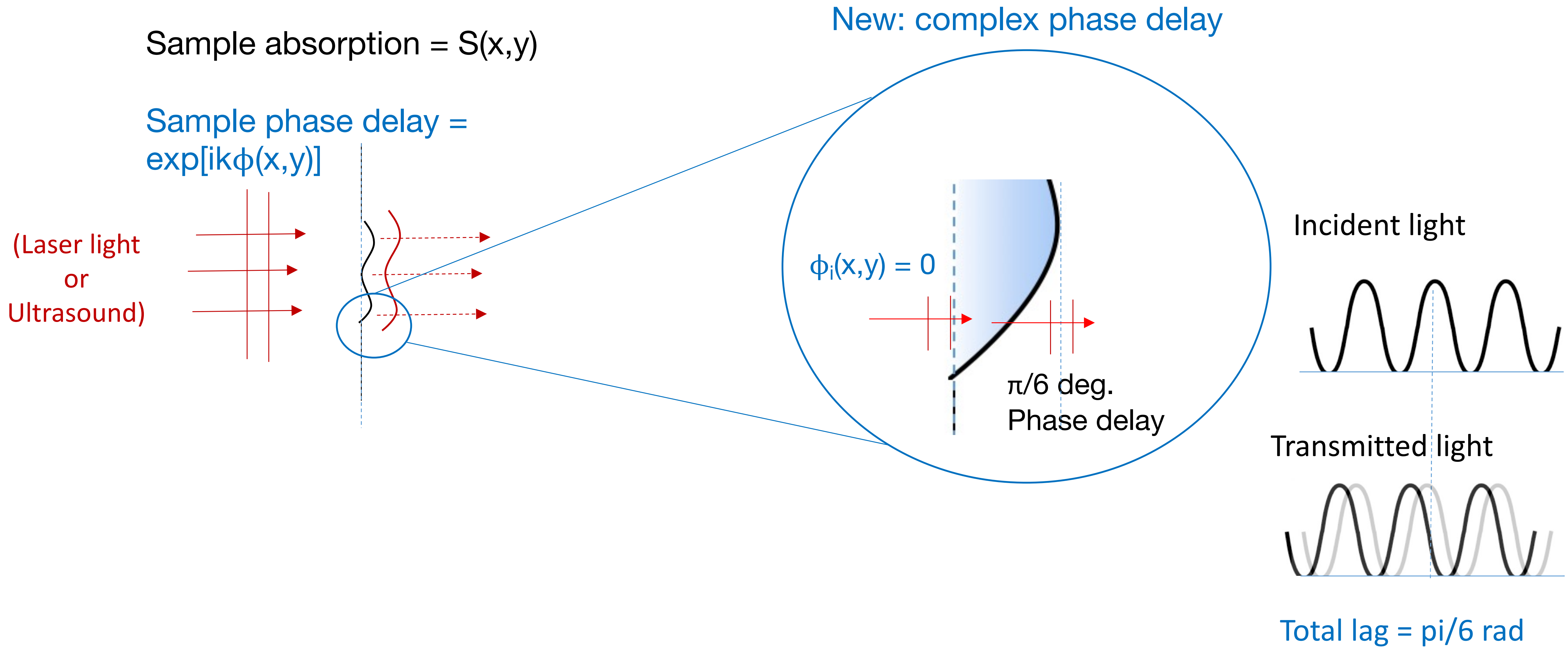

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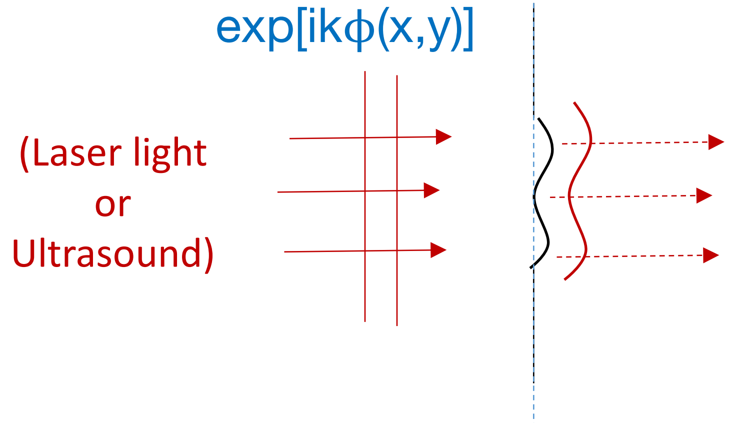

Output phase is sum of phase delays, product of phasors

$$\boxed{\begin{gathered}\phi_t(x,y) = \phi(x,y) + \phi_i(x,y) \\ e^{ik\phi t(x,y)} = e^{ik\phi i(x,y)} \text{ x } e^{ik\phi(x,y)}\end{gathered}}$$

Multiply phases!

Mathematical model of for coherent image formation

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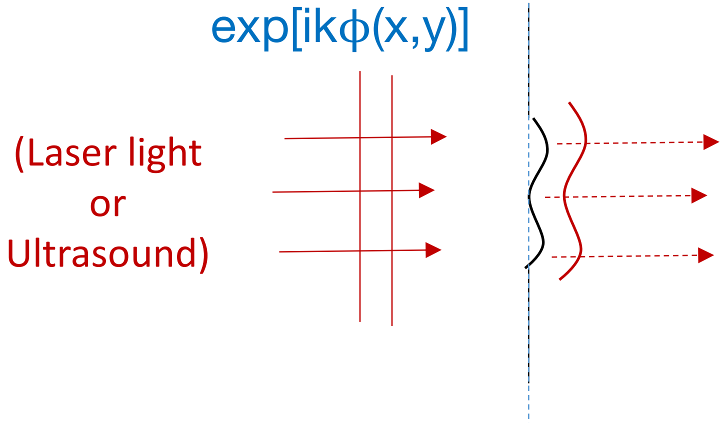

Conclusion:

Transmitted field = incident field x complex sample

$$A_t(x,y)\ \exp[ik\phi_t(x,y)] = A_i(x,y)\ \exp[ik\phi_i(x,y)] \times S(x,y)\ \exp[ik\phi(x,y)]$$

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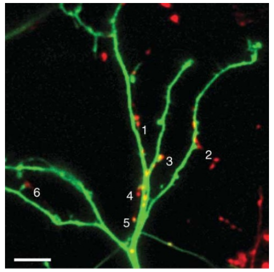

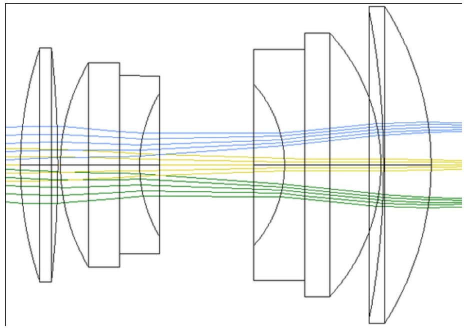

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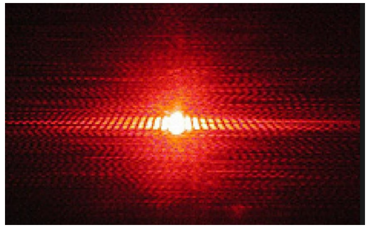

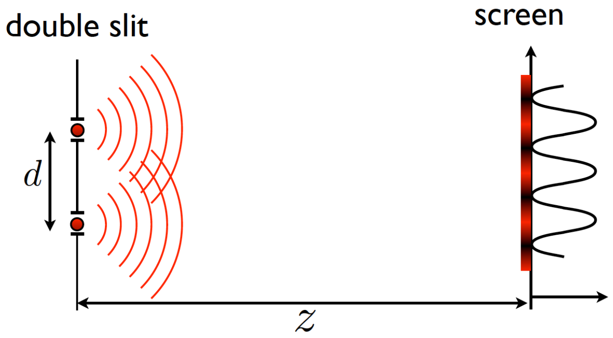

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$$E_{tot} = E_1 + E_2$$

$$\mathbf{U = C\ S_0}$$

U, C and S are complex!

Additional Information about sample index of refraction, spatial frequency and Fourier optics

Microscope illumination and sample index of refraction

So far: illuminate the sample and create a field that is equivalent to the sample's absorption and phase

– why is this true???

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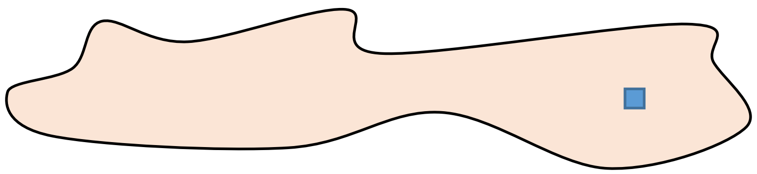
Sample index of refraction $n(x,y,z) = 1 + ia(x) + \varphi(x)$



$a(x=1) = .2$

$\varphi(x=1) = .1$

*For more information, see D. Paganin, Coherent X-Ray Optics, Section 2.2

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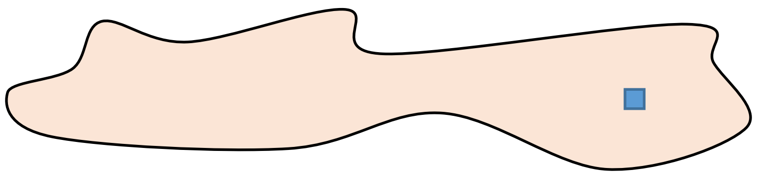

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Sample's effect on light is multiplication with $\exp[-ik * n(x,y)]$

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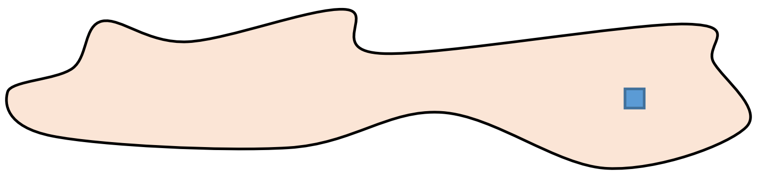

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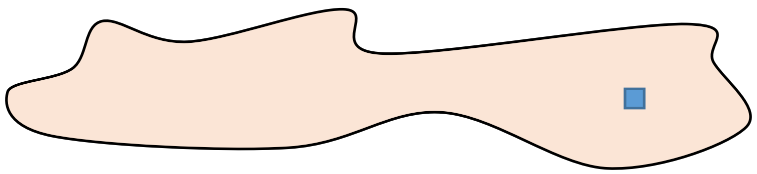

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$$\boxed{U(x) = U_i(x) * \exp[-ik\, n(x)] = U_i(x)\, A(x)\, \exp[ik\varphi(x)]}$$

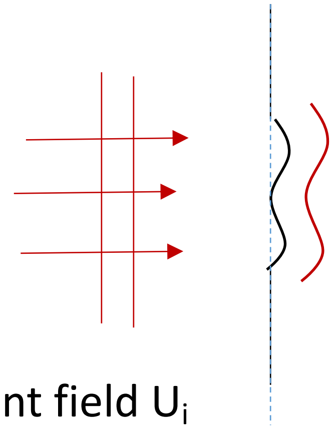
$A(x) = \exp[k\, a(x)]$

absorption

phase shift: new term for laser

Microscope illumination and sample index of refraction

Sample absorption = $A(x)$
Sample phase = $\exp[ik\varphi(x)]$



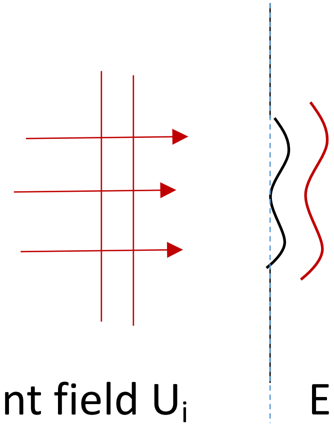
Emerging field U = incident field $U_i(x)$ * sample function $s(x)$

Incident field U_i

Microscope illumination and sample index of refraction

Q: When is the emerging field equal to the absorption and phase?

Sample absorption = A(x)
Sample phase = exp[ikφ(x)]



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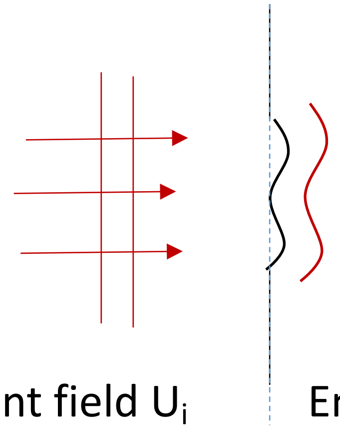

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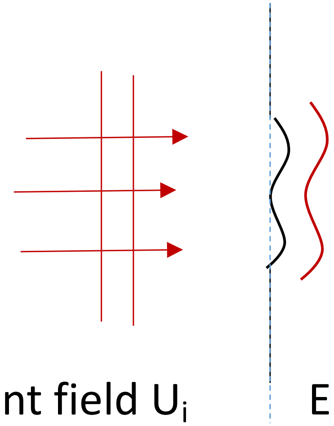


Incident field U_i Emerging field U

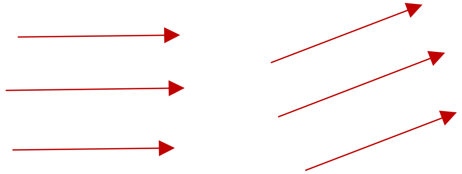
A: When the incident wave = 1, means uniform in amplitude and phase:

$$U_i(x) = 1 \longrightarrow U(x)=A(x)\exp[ik\varphi(x)]$$

Plane wave $U_i(x) = 1 * \exp(ik\bullet x)$

$$U_i(x) = \exp(ikx\sin(\theta))$$

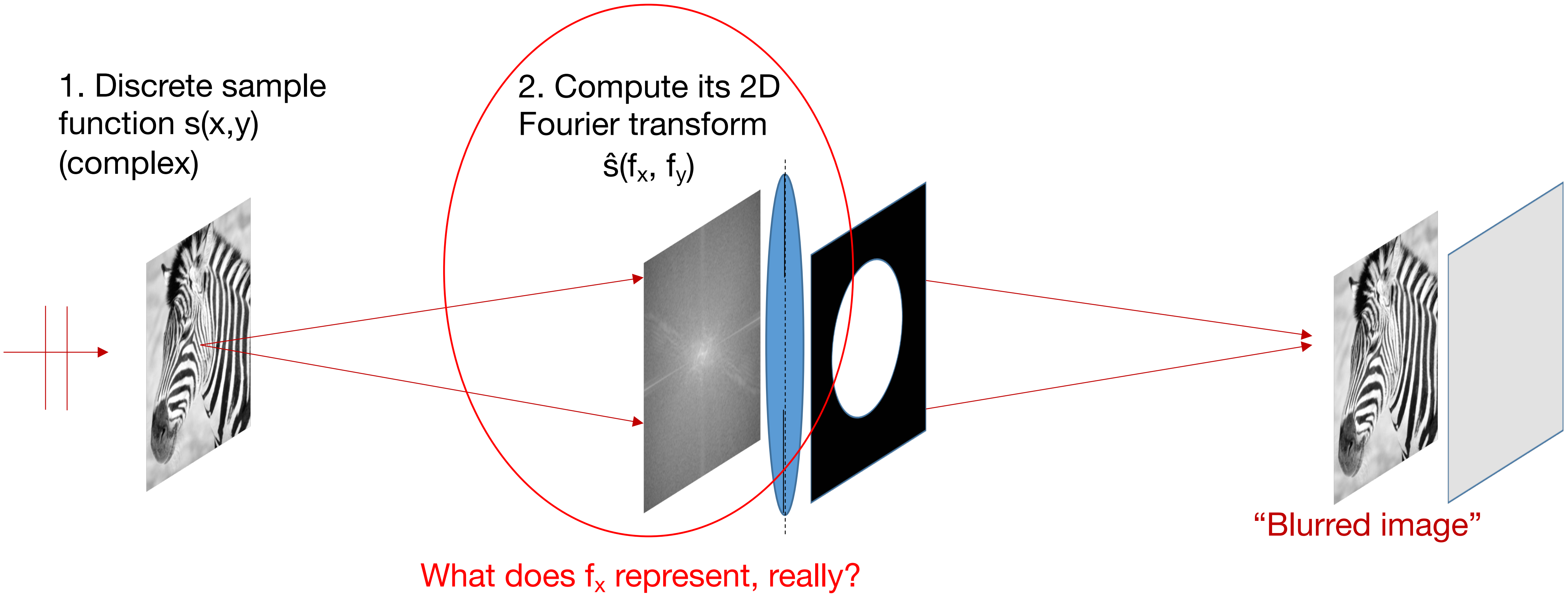
$\theta=0$ $\theta=30$



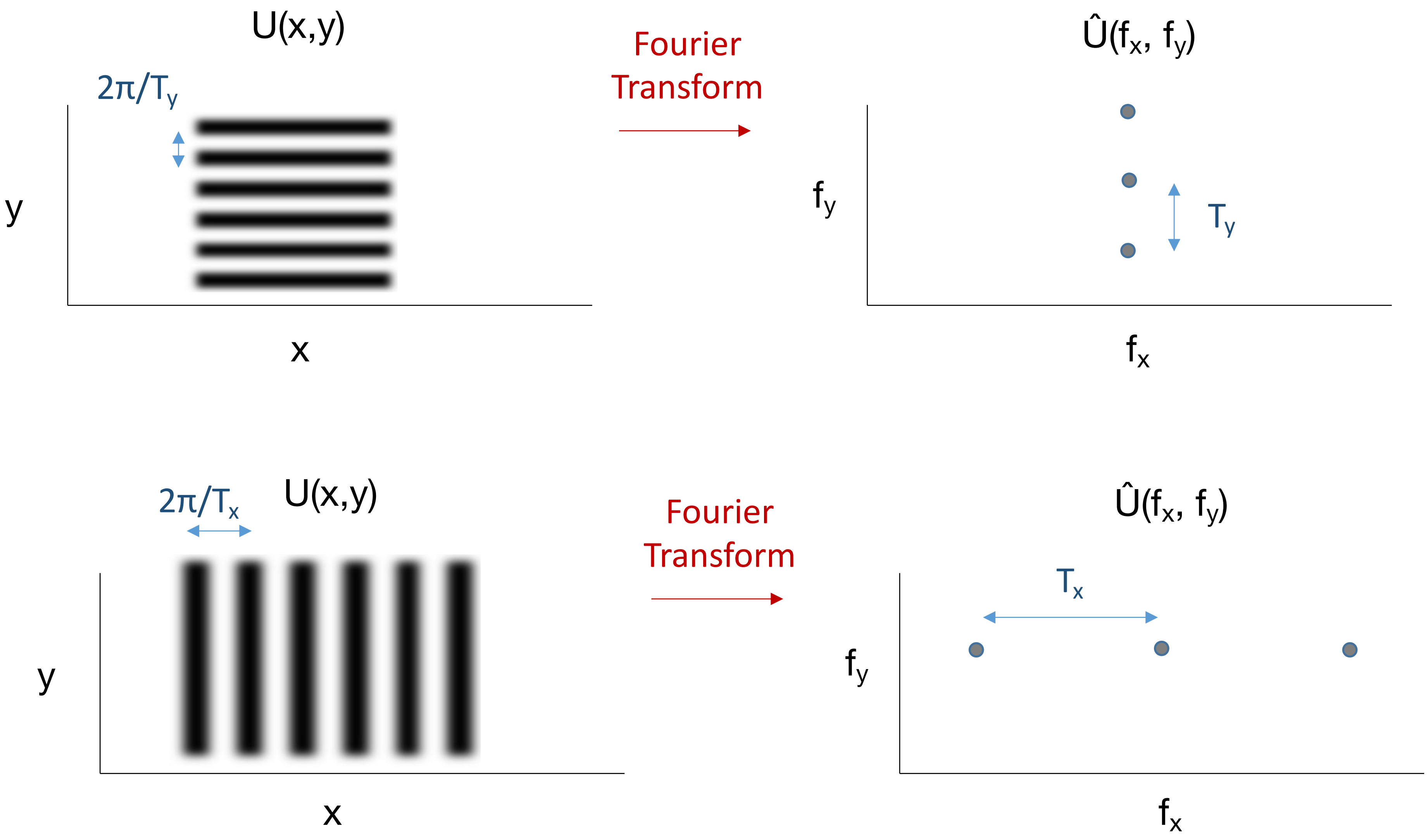
1 everywhere   Not 1 everywhere

This is when incident wave hits the sample with $\theta=0$!

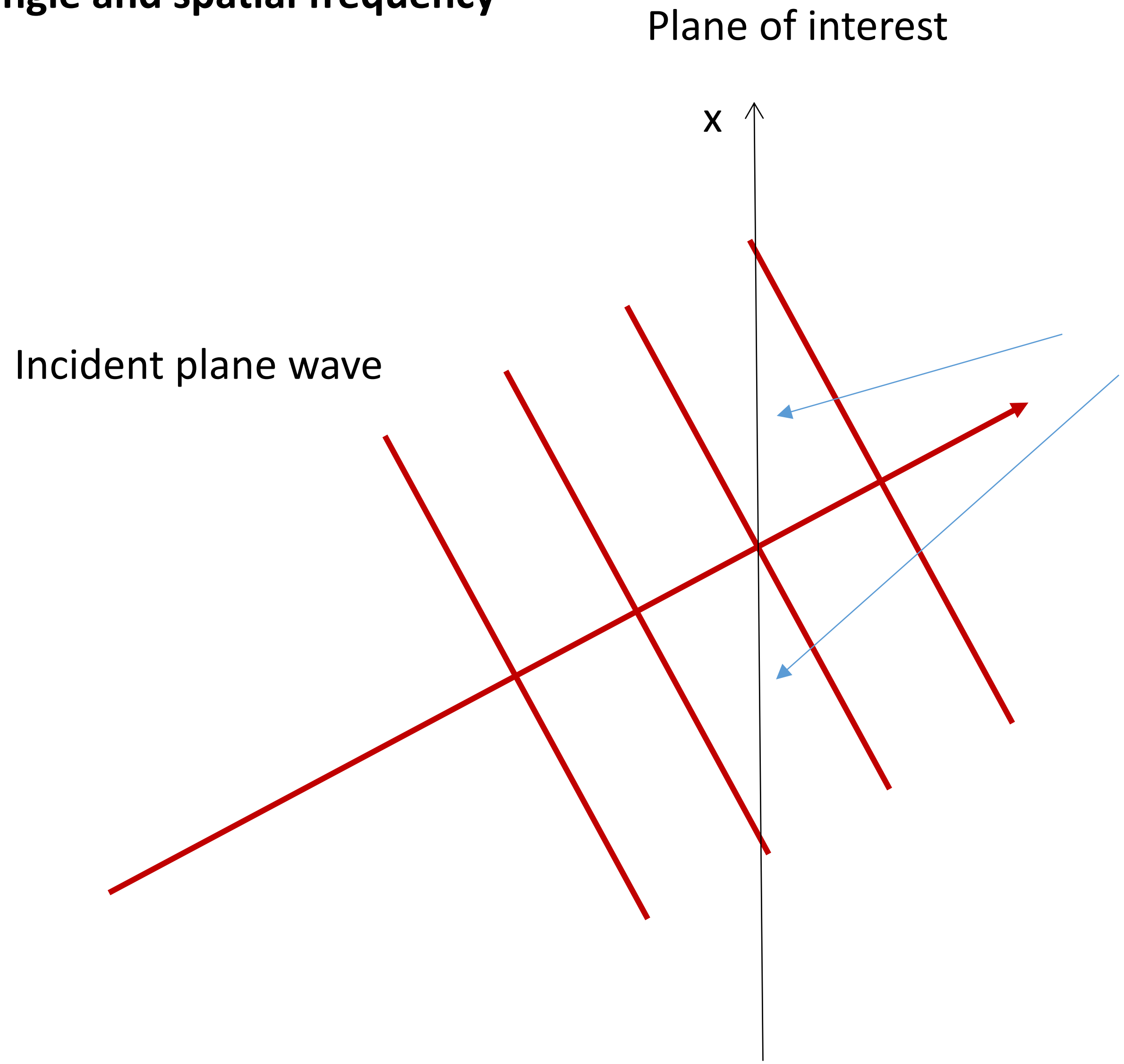
Model of image formation for wave optics (coherent light):



From before: Spatial frequencies = "stripes" within each image



Ray angle and spatial frequency

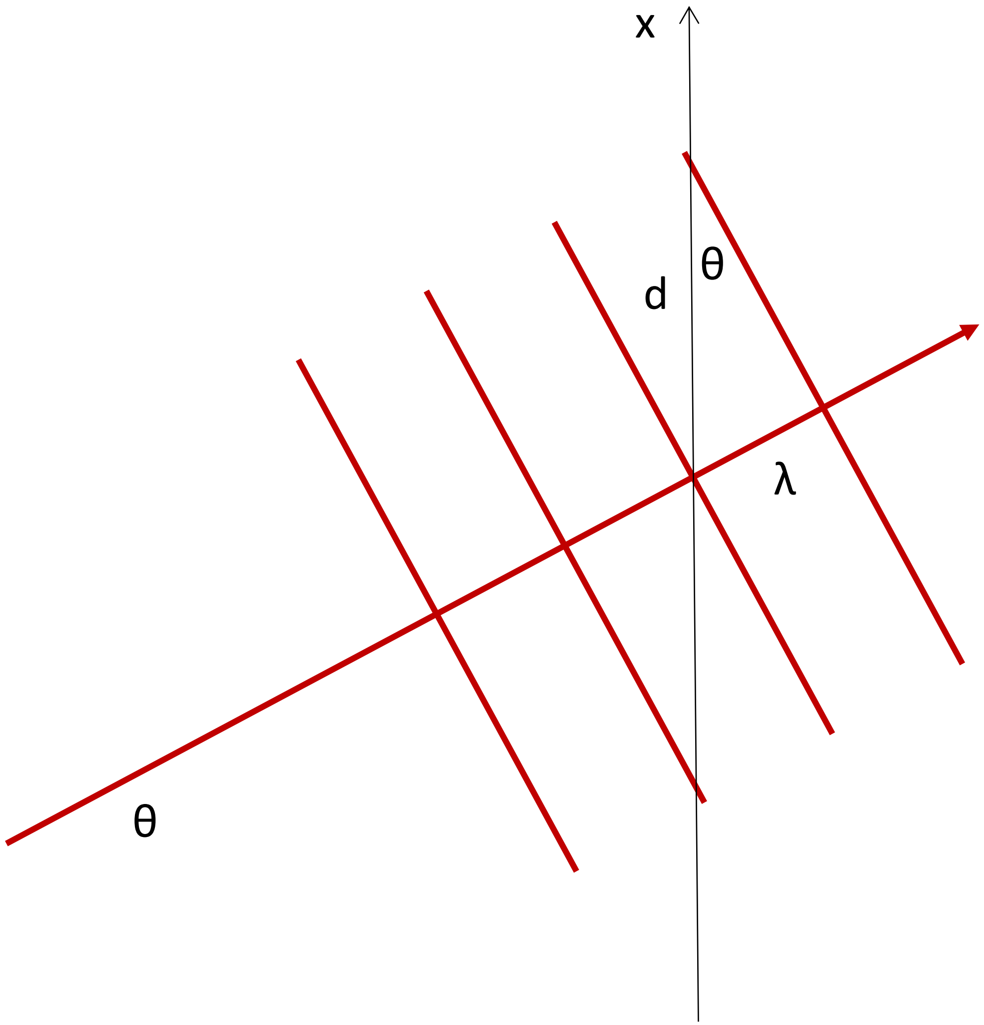


Stripes are for complex fields!

Proportional to distance between subsequent peaks of wave along plane of interest

Ray angle and spatial frequency

Plane of interest

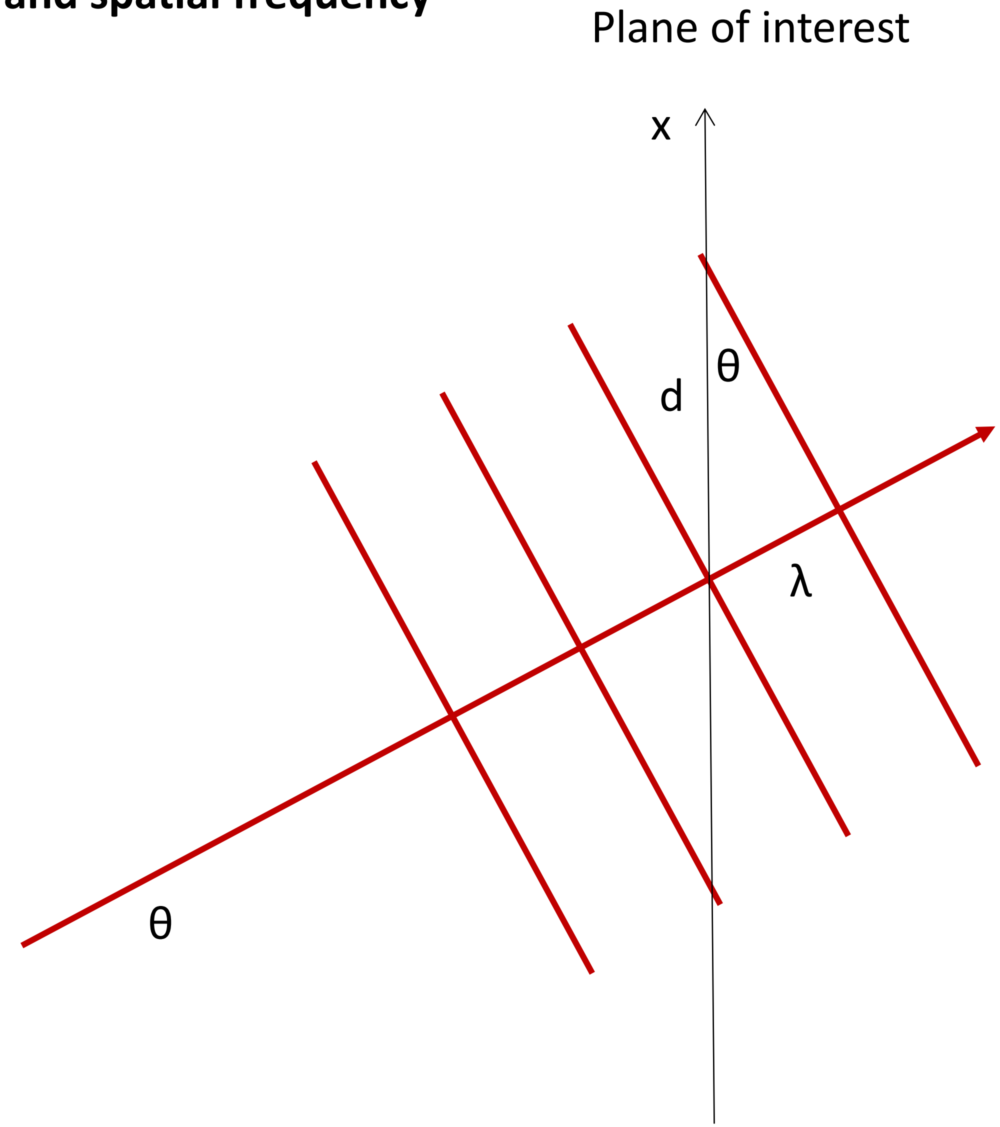


Distance to two crests = spatial period

$$\sin(\theta) = \lambda/d$$

$$d = \lambda/\sin(\theta)$$

Ray angle and spatial frequency



Distance to two crests = spatial period

$\text{Sin}(\theta) = \lambda/d$

$d = \lambda/\text{Sin}(\theta)$

Spatial frequency = 1/spatial period
(number of periods per unit length)

$f_x = 1/d = \text{Sin}(\theta)/\lambda$

Equivalent coordinates in the Fourier domain and at the Fourier plane

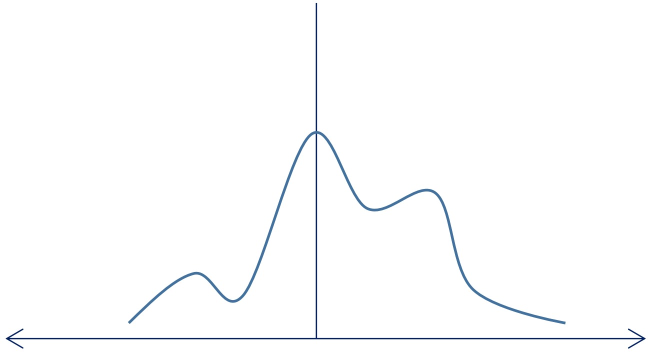


Image plane ray angle θ

$f_x = \text{Sin}(\theta)/\ \lambda$

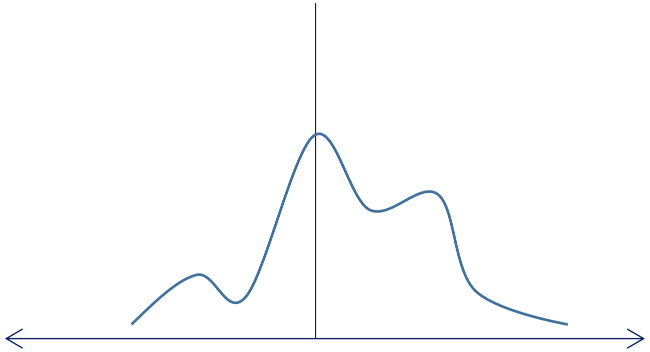


Image plane spatial frequency f_x

$k_x = 2\pi f_x = 2\pi/\ \lambda\ \text{Sin}(\theta)$

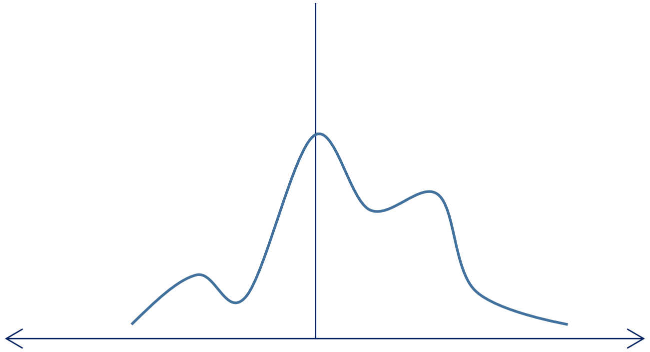
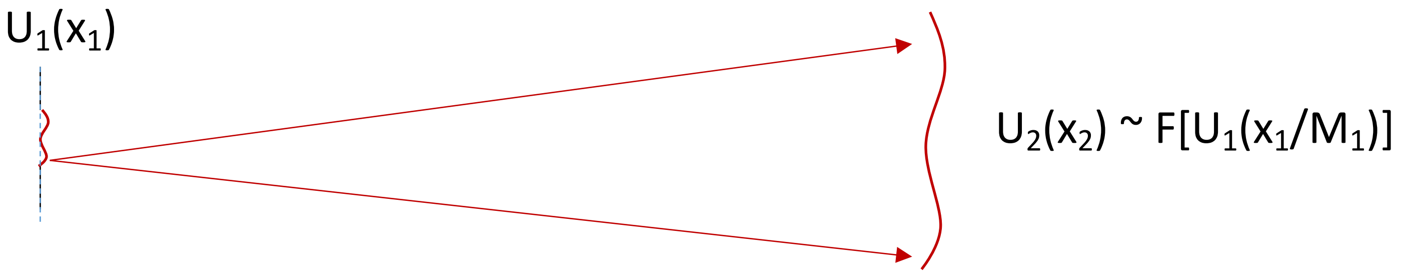


Image plane wavevectors k_x

General rules for applying the Fourier transform in optics

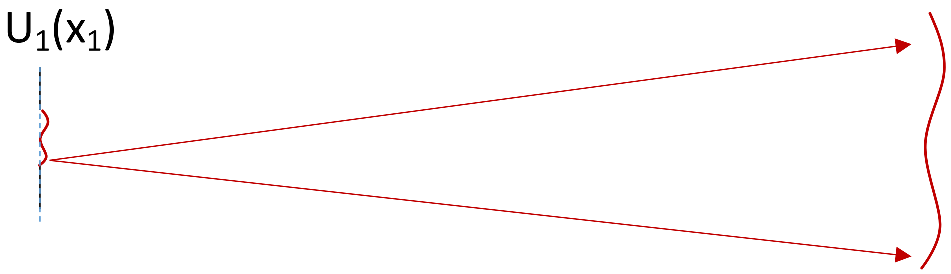
deep imaging

Situation 1: From an object to a plane "really far away"



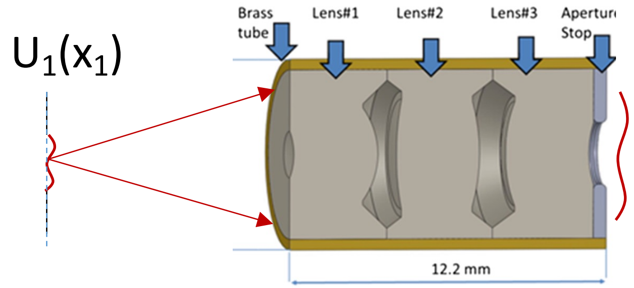
General rules for applying the Fourier transform in optics

Situation 1: From an object to a plane "really far away"



$U_2(x_2) \sim F[U_1(x_1/M_1)]$

Situation 2: From an object to the back focal plane of the microscope objective lens



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General rules for applying the Fourier transform in optics

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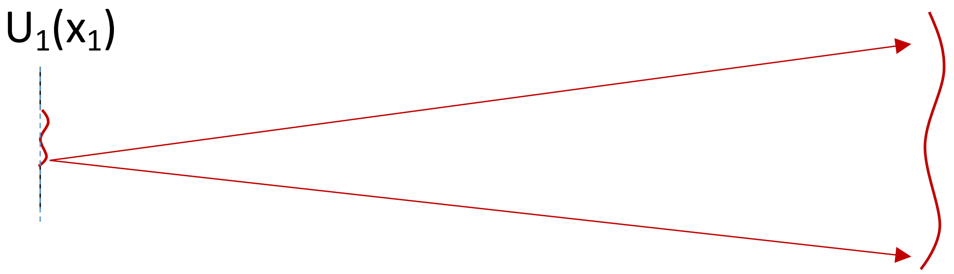

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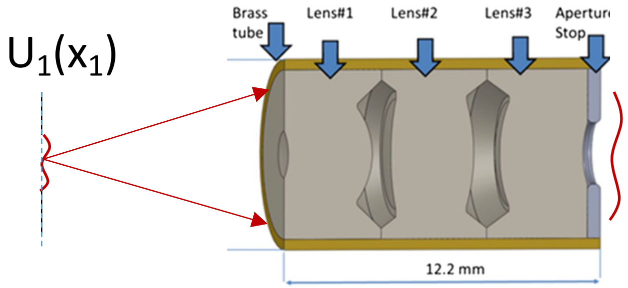
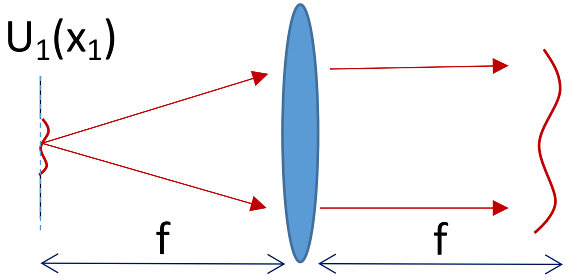


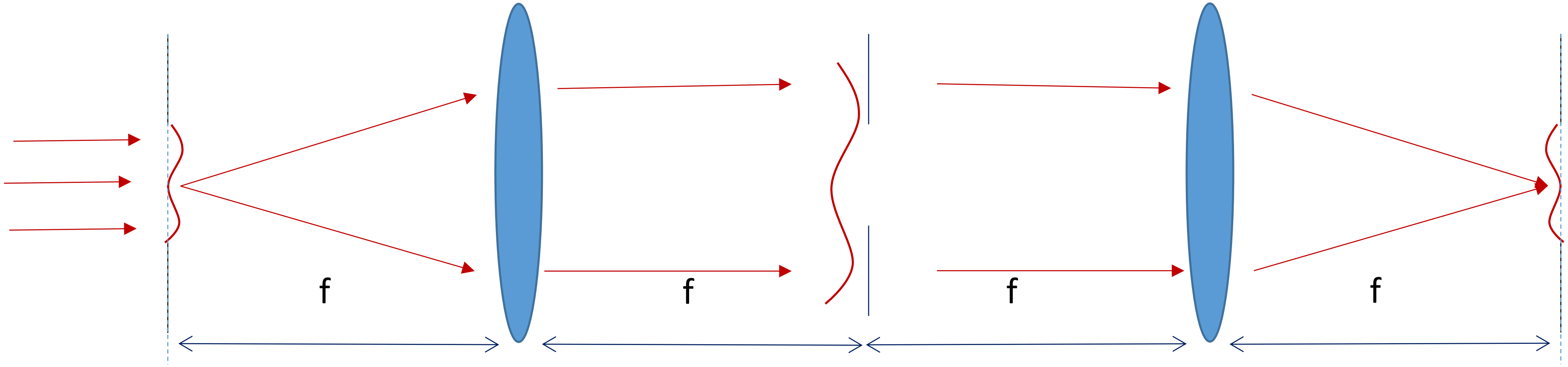
$U_2(x_2) \sim F[U_1(x_1/M_2)]$

Situation 3: From an object to a plane 1 focal length away from a lens (1f-1f system)



$U_2(x_2) \sim F[U_1(x_1/M_3)]$

A more exact model: the 4f optical system



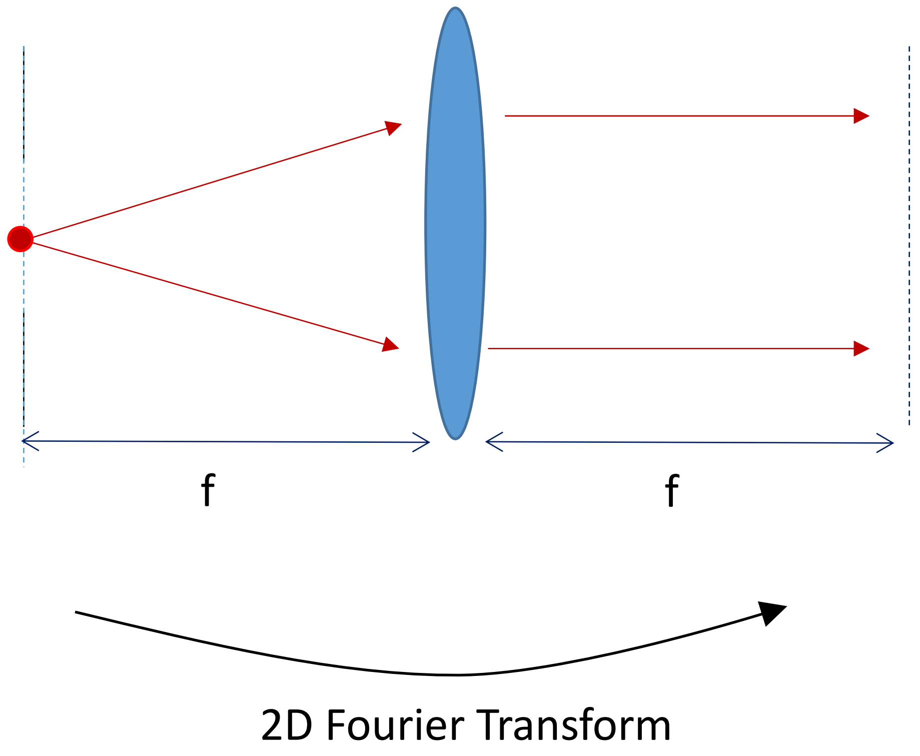
$$E_s(x_s, y_s, 0)$$

$$E_a'(x_d, y_d) = E_a(x_d, y_d) A(x_d, y_d)$$

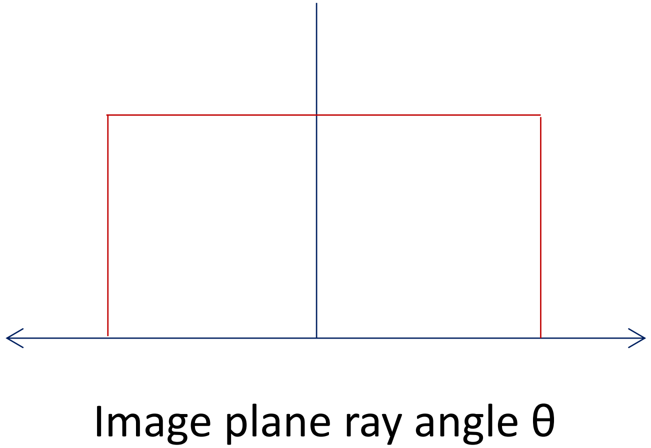
2D Fourier Transform

2D inverse Fourier Transform

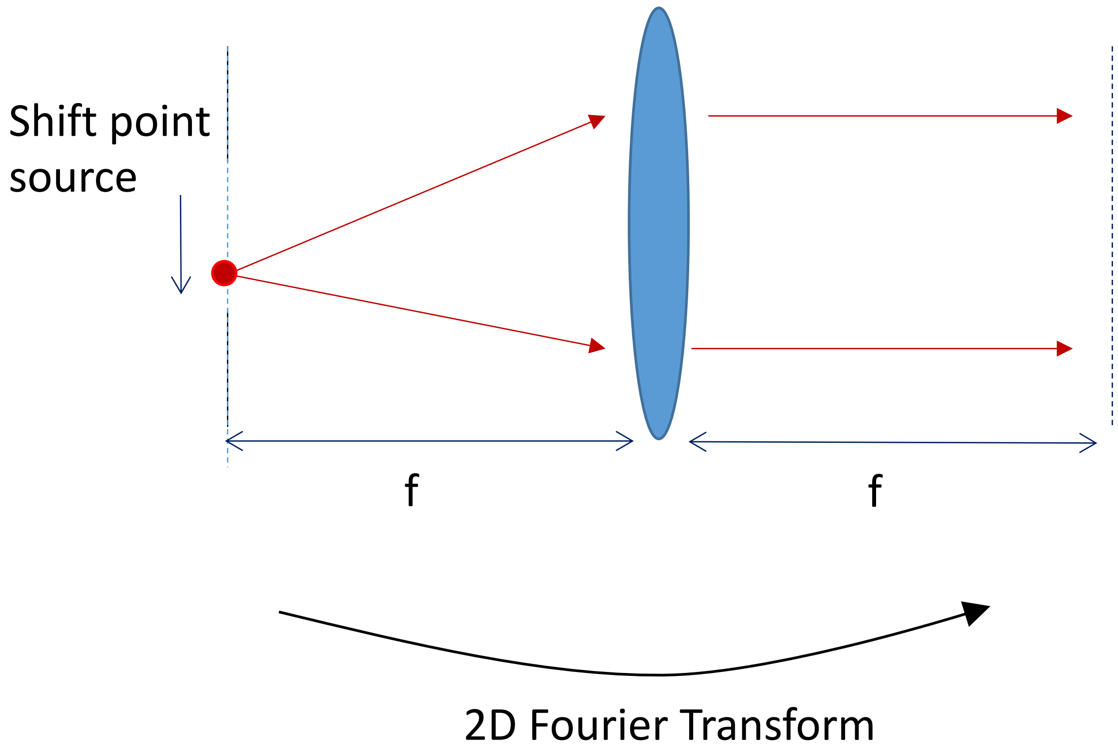
A more exact model: the 4f optical system



The Fourier plane provides a measure of the **ray angles at the image plane**

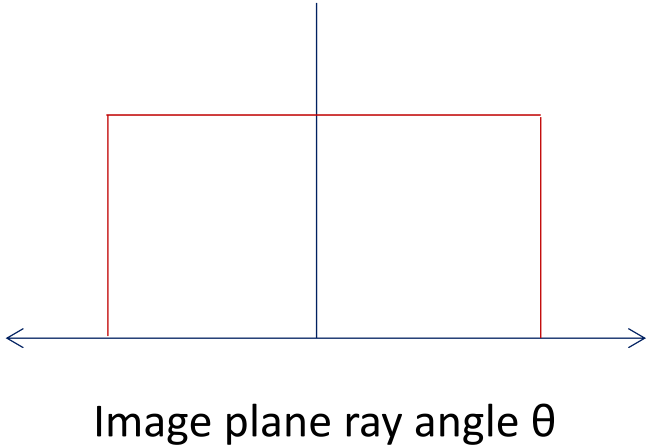


A more exact model: the 4f optical system

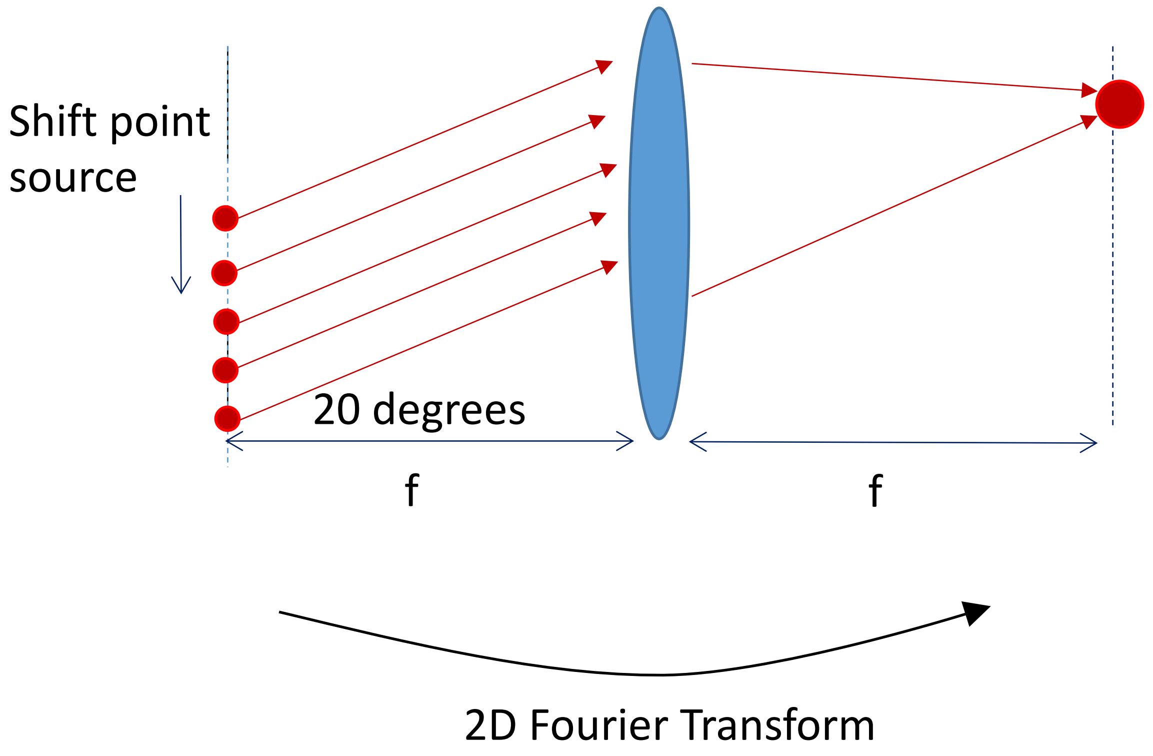


The Fourier plane provides a measure of the **ray angles at the image plane**

Doesn't contain info about spatial distribution light

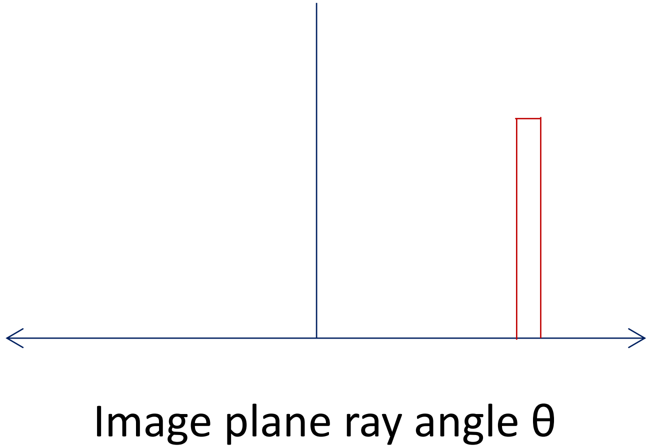


A more exact model: the 4f optical system



The Fourier plane provides a measure of the **ray angles at the image plane**

Rays are leaving image plane at +20 degrees



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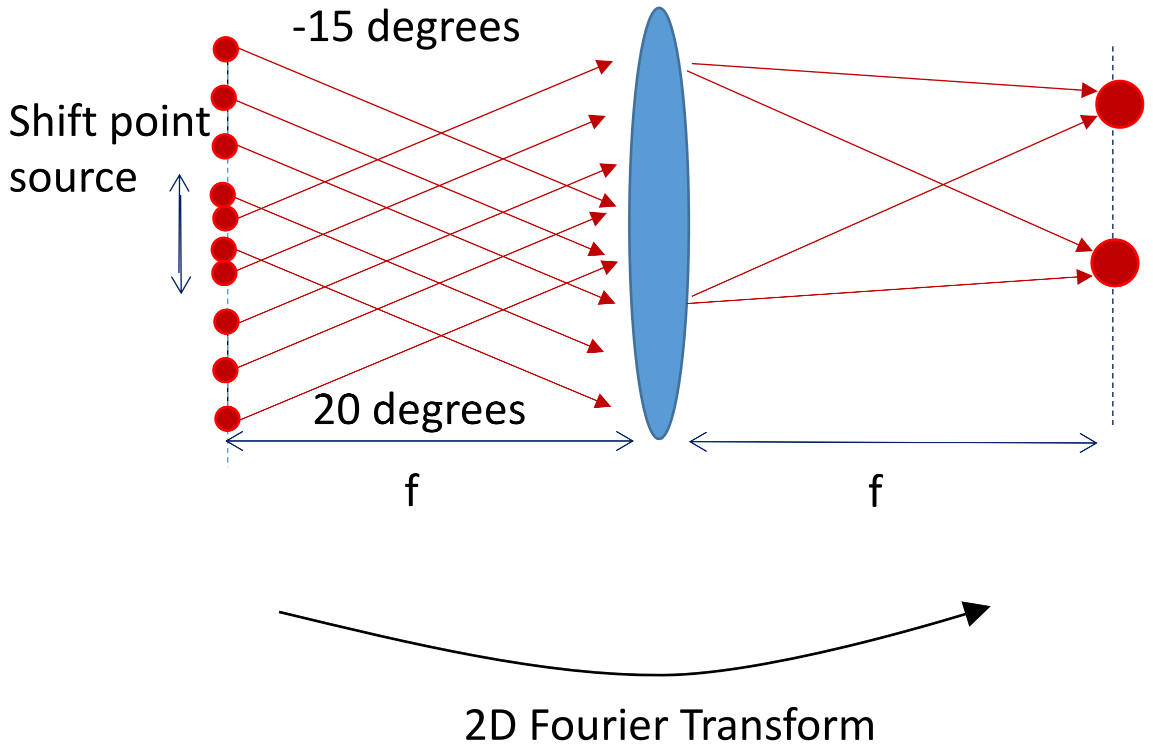
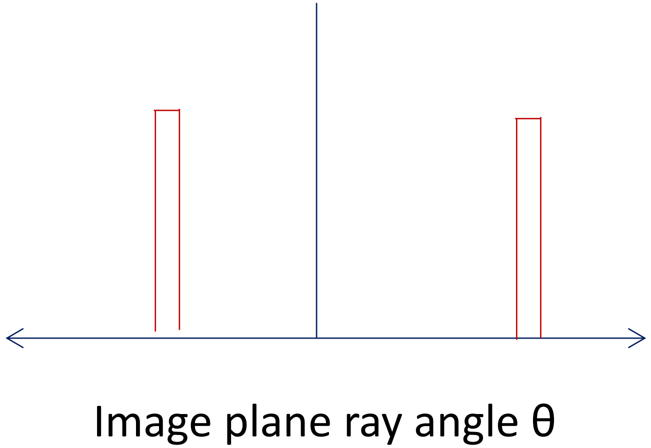


The Fourier plane provides a measure of the **ray angles at the image plane**

Rays are coming in at +20 degrees and -15 degrees



You typically go between 4 functions to describe one imaging system:

Coherent point-spread function $h(x)$

Incoherent point-spread function $h_i(x)$

Coherent transfer function $H(f_x)$

Incoherent transfer function $H_i(f_x)$

$$\begin{tikzcd} h(x) \arrow[r, "|.|^2"] \arrow[d, "F[.]"] & h_i(x) \arrow[d, "F[.]"] \\ H(f_x) \arrow[r, "H^*H"] & H_i(f_x) \end{tikzcd}$$