deep imaging

Lecture 17: Physical layers with coherent fields

Machine Learning and Imaging

BME 590L
Roarke Horstmeyer

Review: how do coherent EM waves propagate?

Maxwell's equations without any charge

$$\nabla \times \vec{\mathcal{E}} = -\mu \frac{\partial \vec{\mathcal{H}}}{\partial t}$$

$$\nabla \times \vec{\mathcal{H}} = \epsilon \frac{\partial \vec{\mathcal{E}}}{\partial t}$$

$$\nabla \cdot \epsilon \vec{\mathcal{E}} = 0$$

$$\nabla \cdot \mu \vec{\mathcal{H}} = 0.$$

1. Take the curl of both sides of first equation
2. Substitute 2nd and 3rd equation
3. Arrive at the wave equation:

$$\nabla^2 \vec{\mathcal{E}} - \frac{n^2}{c^2} \frac{\partial^2 \vec{\mathcal{E}}}{\partial t^2} = 0 \qquad n = \left(\frac{\epsilon}{\epsilon_0}\right)^{1/2} \qquad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

Review: how do coherent EM waves propagate?

Considering light that isn't pulsed over time, we can use the following solution:

$$u(P, t) = A(P)\cos[2\pi\nu t + \phi(P)]$$

$$u(P, t) = Re\{U(P)\exp(-j2\pi\nu t)\},$$

With this particular solution, we get the following important time-independent equation:

Helmholtz Equation

$$\boxed{(\nabla^2 + k^2)U = 0.} \qquad k = 2\pi n\frac{\nu}{c} = \frac{2\pi}{\lambda},$$

This is an important equation in physics. We won't go into the details, but it leads to the Huygen-Fresnel principle:

$$U(P_2) = \frac{1}{j\lambda}\iint_{\Sigma} U(P_1)\frac{\exp(jkr_{21})}{r_{21}}\cos\theta\, ds$$

Plane-to-plane light propagation via the "paraxial approximation"

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We are usually concerned about propagation between two planes (almost always in an optical system):

Paraxial approximation:

$$\nabla_{\perp}^{2} U + 2ik\frac{dU}{dz} = 0$$

Substitute in $U(P) = E(x, y, z)e^{ikz}$ and crank the wheel,

$$\nabla_{\perp}^{2} E + 2ik\frac{dE}{dz} + 2k^2 E = 0$$

Paraxial Helmholtz Equation. This has an exact integral solution:

$$\boxed{E(x, y, z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{+\infty} E(x', y', 0)e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]} dx' dy'}$$

Fresnel diffraction integral

This is how light propagates from one plane to the next. It's a convolution!

From the Fresnel approximation to the Fraunhofer approximation

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Fresnel Approximation:

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{+\infty} E(x',y',0) e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]} dx'\,dy'$$

Lets assume that the second plane is "pretty far away" from the first plane. Then,

$$\boxed{z > \frac{2D^2}{\lambda}}$$

1. Expand the squaring

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint E(x',y',0) e^{\frac{ik}{2z}(x^2+y^2)} e^{\frac{ik}{2z}(x'^2+y'^2)} e^{\frac{ik}{2z}(xx'+yy')} dx'\,dy'$$

2. Front term comes out, assume second term goes away, then,

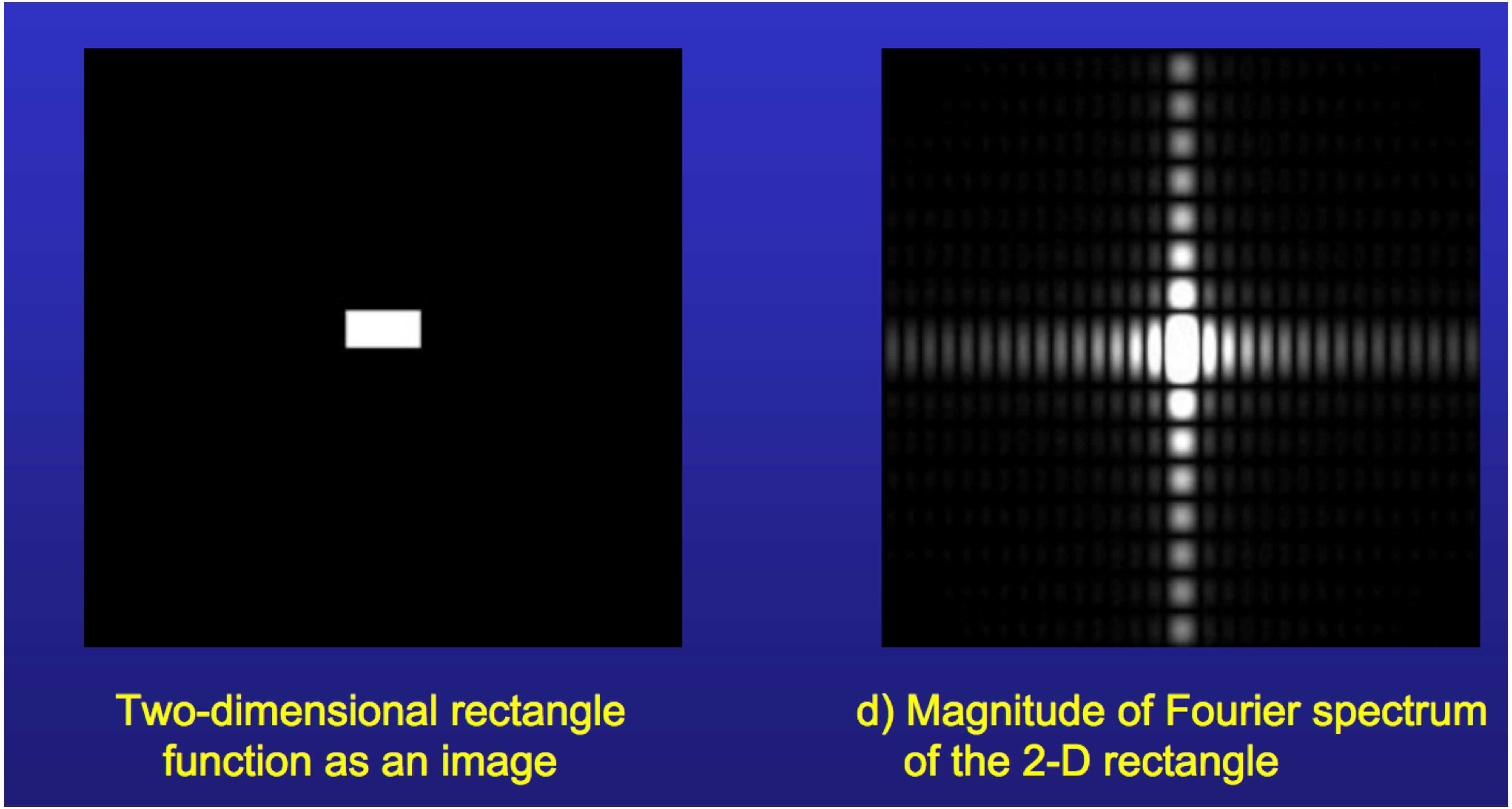
$$\boxed{E(x,y,z) = C \iint E(x',y',0) e^{\frac{ik}{2z}(xx'+yy')} dx'\,dy'}$$

$$C = \frac{e^{ikz}}{i\lambda z} e^{\frac{ik}{2z}(x^2+y^2)}$$

Fraunhofer diffraction is a Fourier transform!!!!!!!

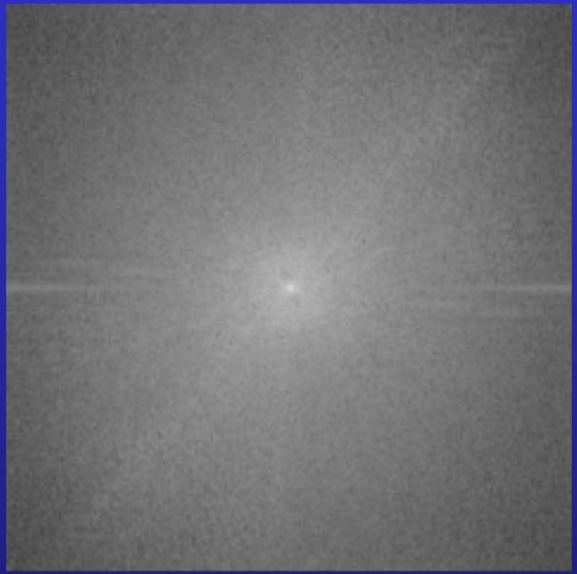
This is the aperture

This is what you see far away

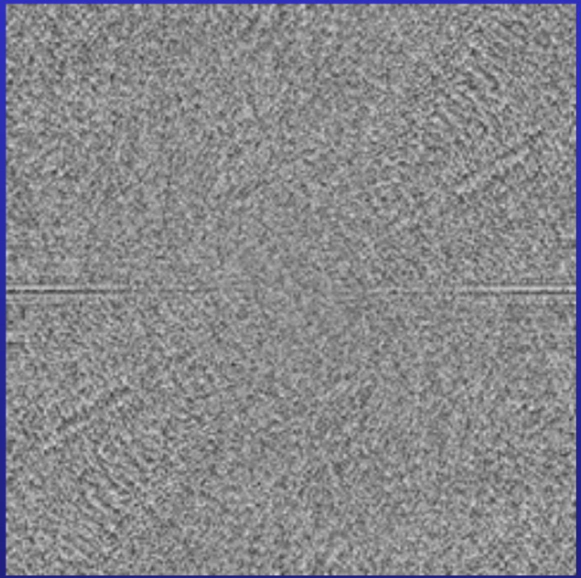


Cheetah

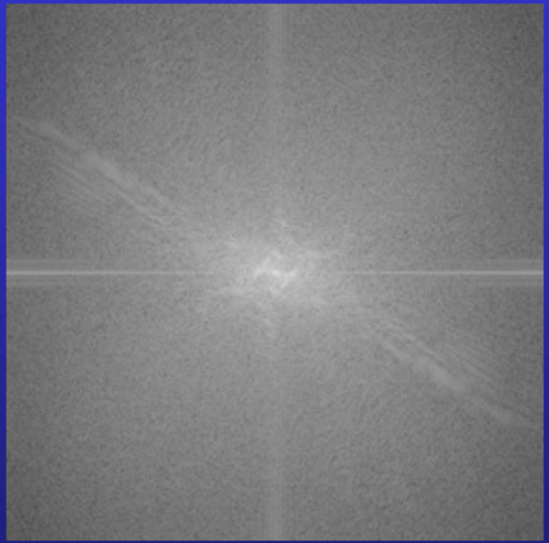
Zebra



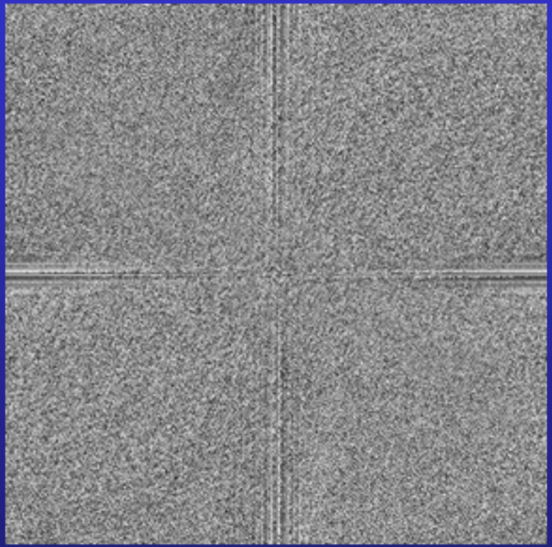
magnitude of cheetah



phase of cheetah



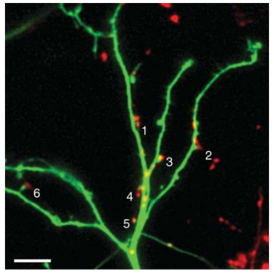
magnitude of zebra

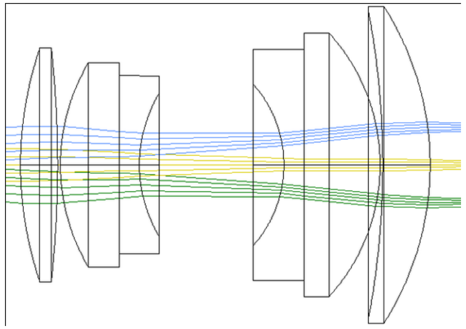


phase of zebra

Summary of two models for image formation

- **Interpretation #1: Radiation (*Incoherent*)**
- Model: Rays





- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

Mathematical model of for incoherent image formation

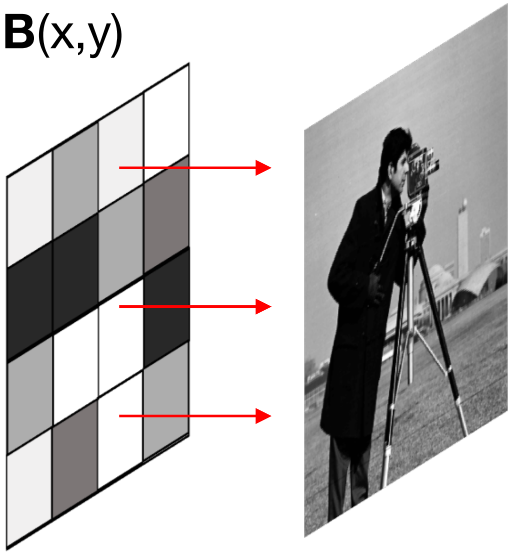
- All quantities are real, and non-negative

Object absorption:

$\mathbf{S_0}(x,y)$

Illumination brightness:

$\mathbf{B}(x,y)$



Mathematical model of for incoherent image formation

- *All quantities **B** , **S₀**, **H** are real and and non-negative*

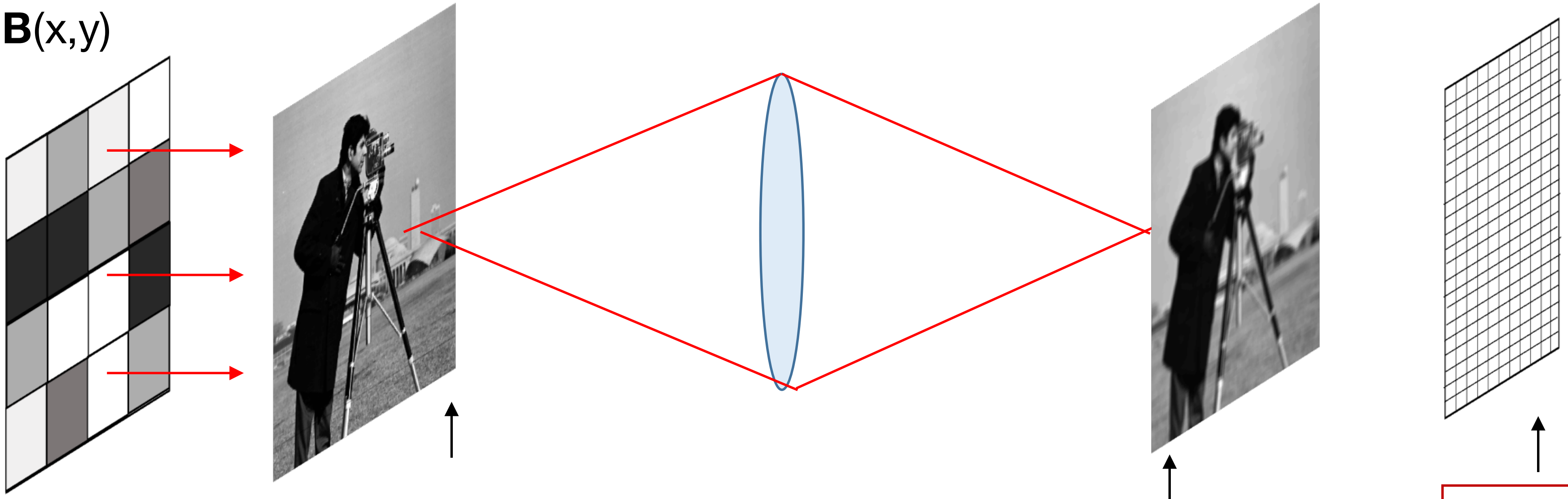
Object absorption:

$\mathbf{S_0}(x,y)$

Illumination brightness:

$\mathbf{B}(x,y)$

Photons (intensity) hits\
detector



$\mathbf{B\ S_0}$

multiplication

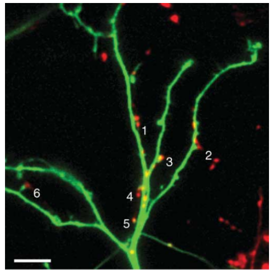
$(\mathbf{B\ S_0}) * \mathbf{h}$

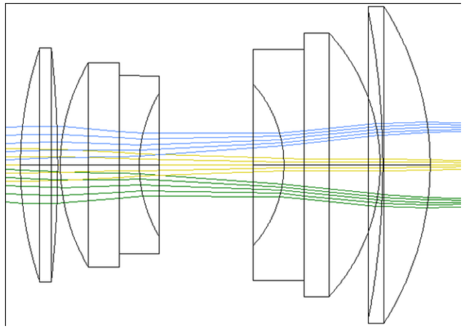
convolution

$$\boxed{\mathbf{I_s = H\ B\ S_0}}$$

Summary of two models for image formation

- **Interpretation #1: Radiation (*Incoherent*)**
- Model: Rays





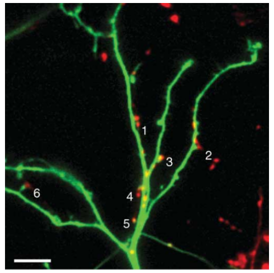
- Real, non-negative
- Models absorption and brightness

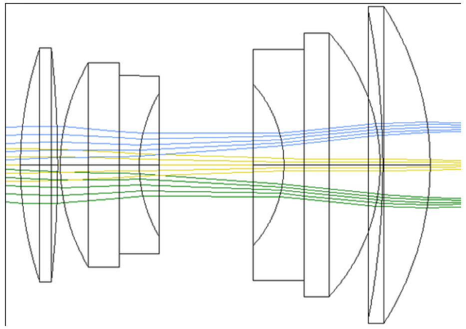
$$I_{tot} = I_1 + I_2$$

$$\mathbf{I_s = H\ B\ S_0}$$

Summary of two models for image formation

- **Interpretation #1: Radiation (*Incoherent*)**
- Model: Rays



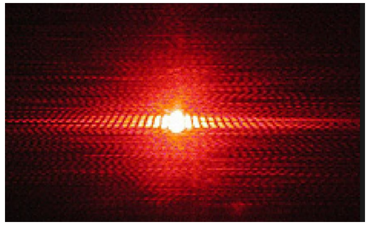


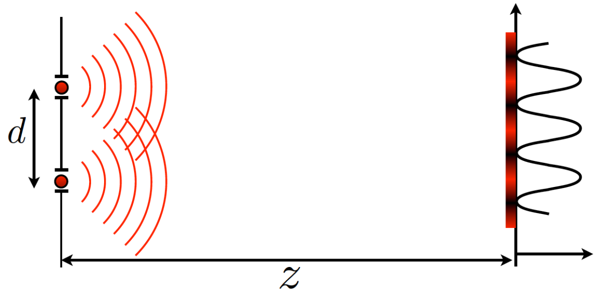
- Real, non-negative
- Models absorption and brightness

$I_{tot} = I_1 + I_2$

$\mathbf{I_s = H\,B\,S_0}$

- **Interpretation #2: Electromagnetic wave (*Coherent*)**
- Model: Waves





- Complex field
- Models Interference

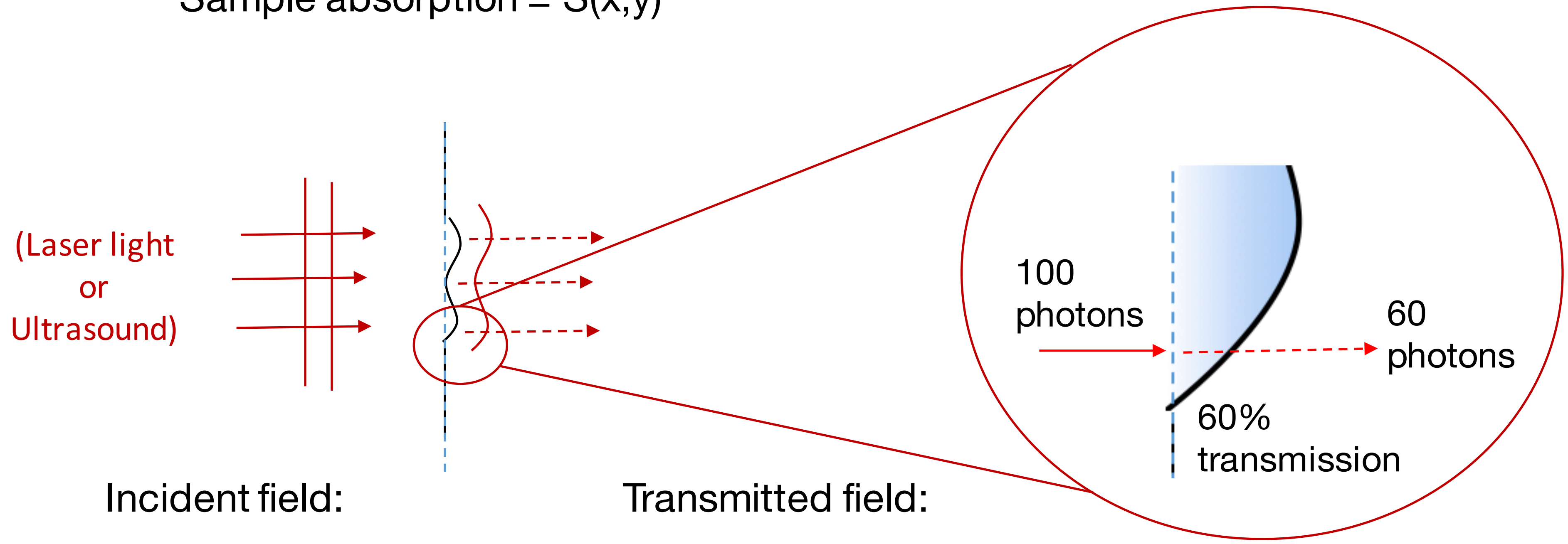
$E_{tot} = E_1 + E_2$

Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = S(x,y)

Point #1: Amplitudes behave just like before



$$A_t(x,y) = A_i(x,y)\,S(x,y)$$

Incident field:

$C(x,y) = A_i(x,y)$

Transmitted field:

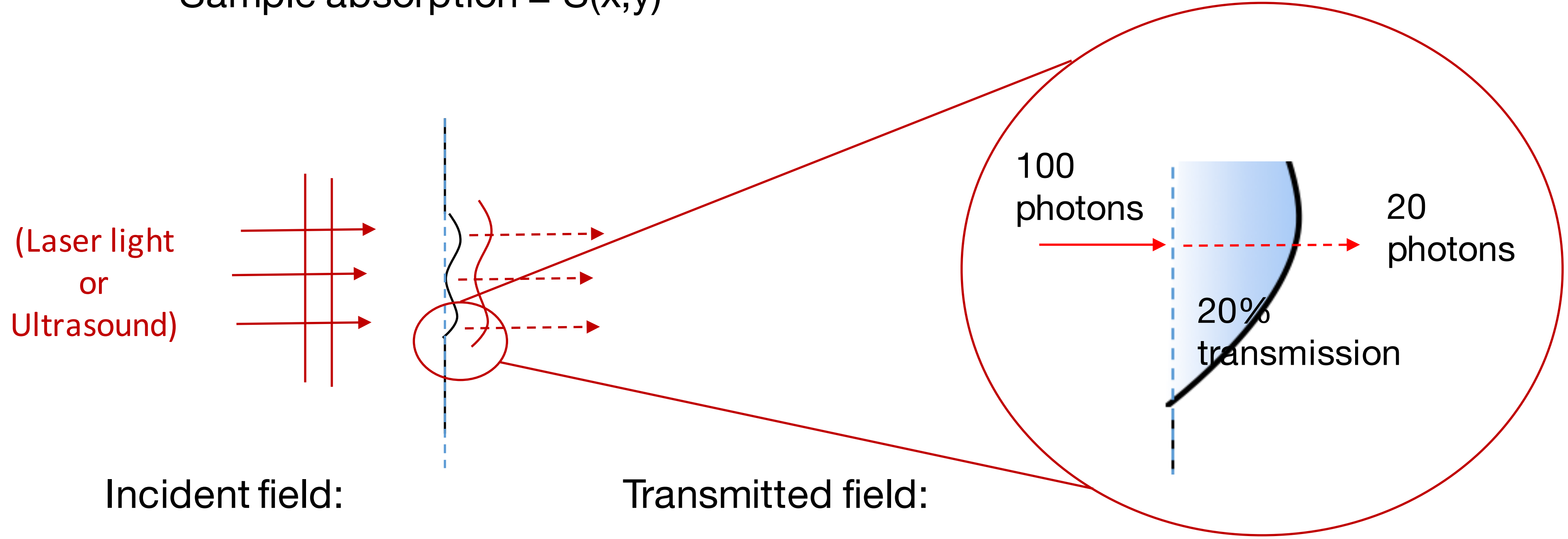
$U(x,y) = A_t(x,y) = ?$

Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = S(x,y)

Point #1: Amplitudes behave just like before



$\boxed{A_t(x,y) = A_i(x,y)\, S(x,y)}$

Incident field:

$C(x,y) = A_i(x,y)$

Transmitted field:

$U(x,y) = A_t(x,y) = A_i(x,y)\, S(x,y)$

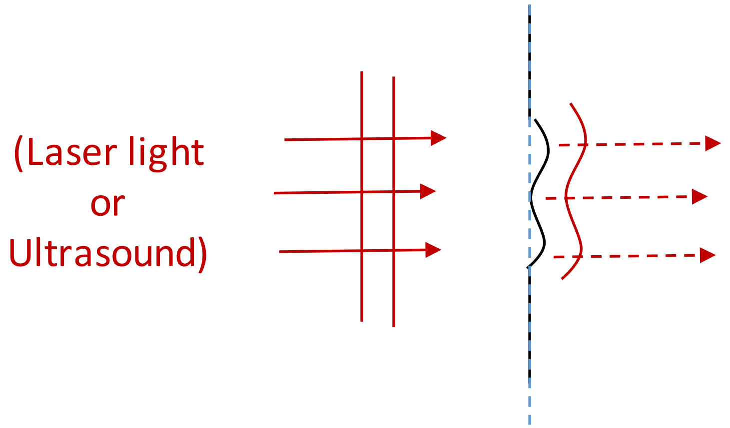
Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = $S(x,y)$

Sample phase delay = $\exp[ik\varphi(x,y)]$

New: complex phase delay
- Needed to represent wave
- Represents wave delay across space



Incident field:

$C(x,y) = A_i(x,y) \exp[ik\varphi_i(x,y)]$

Transmitted field:

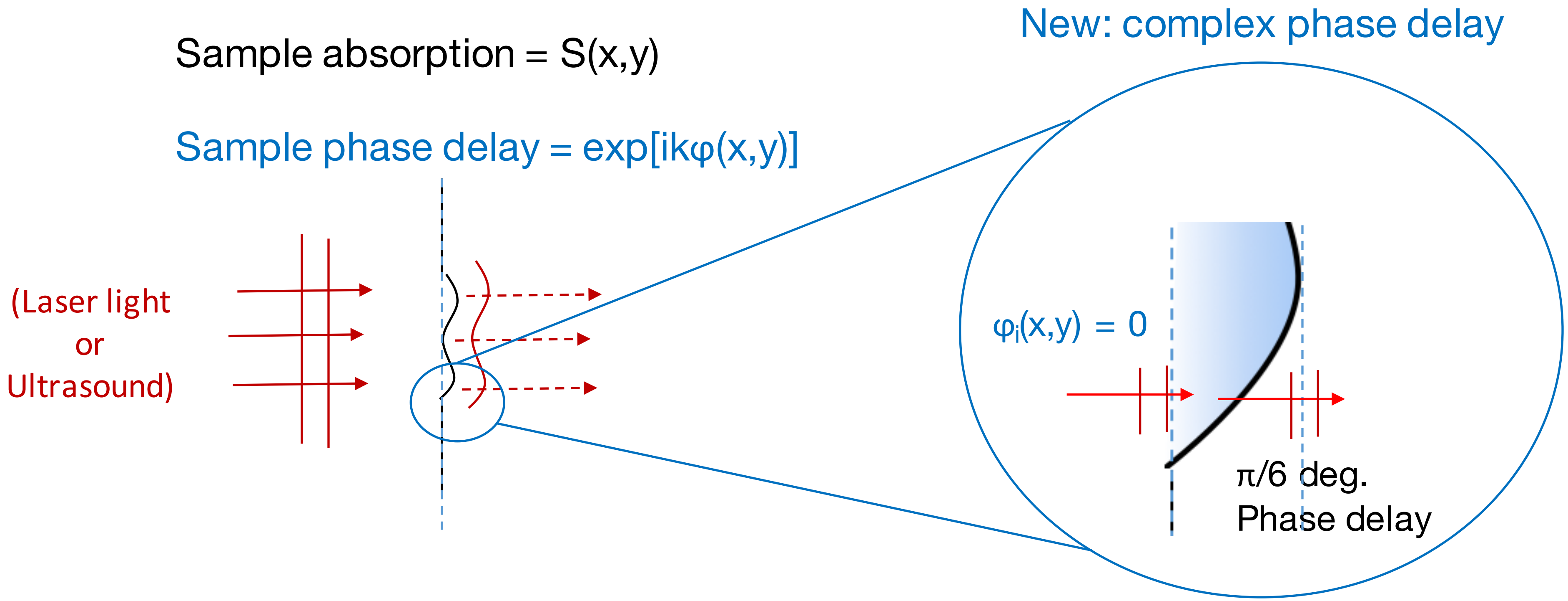
$U(x,y) = A_i(x,y)\, S(x,y) \exp[ik\varphi_t(x,y)]$

Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

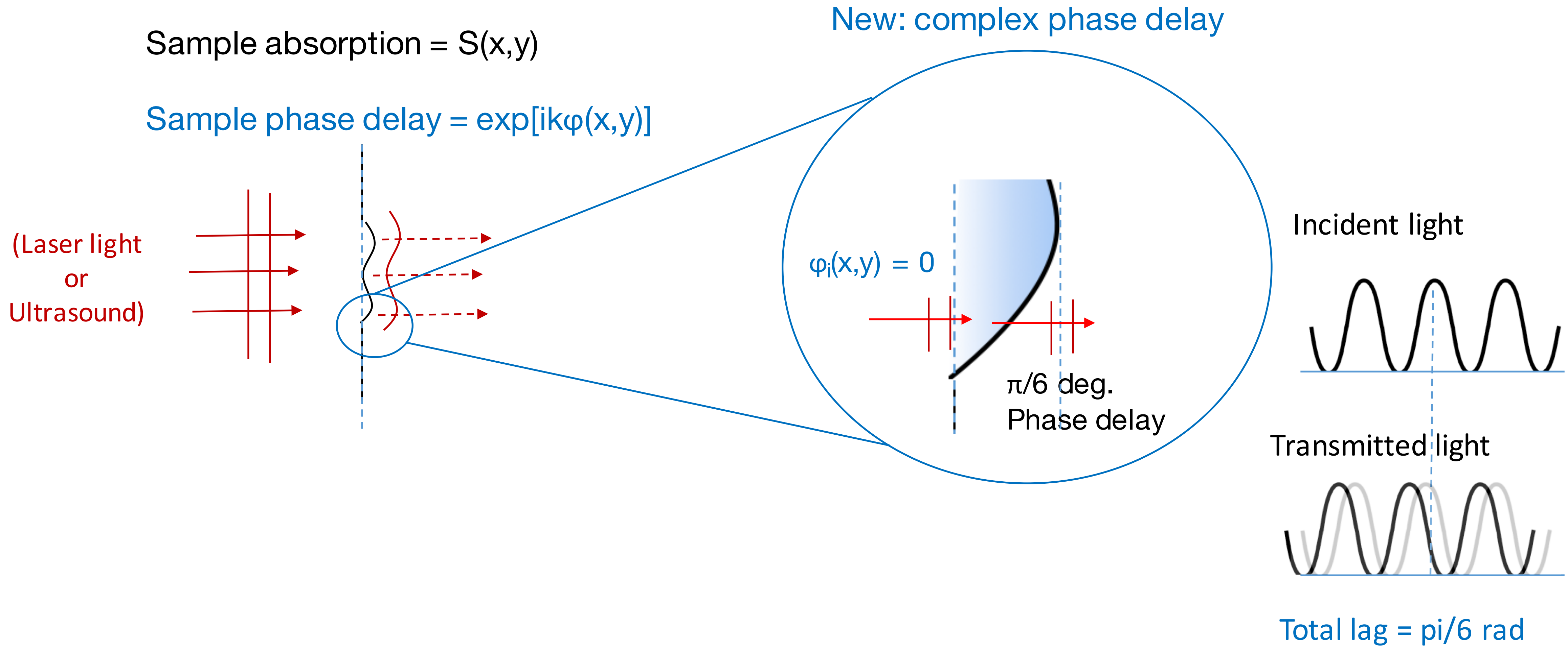
Sample absorption = S(x,y)

Sample phase delay = exp[ikφ(x,y)]



Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase



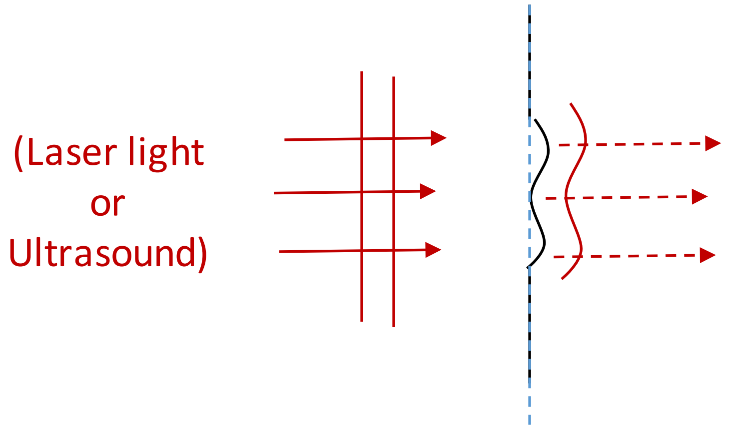
Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = S(x,y)

Sample phase delay = exp[ikφ(x,y)]

Output phase is sum of phase delays, product of phasors



$$\varphi_t(x,y) = \varphi(x,y) + \varphi_i(x,y)$$

$$\exp[ik\varphi_t(x,y)] = \exp[ik\varphi_i(x,y)]\ \exp[ik\varphi(x,y)]$$

Incident field:

$C(x,y) = A_i(x,y)\ \exp[ik\varphi_i(x,y)]$

Transmitted field:

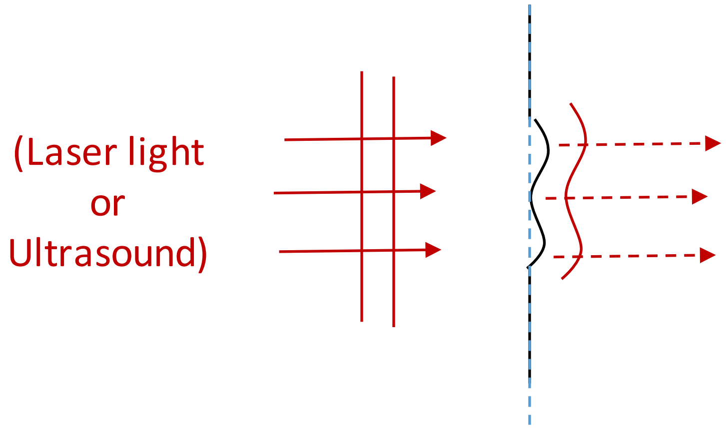
$U(x,y) = A_i(x,y)\ S(x,y)\ \exp[ik\varphi_i(x,y)]\ \exp[ik\varphi(x,y)]$

Mathematical model of for coherent image formation

- *Summary: coherent light hitting a thin object also modeled as a multiplication, but now with complex-valued matrices C(x,y) and S(x,y)*

Sample absorption = S(x,y)

Sample phase delay = $\exp[ik\varphi(x,y)]$



Conclusion:

Transmitted field = incident field x complex sample :

$U(x,y) = C(x,y)\, S(x,y)\, \exp[ik\varphi(x,y)]$

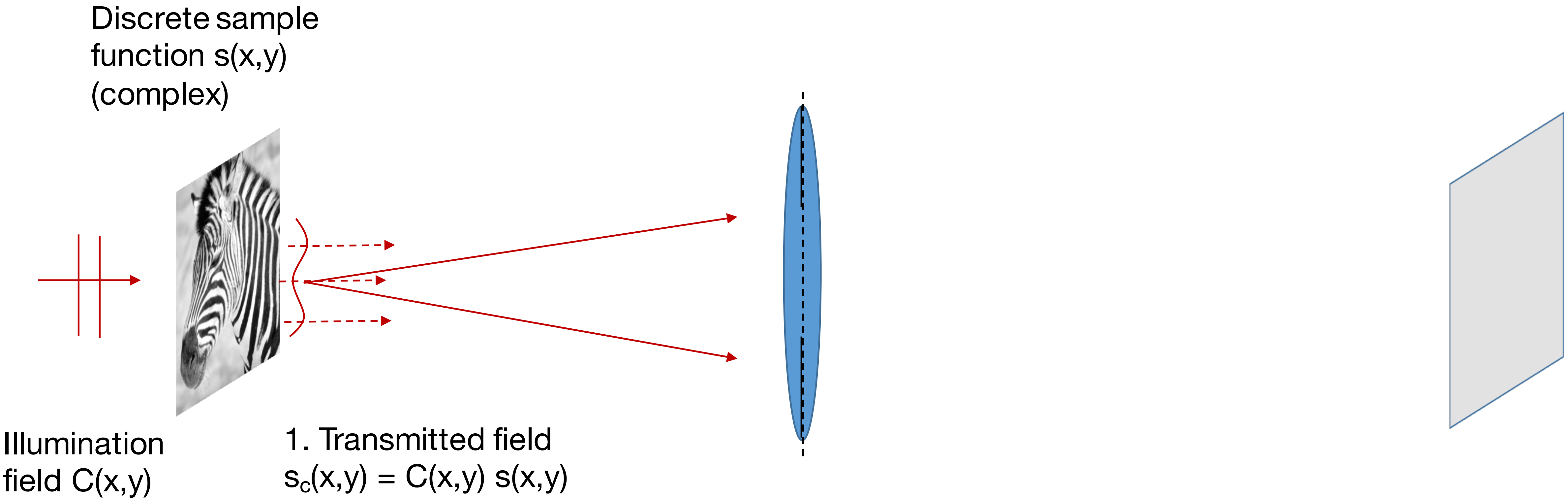
Incident field:

$C(x,y) = A_i(x,y) \exp[ik\varphi_i(x,y)]$

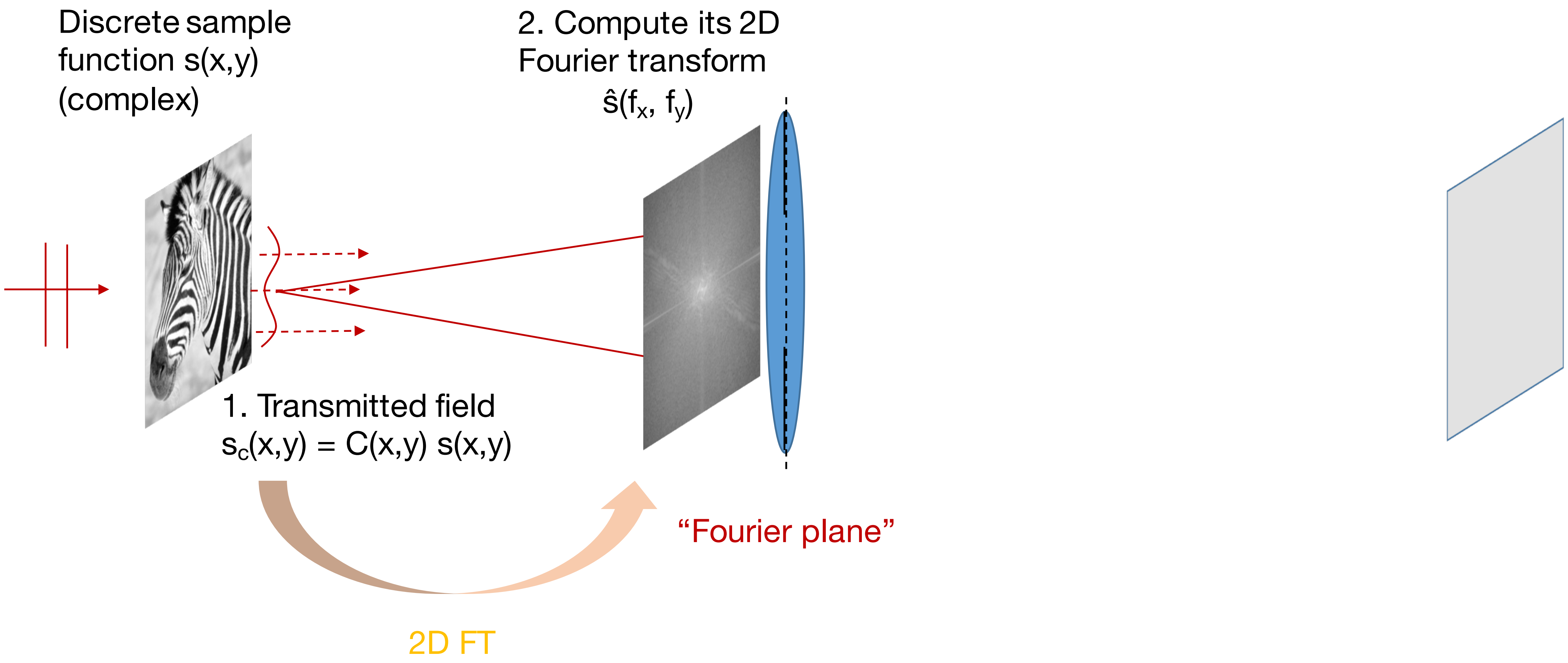
Transmitted field:

$U(x,y) = A_i(x,y)\, S(x,y) \exp[ik\varphi_i(x,y)] \exp[ik\varphi(x,y)]$

Model of image formation for wave optics (coherent light):

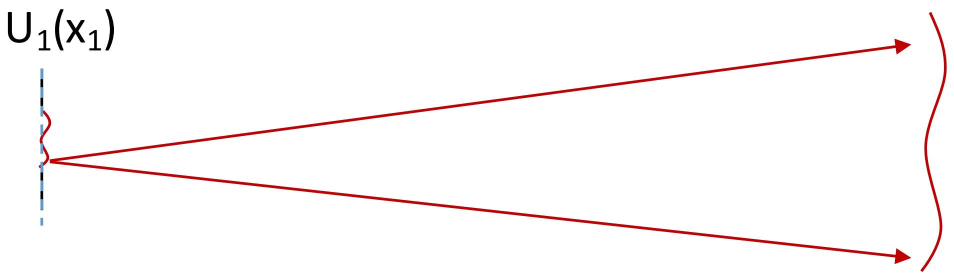


Model of image formation for wave optics (coherent light):



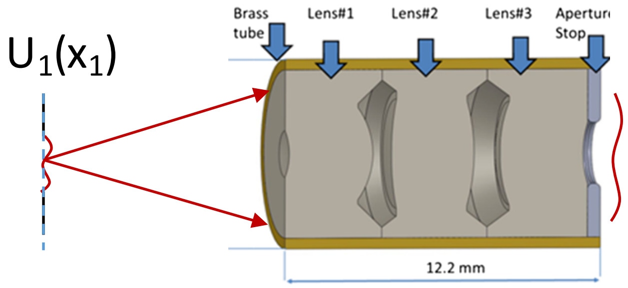
General rules for applying the Fourier transform in optics

Situation 1: From an object to a plane "really far away"



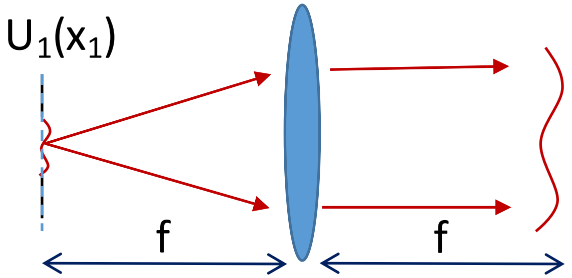
$U_2(x_2) \sim F[U_1(x_1/M_1)]$

Situation 2: From an object to the back focal plane of the microscope objective lens



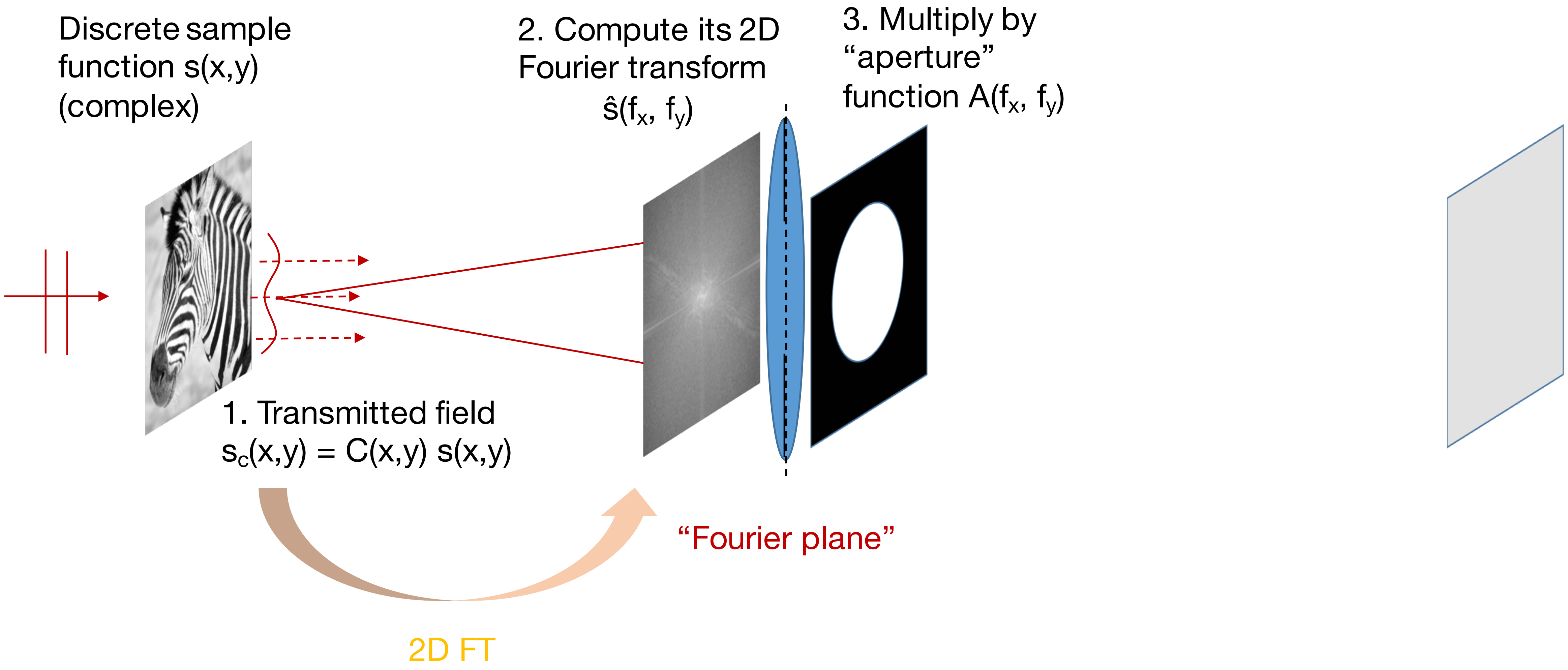
$U_2(x_2) \sim F[U_1(x_1/M_2)]$

Situation 3: From an object to a plane 1 focal length away from a lens (1f-1f system)

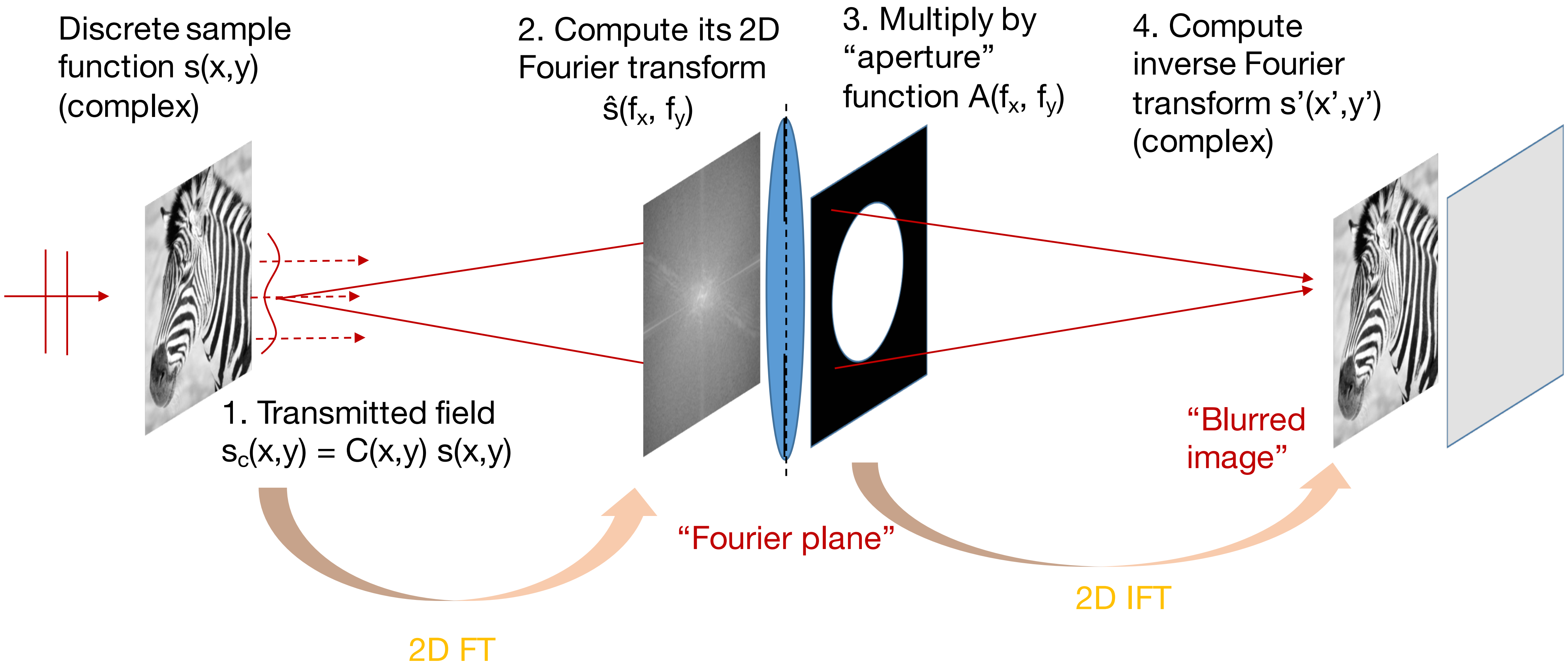


$U_2(x_2) \sim F[U_1(x_1/M_3)]$

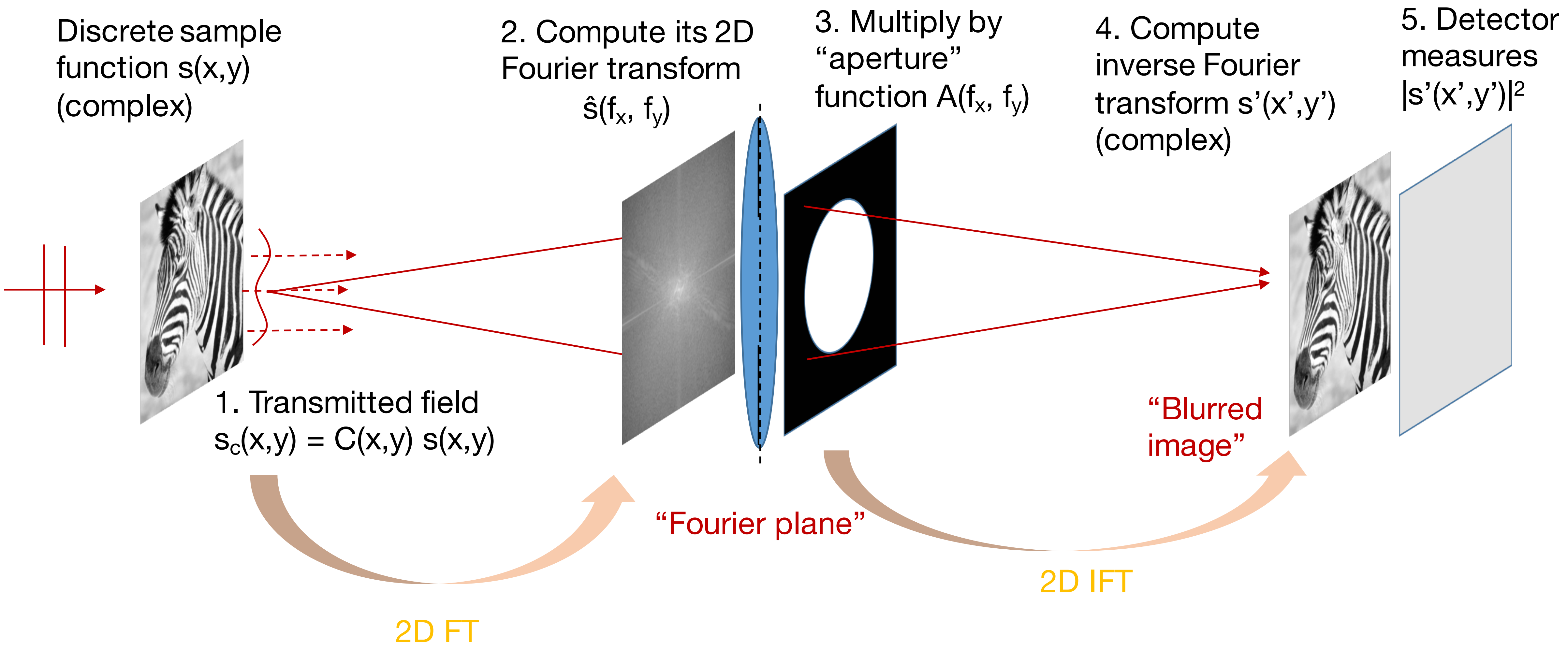
Model of image formation for wave optics (coherent light):



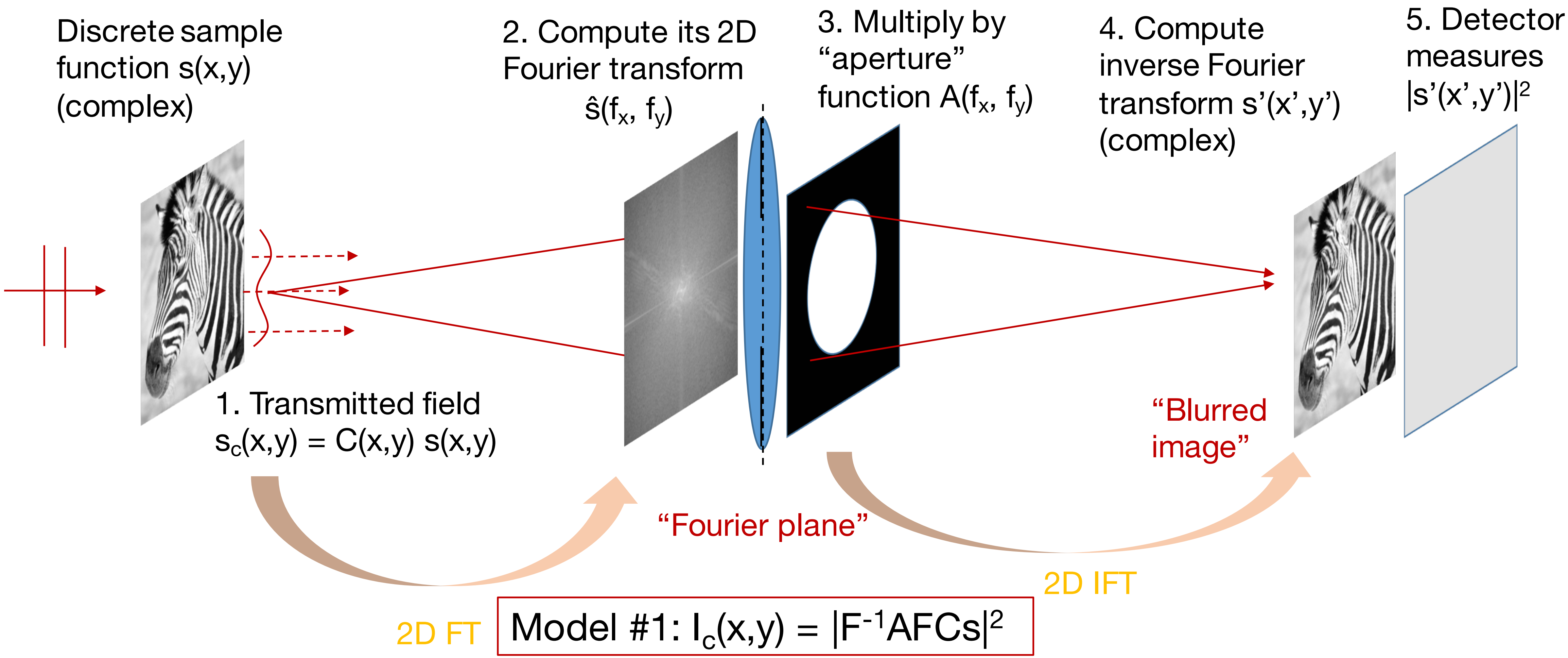
Model of image formation for wave optics (coherent light):



Model of image formation for wave optics (coherent light):

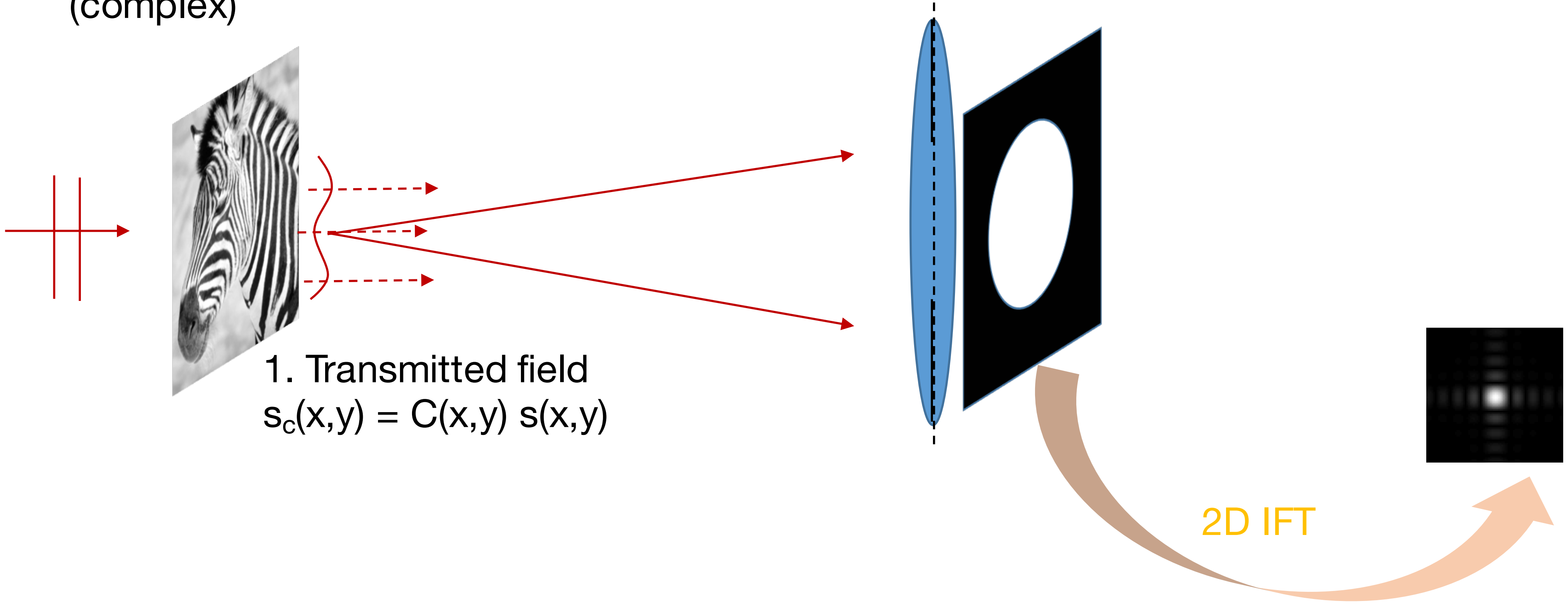


Model of image formation for wave optics (coherent light):

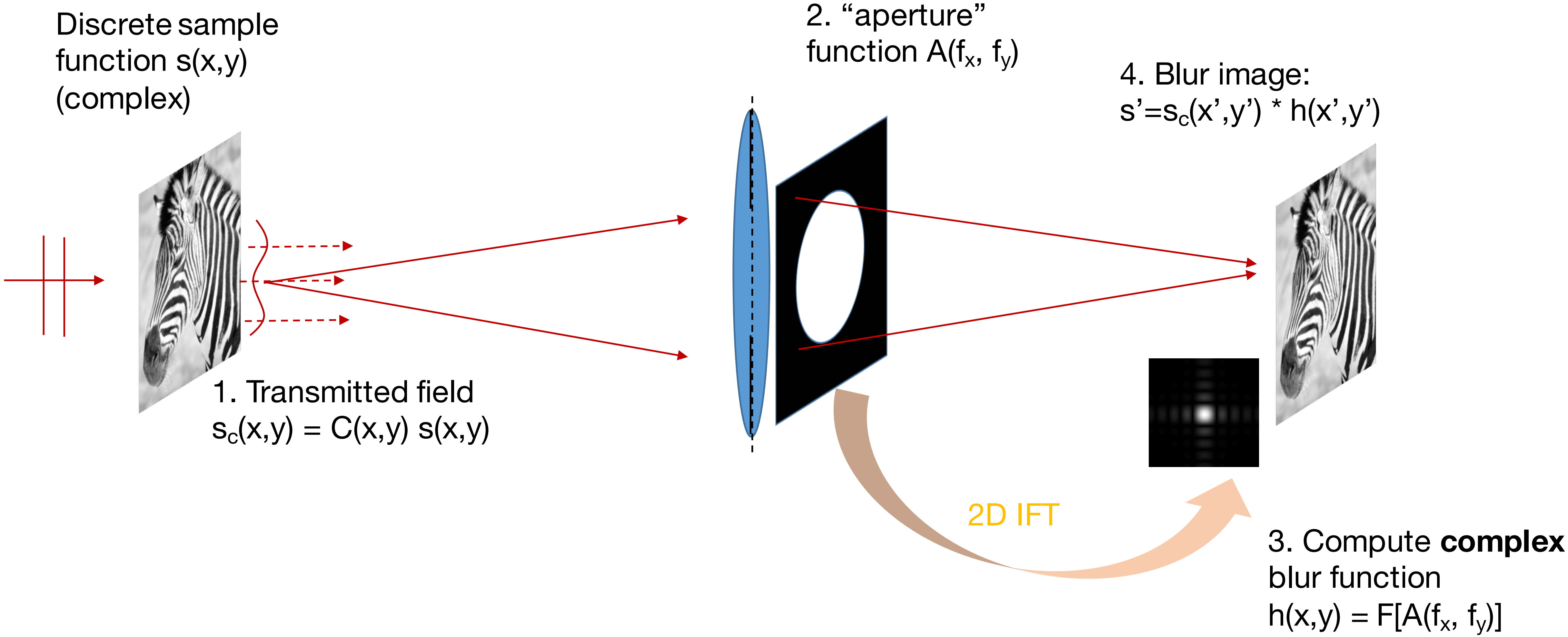


$$\text{Model \#1: } I_c(x,y) = |F^{-1}AFC s|^2$$

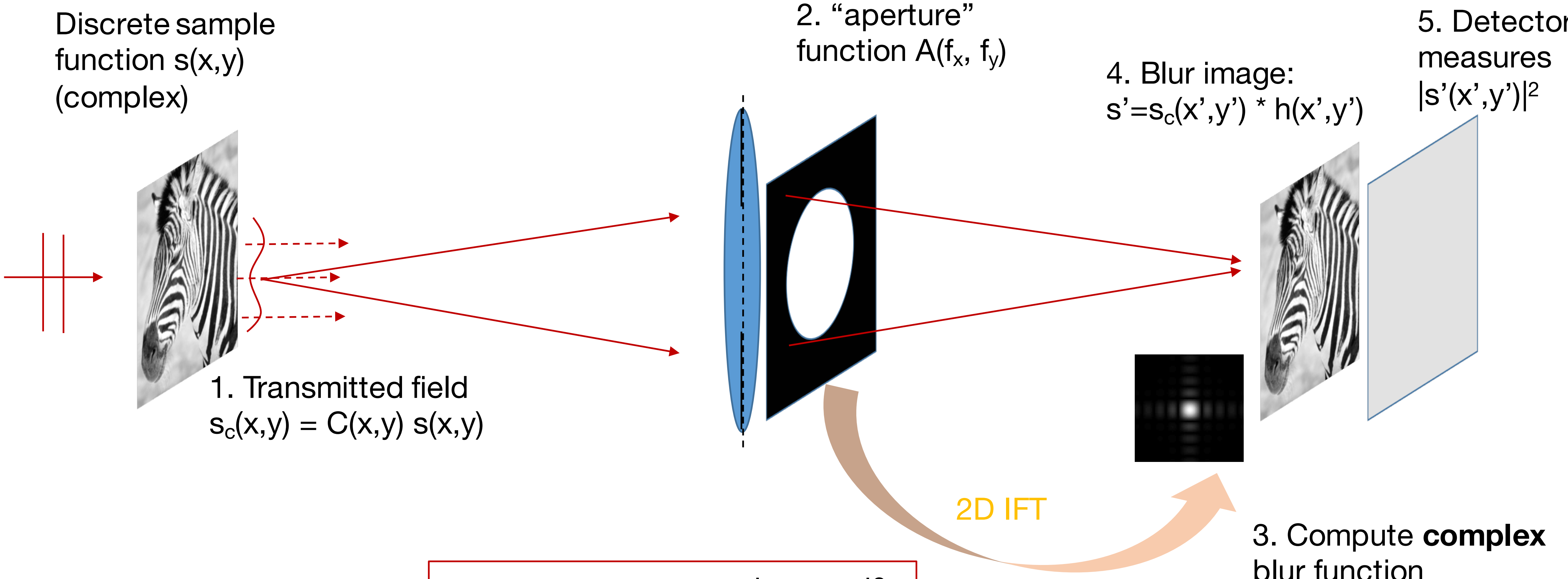
Model of image formation for wave optics (coherent light):



Model of image formation for wave optics (coherent light):



Model of image formation for wave optics (coherent light):



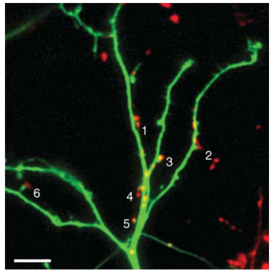
$$\boxed{\text{Model \#2: } I_c(x,y) = |h * Cs|^2}$$

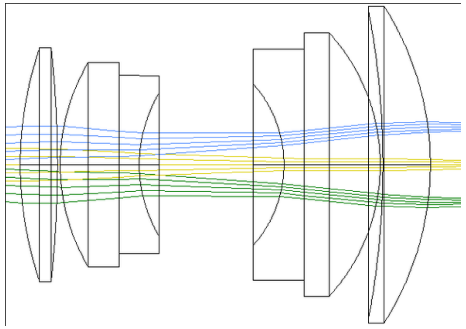
You typically go between 4 functions to describe one imaging system:

$$\begin{tikzcd}
\boxed{\begin{array}{c}\text{Coherent point-}\\ \text{spread function}\\ h(x)\end{array}} \arrow[r, "|.|^2"'] \arrow[d, "F[.]"] & \boxed{\begin{array}{c}\text{Incoherent point-}\\ \text{spread function}\\ h_i(x)\end{array}} \arrow[d, "F[.]"] \\
\boxed{\begin{array}{c}\text{Coherent}\\ \text{transfer function}\\ H(f_x)\end{array}} \arrow[r, "H*H"] & \boxed{\begin{array}{c}\text{Incoherent}\\ \text{transfer function}\\ H_i(f_x)\end{array}}
\end{tikzcd}$$

Summary of two models for image formation

- **Interpretation #1: Radiation (*Incoherent*)**
- Model: Rays



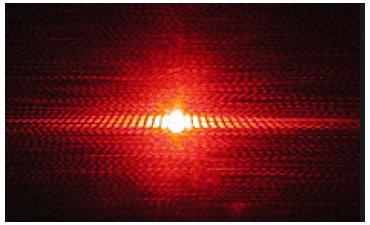


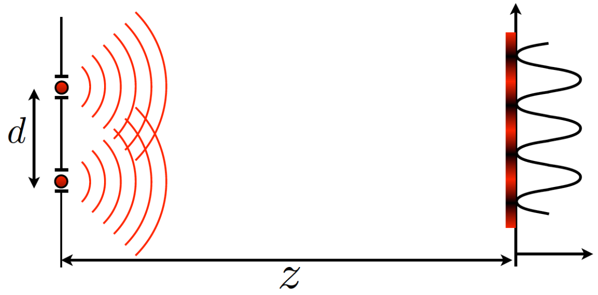
- Real, non-negative

$$\boxed{I_s = H\, B\, S_0}$$

- Sample absorption **S**
- Illumination brightness **B**
- Blur in **H**

- **Interpretation #2: Electromagnetic wave (*Coherent*)**
- Model: Waves





- Complex-valued

$$\boxed{I_C = |\, H\, C\, S_C\, |^2}$$

- Sample abs./phase **S**
- Illumination wave **B**
- Blur in **H**

Coherent image formation equation as CNN operations

$$\boxed{I_C = D\,|\,H\,C\,S_C\,|^2}$$

Step 1: Multiply with weights

Step 2: Convolution

Step 3: Absolute value square (non-linearity)

Step 4: Down-sampling by detector

CNN layer

(Step 1: Normalization)

Step 2: Convolution

Step 3: Non-linearity

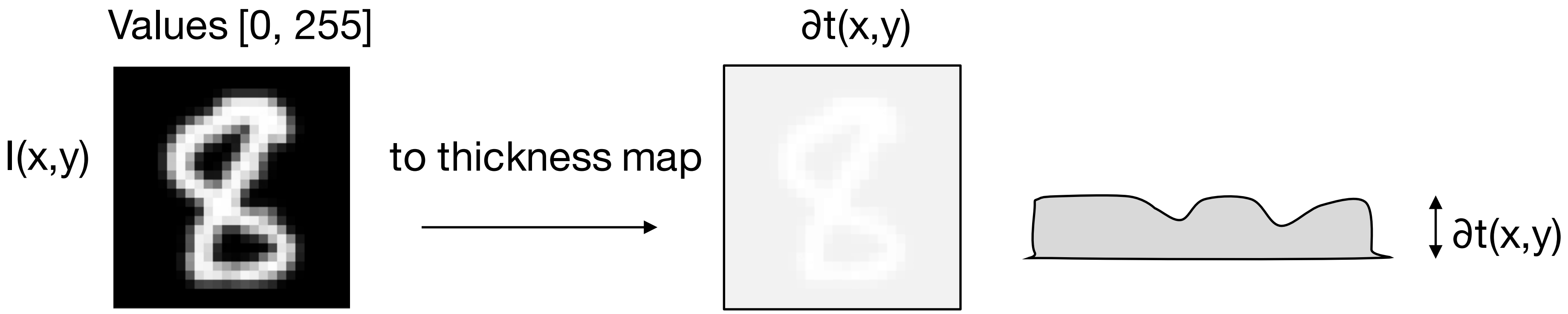
Step 4: Down-sampling by max pooling

Example #1: Optimizing coherent illumination pattern for improved classification

Example future situation: Hacking has brought online banking to a halt. We now rely on a special form of physical check that is made of visibly transparent plastic. To write the amount in, you press down with a pen-like instrument, and then the check is read out by shining a particular pattern of laser light onto it, and then imaging it with a lens.

Question: What type of illumination should you use to maximize the classification accuracy of the numbers on the check?

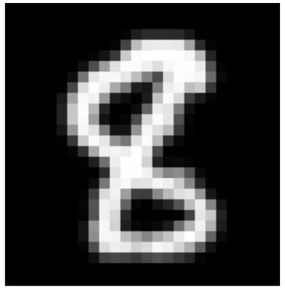
Step 1: Transform MNIST image data set into transparent plastic sheets with varying thickness



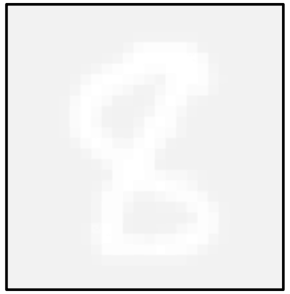
Values [0, 255]

t(x,y)

I(x,y)



to thickness map



1. Normalize intensity map to 1

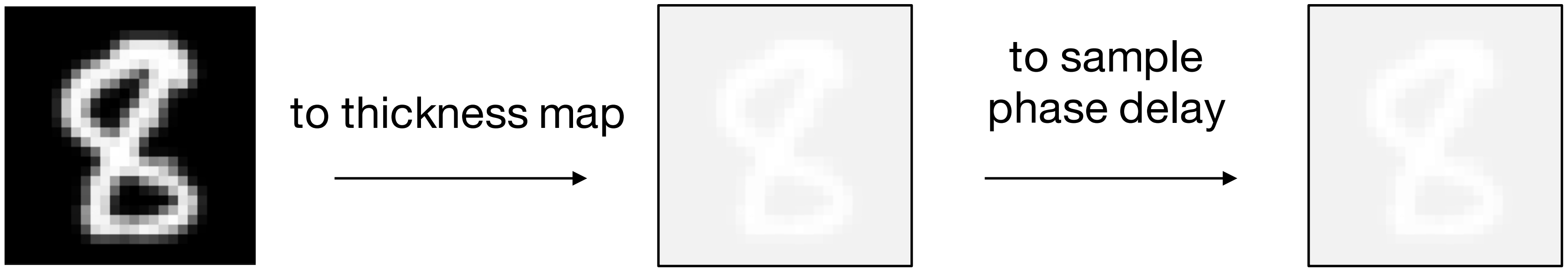
2. Define thickness map at some reasonable amount (100 μm max change)

Values [0, 255]

t(x,y)

$\partial\phi(x,y)$

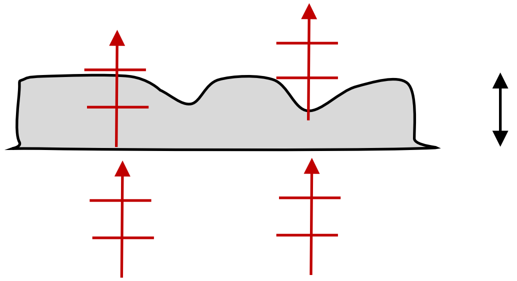
I(x,y)



to thickness map

to sample phase delay

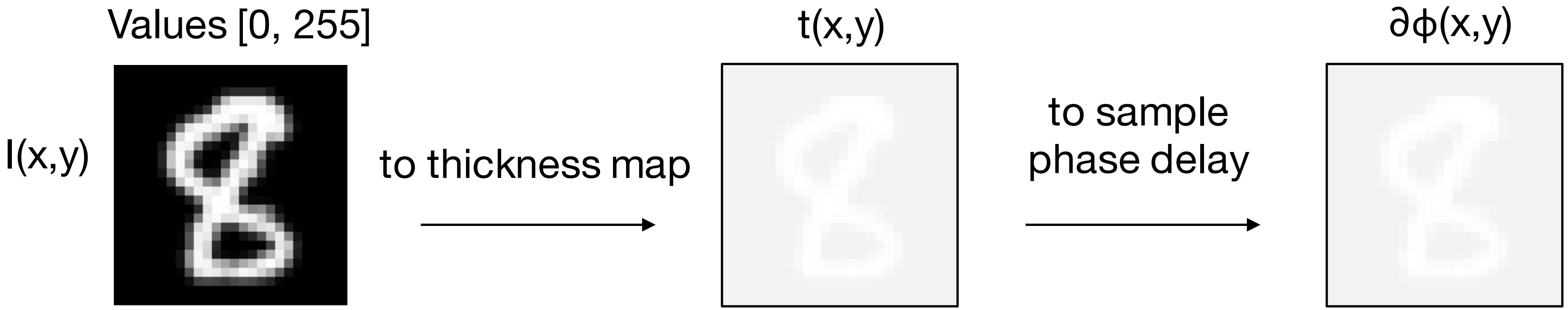
1. Normalize intensity map to 1
2. Define thickness map at some reasonable amount (100 µm max change)
3. Convert thickness map into optical phase delay:



$$\boxed{\partial\phi(x,y) = \exp[\, j\, n\, t(x,y) / \lambda \,]}$$

nt = Optical path length

deep imaging



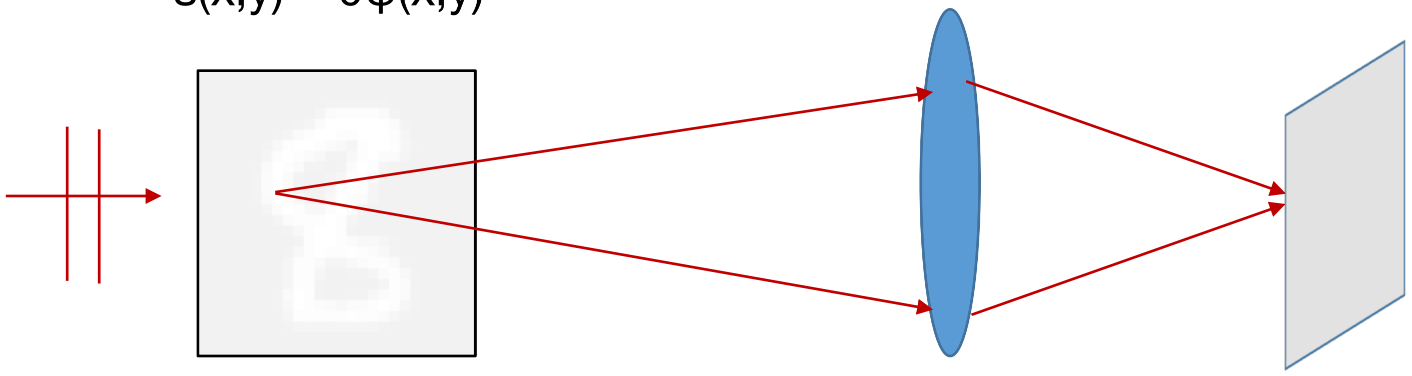
1. Normalize intensity map to 1
2. Define thickness map at some reasonable amount (100 μm max change)
3. Convert thickness map into optical phase delay:

```
n = 1
wavelength = 0.5e-3
mnist_raw_images = tf.placeholder(tf.float32, [image_size, None])
thickness_map = mnist_raw_images/np.amax(mnist_raw_images)
mnist_phase_delay_real = cos(thickness_map * n/wavelength)
mnist_phase_delay_imag = sin(thickness_map * n/wavelength)
mnist_phase_delay = tf.complex(mnist_phase_delay_real,mnist_phase_delay_imag)
```

Example #1: Optimizing coherent illumination pattern for improved classification

Coherent image Model: $I_c(x,y) = |h * Cs|^2$

$s(x,y) = \partial\phi(x,y)$



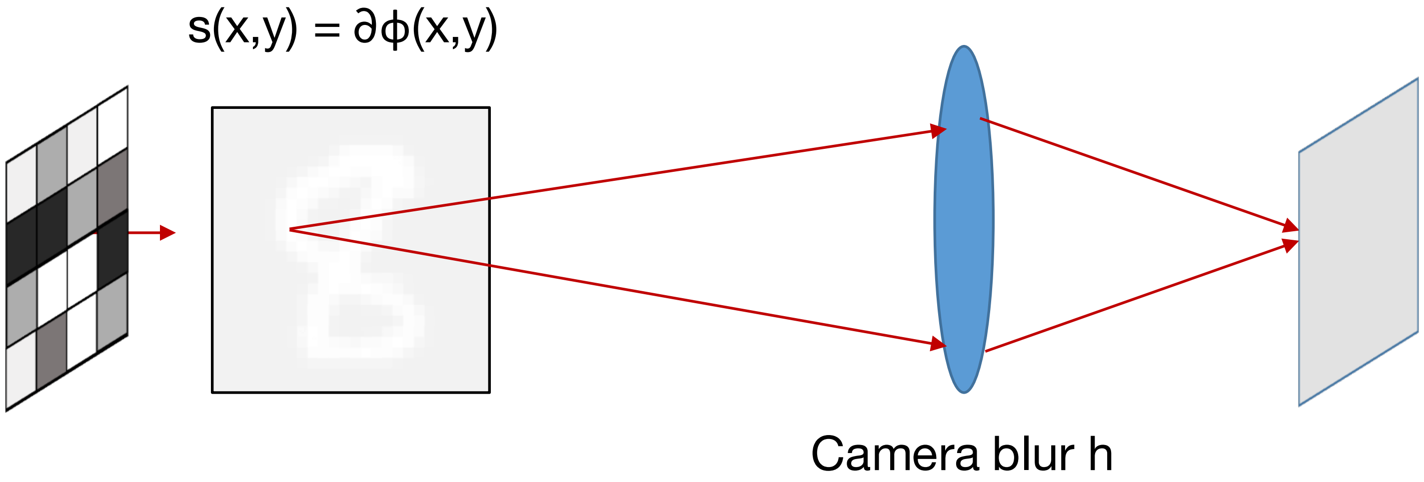
Example #1: Optimizing coherent illumination pattern for improved classification

Coherent image Model: $I_c(x,y) = |h * Cs|^2$

Unknown

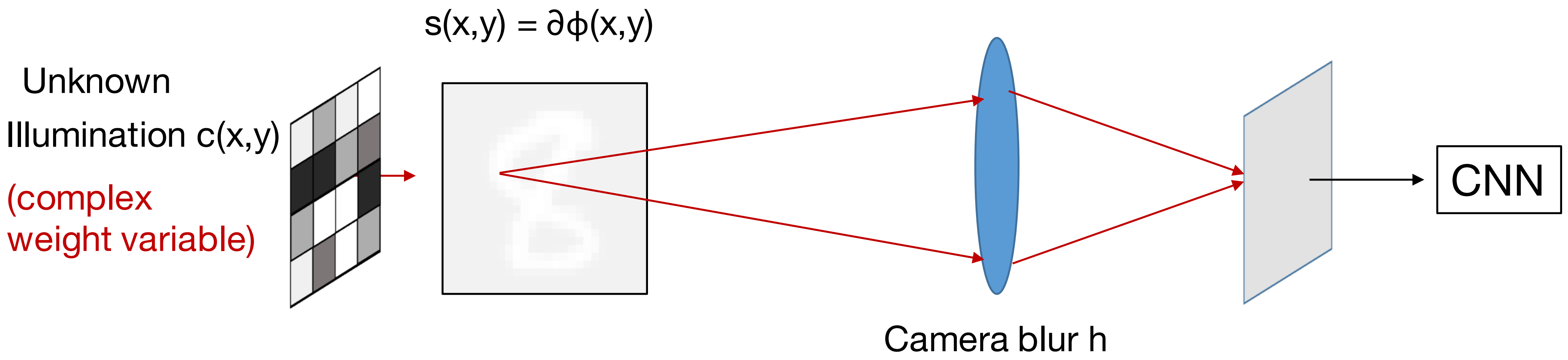
Illumination c(x,y)

(complex weight variable)



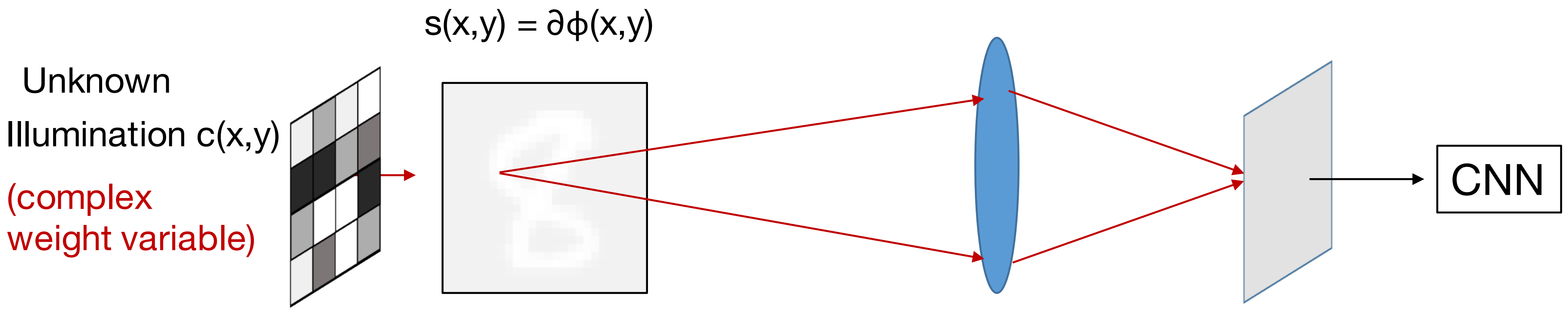
Example #1: Optimizing coherent illumination pattern for improved classification

Coherent image Model: $I_c(x,y) = |h * Cs|^2$



Example #1: Optimizing coherent illumination pattern for improved classification

Coherent image Model: $I_c(x,y) = |h * Cs|^2$




```
mnist_phase_delay = tf.reshape(mnist_phase_delay, [-1, image_size, image_size])
C0_real = tf.Variable([image_size, image_size])
C0_imag = tf.Variable([image_size, image_size])
C0_complex = tf.complex(C0_real, C0_imag)
x_C_complex = tf.mul(mnist_phase_delay, C0_complex)
image_complex = conv2d(x_C_complex, camera_blur)
detected_image = tf.complex_abs(image_complex)
```

detected_image then enters standard CNN classification pipeline

Example #2: Optimizing aperture shape for improved digit classification

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Example future situation: Hacking has brought online banking to a halt. We now rely on a special form of physical check that is made of visibly transparent plastic. To write the amount in, you press down with a pen-like instrument, and then the check is read out by shining a particular pattern of laser light onto it, and then imaging it with a lens.

Question #2: What type of aperture shape should you use to maximize classification accuracy?

Example #2: Optimizing aperture shape for improved digit classification

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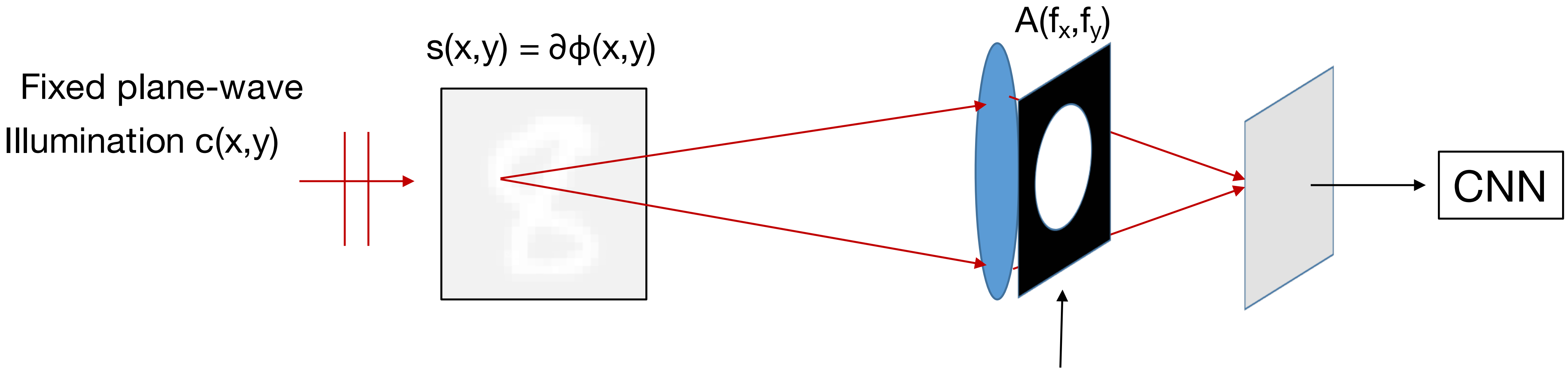
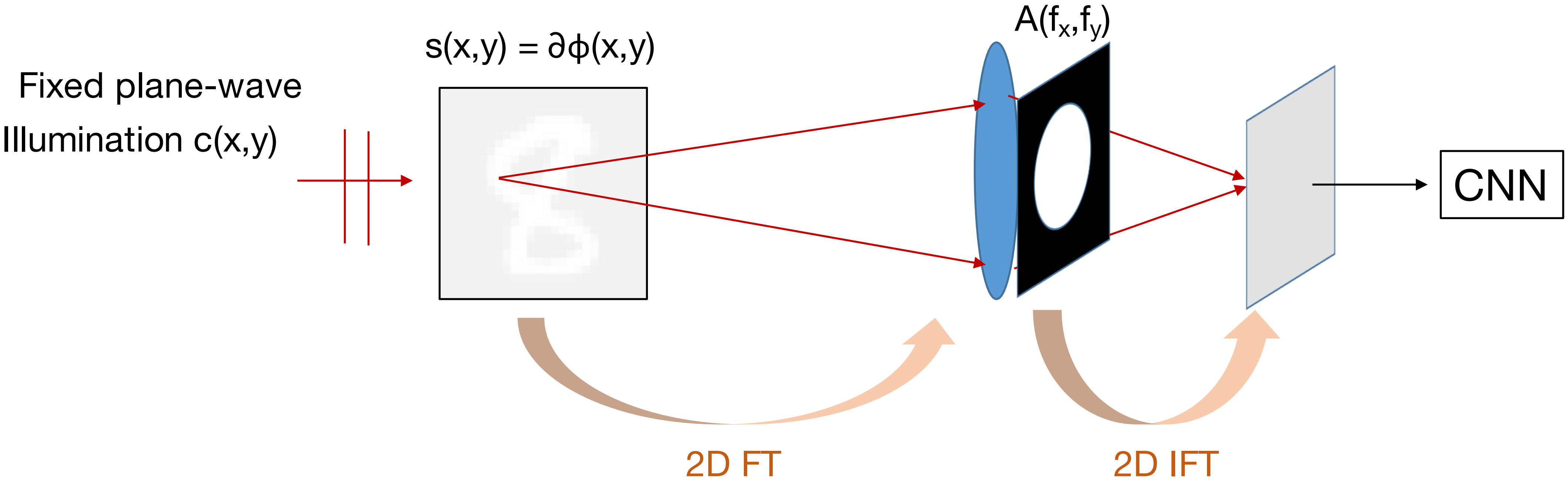
<u>Question #2</u>: What type of aperture shape should you use to maximize classification accuracy?

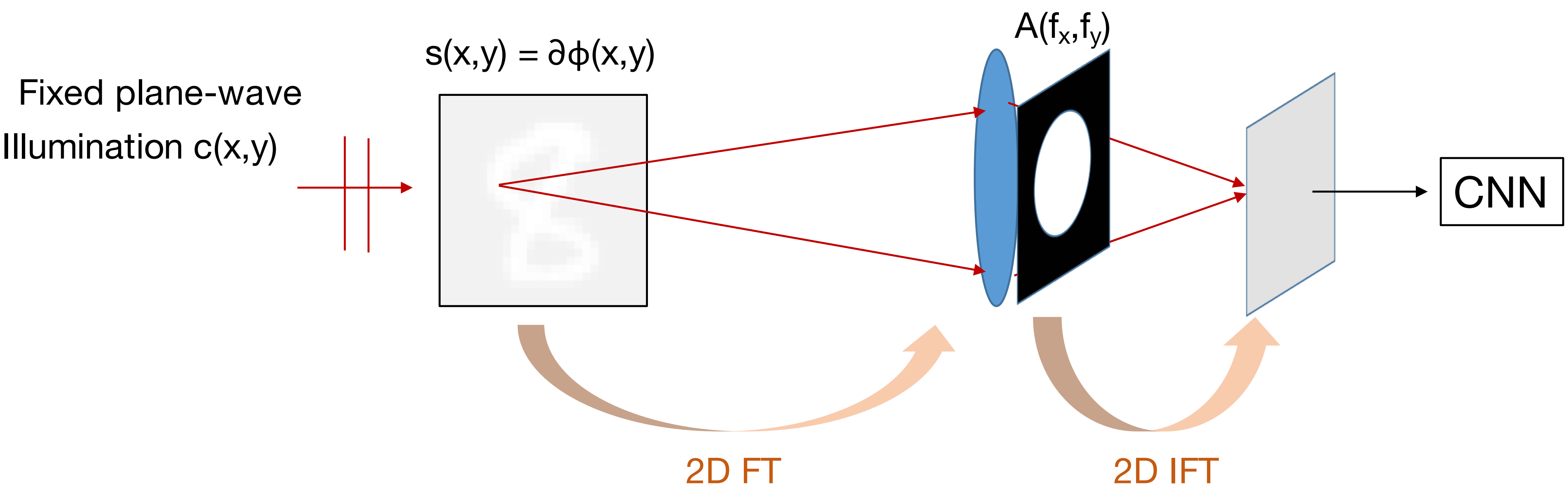


Let's make $A(f_x,f_y)$ any shape – it becomes a weight variable

Example #2: Optimizing aperture shape for improved digit classification



Example #2: Optimizing aperture shape for improved digit classification




```
mnist_phase_delay = tf.reshape(mnist_phase_delay, [-1, image_size, image_size])
C0 = np.ones(image_size, image_size)
C0 = tf.constant(C0)
x_C_complex = tf.mul(mnist_phase_delay, C0)
fx_C_complex = tf.fft2d(x_C_complex)
ap_filter = tf.Variable([image_size, image_size])
filtered_x_C =  tf.mul(fx_C_complex, ap_filter)
image_complex = tf.ifft2d(filtered_x_C)
detected_image = tf.complex_abs(image_complex)
```

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Remaining questions to address about physical layers:

- Where and how should I implement my physical layer?
  - Simulation data
  - Experimental data
- How can I add some constraints to the physical weights that I'm optimizing?
- What are some common issues and pitfalls?