deep imaging

Lecture 16: Wave optics and Fourier optics

Machine Learning and Imaging

BME 590L
Roarke Horstmeyer

Announcements:

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Welcome to BME590 Online! Things shouldn't change much, but here's a summary:

0. I hope that everyone is safe and is doing ok!

1. See summary of all changes to transition things online here

a. Lectures are now held on Zoom (live, at same class time) and will be recorded and posted online

b. Lab session are now held on Zoom (live at same lab time) and will be recorded

c. Office hours are now held on Zoom

*My Zoom ID: 417-271-8775

2. Homework #3 is due today – please submit via class website

3. Homework #4 will be posted online today – it is due it

4. I have sent some of you feedback about your project proposal via email. If so, please send me an updated project proposal by this Thursday 3/26 at midnight. For the rest of the received proposals, I have input your grade on Sakai. If you didn't get an email or grade on Sakai, I didn't get your proposal.

Rough schedule for the rest of the semester:

Homework #4 assigned: March 24
Homework #4 due: April 7
Homework #5 assigned: April 7
Homework #5 due: April 21
Final projects due: (original) Thurs 4/23, Friday 4/24, or Monday 4/27
Final project presentations: (original) Thurs 4/23, Friday 4/24, or Monday 4/27

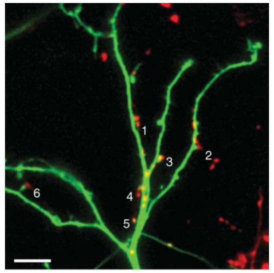
Questions from class survey:

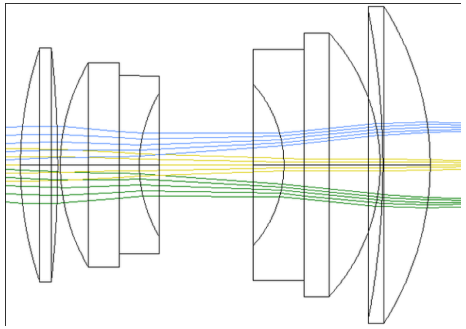
Q: Are we maintaining original final project presentation dates or if these are getting shifted by 1 week to account for the extension of spring break?
A: I would prefer to maintain the original presentation dates. It may be possible to shift it back a few days into finals week, but it depends on what I hear about regarding the format of other finals

Q: Would the recorded video of lectures be uploaded right after the class? Or it would be upload several days after the class?
A: Right after the class (i.e., as soon as I can after the class)

First - what is light and how can we model it?

- Interpretation #1: Radiation (*Incoherent*)
- Model: Rays

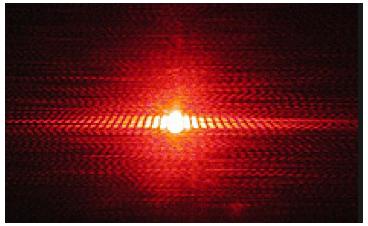


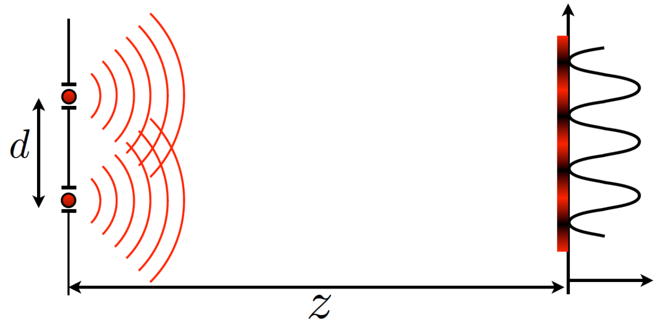


- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

- Interpretation #2: Electromagnetic wave (*Coherent*)
- Model: Waves



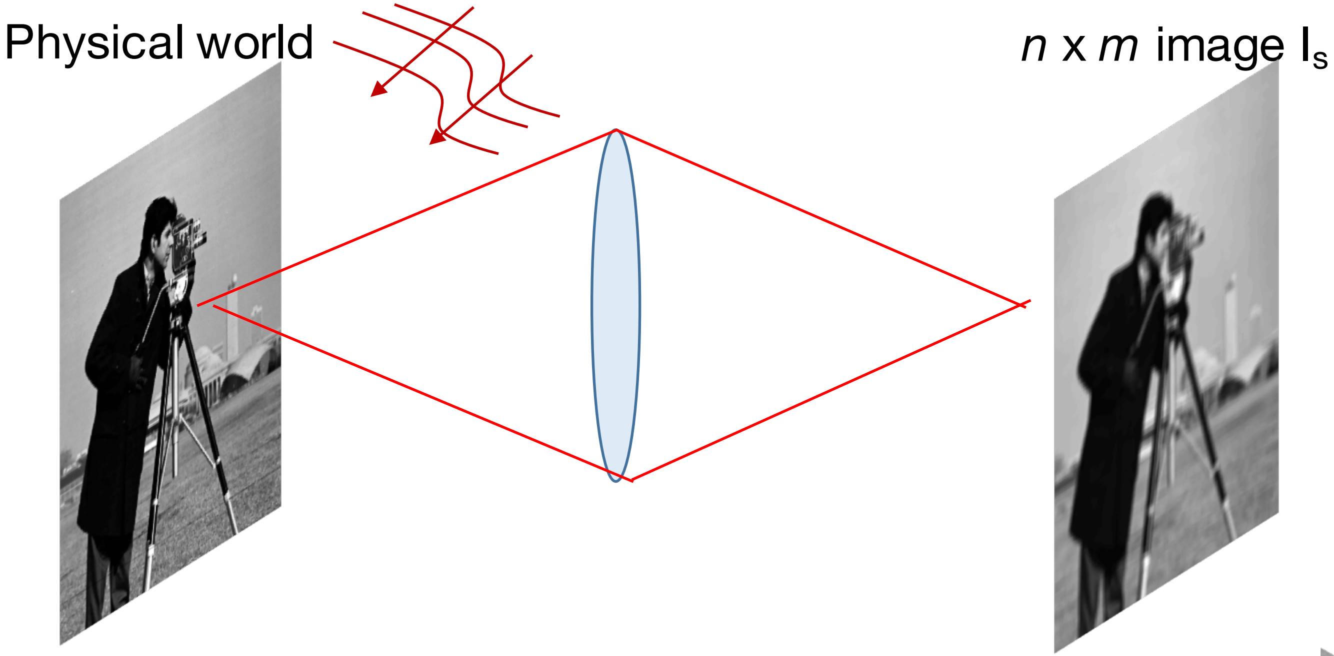


- Complex field
- Models Interference

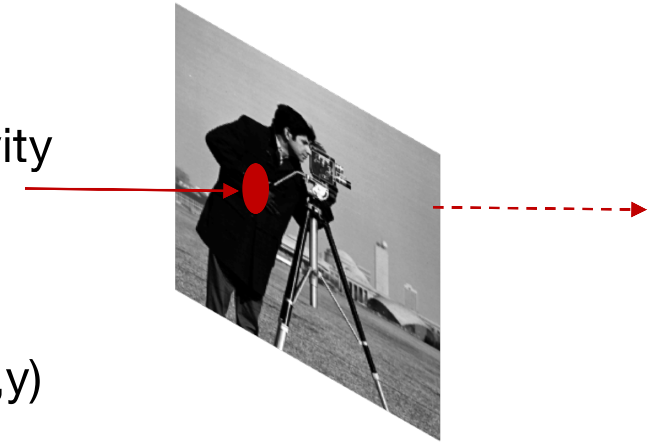
$$E_{tot} = E_1 + E_2$$

- Interpretation #3: Particle
- Model: Photons

Simple mathematical model of incoherent image formation

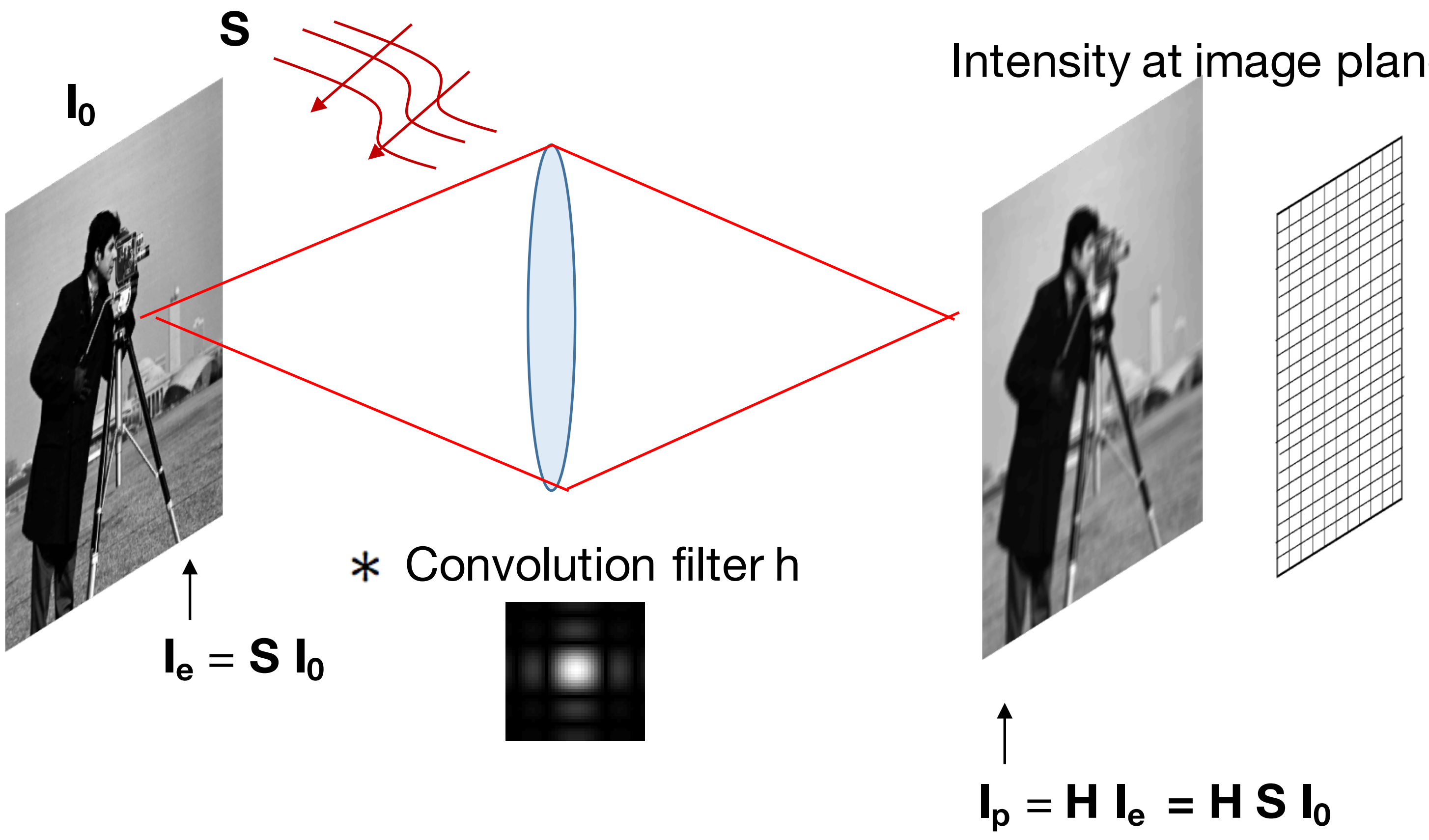


- Assume incoherent illumination
- Assume thin 2D object
- Object is real, non-negative map of absorption/reflectivity



Object absorption: $\mathbf{I_0}(x,y)$
Illumination pattern: $\mathbf{s}(x,y)$
Light exiting object surface: $\mathbf{I_e}(x,y) = \mathbf{I_0}(x,y) \circ \mathbf{s}(x,y)$

Simple mathematical model of image formation



Summary: simple physical layers for incoherent imaging

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- Deal with sample/image intensities I, real and non-negative
- Effect of illumination is element-wise multiplication $I_e(x,y) = \mathbf{S}\ I_0(x,y)$

λ

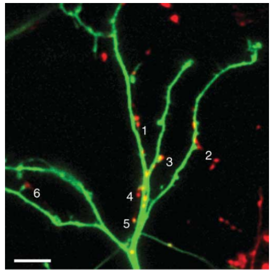
- Imaging systems blur the object via point-spread function matrix **H**

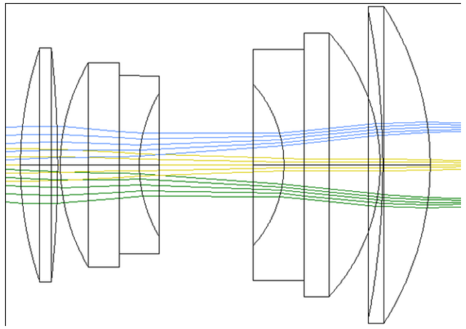
$I_b(x,y) = \mathbf{H}\ I_0(x,y)$

- Discrete pixels down-sample the object via $I_d(x,y) = \mathbf{D}\ I_0(x,y)$
- Add noise into measurement $I_N(x,y) = \mathbf{D}\ I_0(x,y) + \mathbf{N}$
- Different colors add linearly $I_s(x, y) = \Sigma\ I_0(x, y, \lambda)$

What is light and how can we model it?

- Interpretation #1: Radiation (*Incoherent*)
- Model: Rays





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$I_{tot} = I_1 + I_2$

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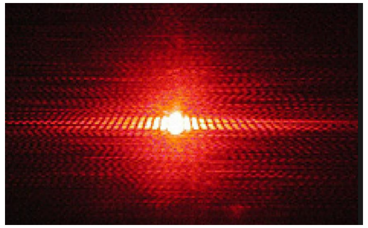

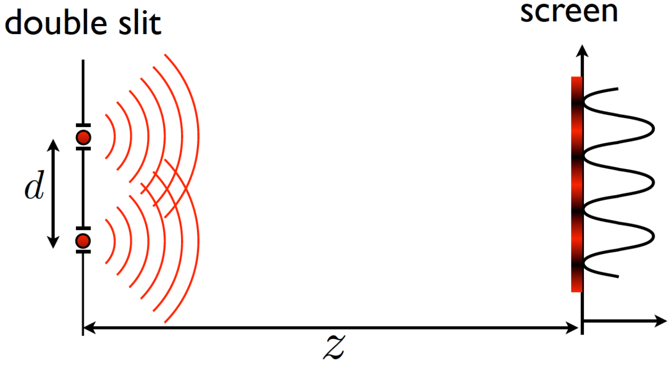

- Complex field
- Models Interference

$E_{tot} = E_1 + E_2$

This class: Modeling coherent radiation as a wave

Let's take a step back: how does light propagate?

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Maxwell's equations
without any charge

$$\nabla \times \vec{\mathcal{E}} = -\mu \frac{\partial \vec{\mathcal{H}}}{\partial t}$$

$$\nabla \times \vec{\mathcal{H}} = \epsilon \frac{\partial \vec{\mathcal{E}}}{\partial t}$$

$$\nabla \cdot \epsilon \vec{\mathcal{E}} = 0$$

$$\nabla \cdot \mu \vec{\mathcal{H}} = 0.$$

Let's take a step back: how does light propagate?

Maxwell's equations without any charge

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$$\nabla \times \vec{\mathcal{H}} = \epsilon \frac{\partial \vec{\mathcal{E}}}{\partial t}$$

$$\nabla \cdot \epsilon \vec{\mathcal{E}} = 0$$

$$\nabla \cdot \mu \vec{\mathcal{H}} = 0.$$

1. Take the curl of both sides of first equation
2. Substitute 2nd and 3rd equation
3. Arrive at the wave equation:

$$\nabla^2 \vec{\mathcal{E}} - \frac{n^2}{c^2} \frac{\partial^2 \vec{\mathcal{E}}}{\partial t^2} = 0 \qquad n = \left(\frac{\epsilon}{\epsilon_0}\right)^{1/2} \qquad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

Let's take a step back: how does light propagate?

Considering light that isn't pulsed over time, we can use the following solution:

$$u(P,t) = A(P)\cos[2\pi\nu t + \phi(P)]$$

$$u(P,t) = Re\{U(P)\exp(-j2\pi\nu t)\},$$

Let's take a step back: how does light propagate?

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With this particular solution, we get the following important time-independent equation:

Helmholtz Equation

$$(\nabla^2 + k^2)U = 0.$$

$$k = 2\pi n\frac{\nu}{c} = \frac{2\pi}{\lambda},$$

Let's take a step back: how does light propagate?

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Helmholtz Equation

$$(\nabla^2 + k^2)U = 0. \qquad k = 2\pi n\frac{\nu}{c} = \frac{2\pi}{\lambda},$$

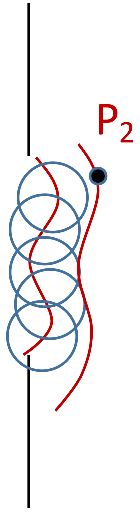
This is an important equation in physics. We won't go into the details, but it leads to the Huygen-Fresnel principle:

$$U(P_2) = \frac{1}{j\lambda}\iint_{\Sigma} U(P_1)\frac{\exp(jkr_{21})}{r_{21}}\cos\theta\, ds$$

Plane-to-plane light propagation via the ”paraxial approximation”

The Huygens-Fresnel Equation

$$U(P_2) = \frac{1}{j\lambda} \iint_{\Sigma} U(P_1) \frac{\exp(jkr_{21})}{r_{21}} \cos\theta \, ds$$

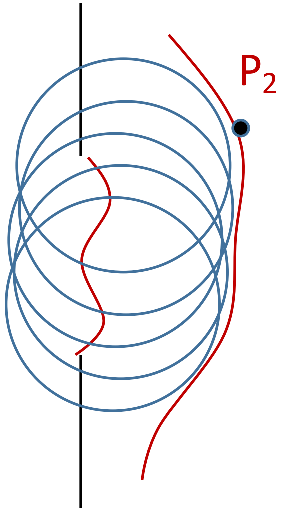


Aperture

Plane-to-plane light propagation via the "paraxial approximation"

The Huygens-Fresnel Equation

$$U(P_2) = \frac{1}{j\lambda} \iint_{\Sigma} U(P_1) \frac{\exp(jkr_{21})}{r_{21}} \cos\theta \, ds$$



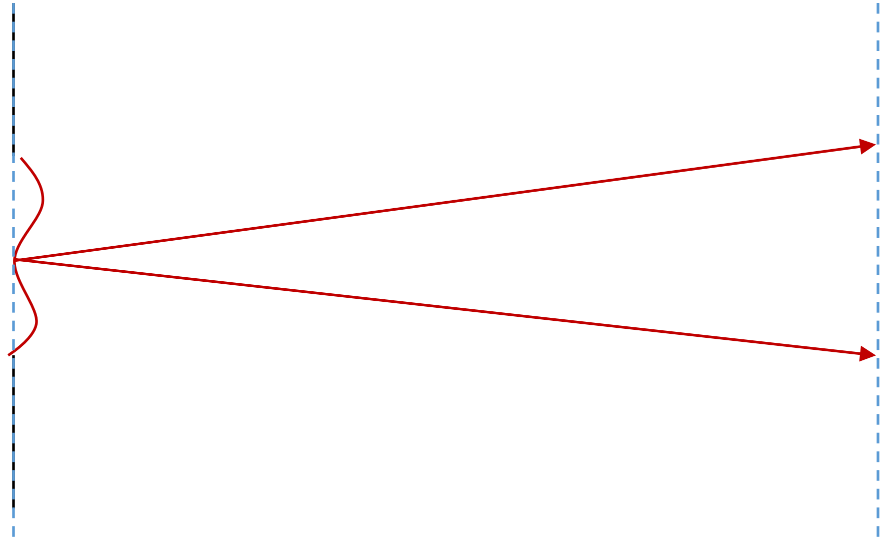
Generally connects two points in 3D:

$$U(P_1) = U(x_1, y_1, z_1)$$

$$U(P_2) = U(x_2, y_2, z_2)$$

Plane-to-plane light propagation via the "paraxial approximation"

We are usually concerned about propagation between two planes (almost always in an optical system):

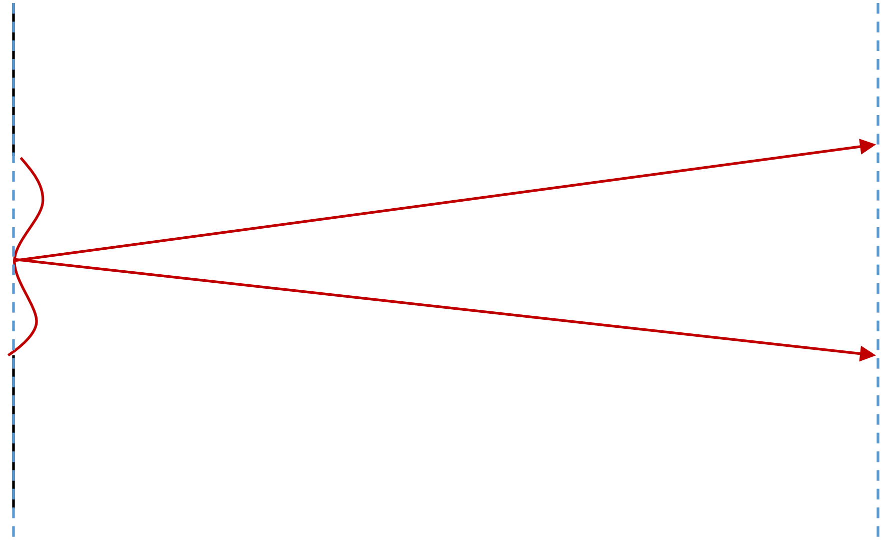


$$U(P_1) = U(x_1, y_1, z_1 = z_{p1}) \qquad U(P_2) = U(x_2, y_2, z_2 = z_{p2})$$

$$U(P) = E(x, y, z)e^{ikz}$$

Plane-to-plane light propagation via the ”paraxial approximation”

We are usually concerned about propagation between two planes (almost always in an optical system):



Paraxial approximation:

$$\nabla_\perp^2 U + 2ik\frac{dU}{dz} = 0$$

$$\nabla_\perp^2 \stackrel{\text{def}}{=} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$U(P_1) = U(x_1, y_1, z_1 = z_{p1}) \qquad U(P_2) = U(x_2, y_2, z_2 = z_{p2})$$

$$U(P) = E(x, y, z)e^{ikz}$$

Plane-to-plane light propagation via the ”paraxial approximation”

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Paraxial approximation:

$$\nabla_{\perp}^{2} U + 2ik\frac{dU}{dz} = 0$$

Substitute in $U(P) = E(x, y, z)e^{ikz}$ and crank the wheel,

$$\nabla_{\perp}^{2} E + 2ik\frac{dE}{dz} + 2k^{2}E = 0$$

Paraxial Helmholtz Equation. This has an exact integral solution:

Plane-to-plane light propagation via the ”paraxial approximation”

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Paraxial Helmholtz Equation. This has an exact integral solution:

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z}\iint_{-\infty}^{+\infty} E(x',y',0)e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]}dx'dy'$$

Fresnel diffraction integral

This is how light propagates from one plane to the next. It’s a convolution!

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Fresnel light propagation as a convolution

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{+\infty} E(x',y',0) e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]} dx'dy'$$

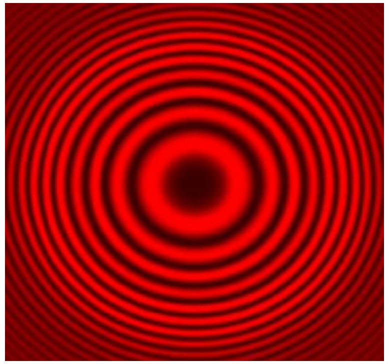
$$h(x,y,z) = \frac{e^{ikz}}{i\lambda z} e^{i\frac{k}{2z}(x^2+y^2)}$$

$$E(x,y,z) = E(x,y,0) * h(x,y,z)$$

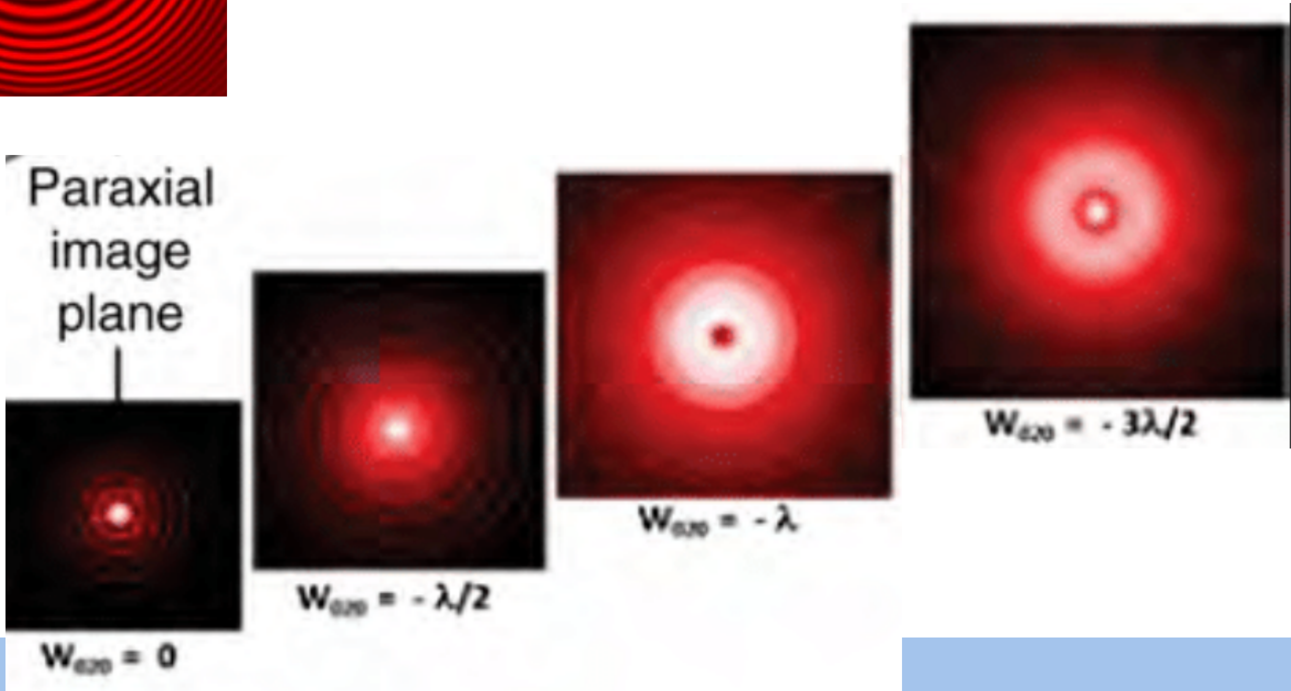
Fresnel light propagation as a convolution

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{+\infty} E(x',y',0)\, e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]}\, dx'\, dy'$$

$$h(x,y,z) = \frac{e^{ikz}}{i\lambda z} e^{i\frac{k}{2z}(x^2+y^2)}$$



$$E(x,y,z) = E(x,y,0) * h(x,y,z)$$



From the Fresnel approximation to the Fraunhofer approximation

Fresnel Approximation:

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{+\infty} E(x',y',0) e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]} dx'\,dy'$$

Lets assume that the second plane is “pretty far away” from the first plane. Then,

$$z > \frac{2D^2}{\lambda}$$

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From the Fresnel approximation to the Fraunhofer approximation

Fresnel Approximation:

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Lets assume that the second plane is "pretty far away" from the first plane. Then,

$$z > \frac{2D^2}{\lambda}$$

1. Expand the squaring

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint E(x',y',0) e^{\frac{ik}{2z}(x^2+y^2)} e^{\frac{ik}{2z}(x'^2+y'^2)} e^{\frac{ik}{2z}(xx'+yy')} dx'dy'$$

From the Fresnel approximation to the Fraunhofer approximation

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2. Front term comes out, assume second term goes away, then,

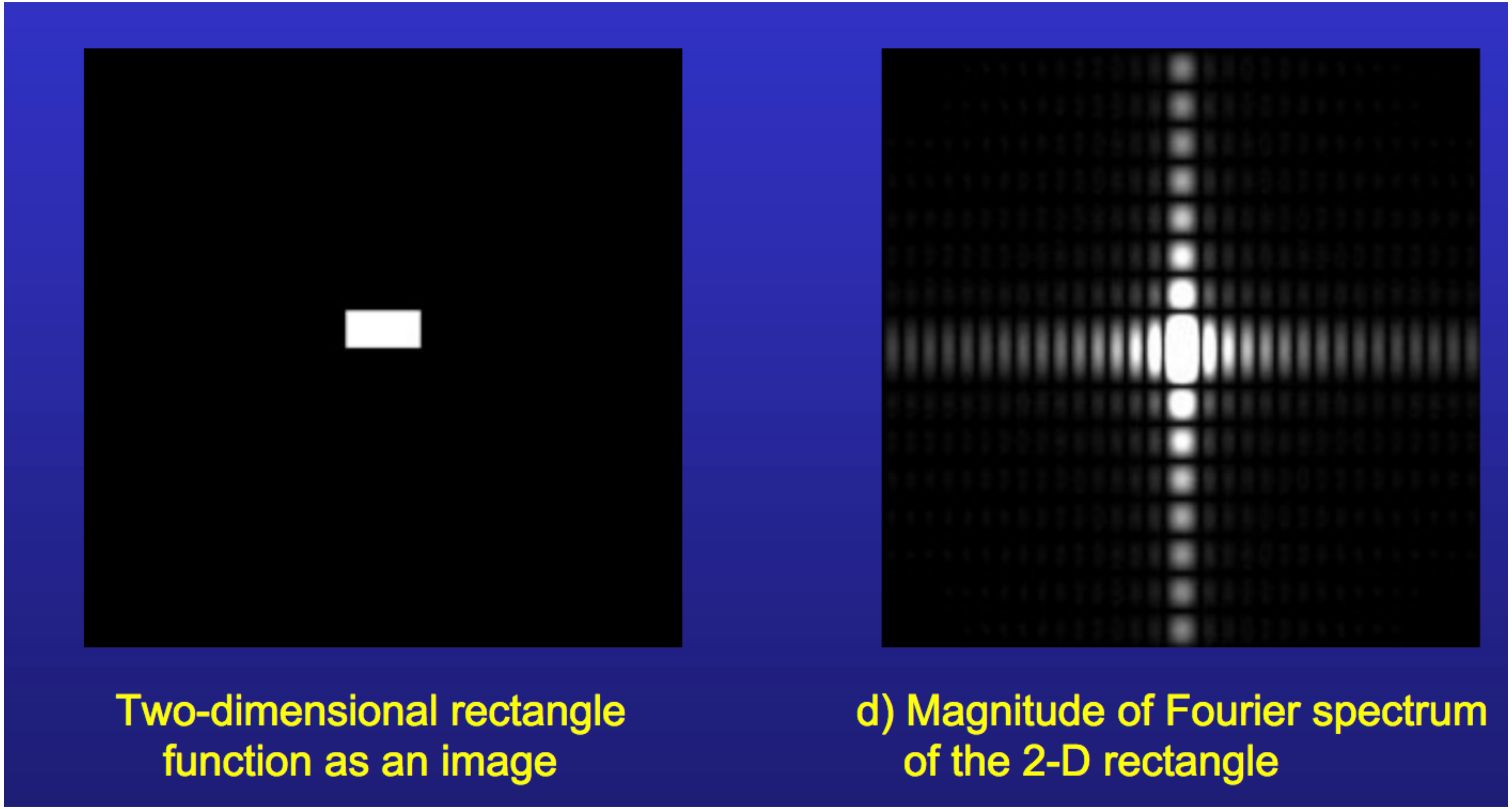
$$E(x,y,z) = C \iint E(x',y',0) e^{\frac{ik}{2z}(xx'+yy')} dx'\,dy'$$

$$C = \frac{e^{ikz}}{i\lambda z} e^{\frac{ik}{2z}(x^2+y^2)}$$

Fraunhofer diffraction is a Fourier transform!!!!!!!

This is the aperture

This is what you see far away



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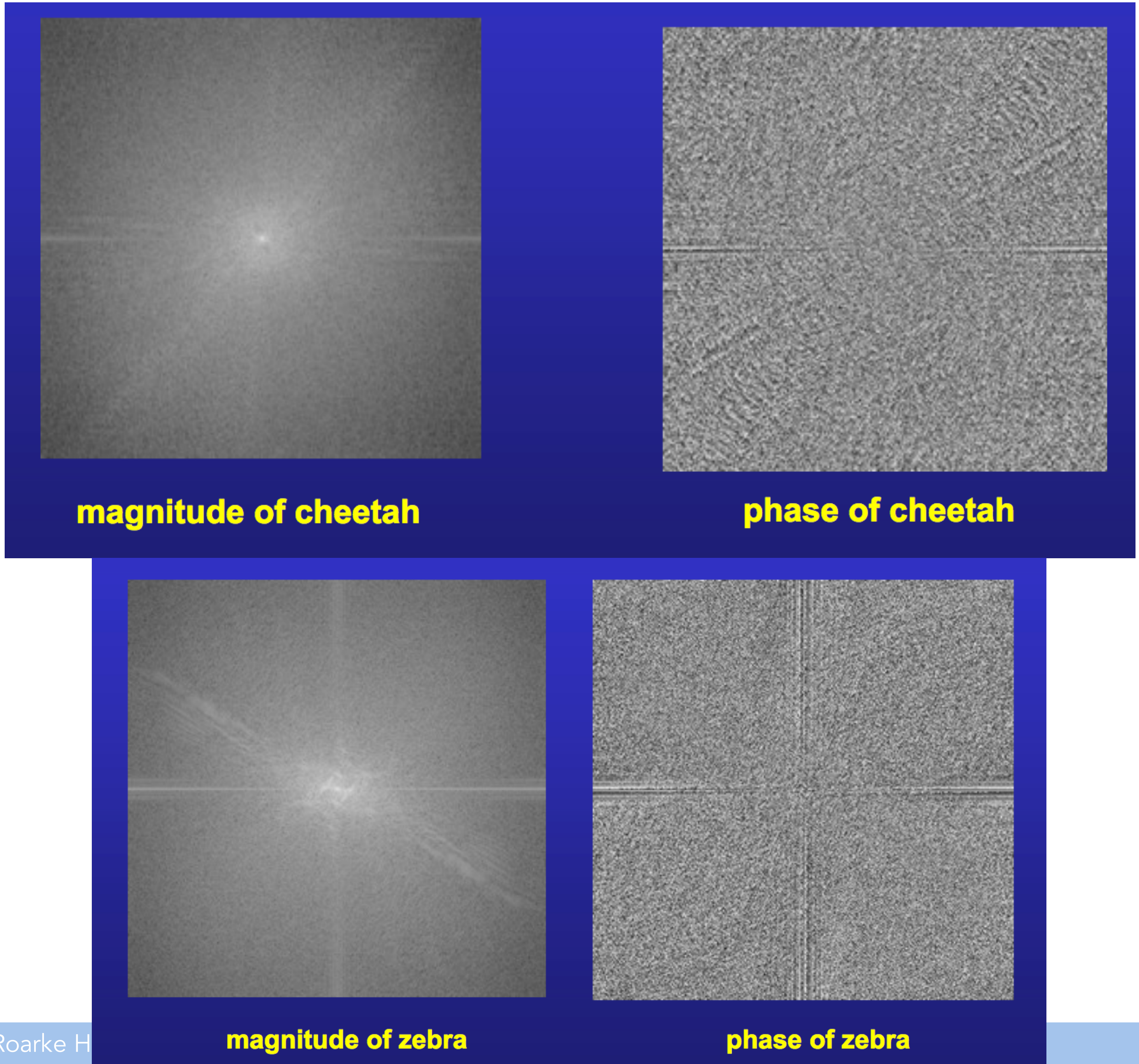
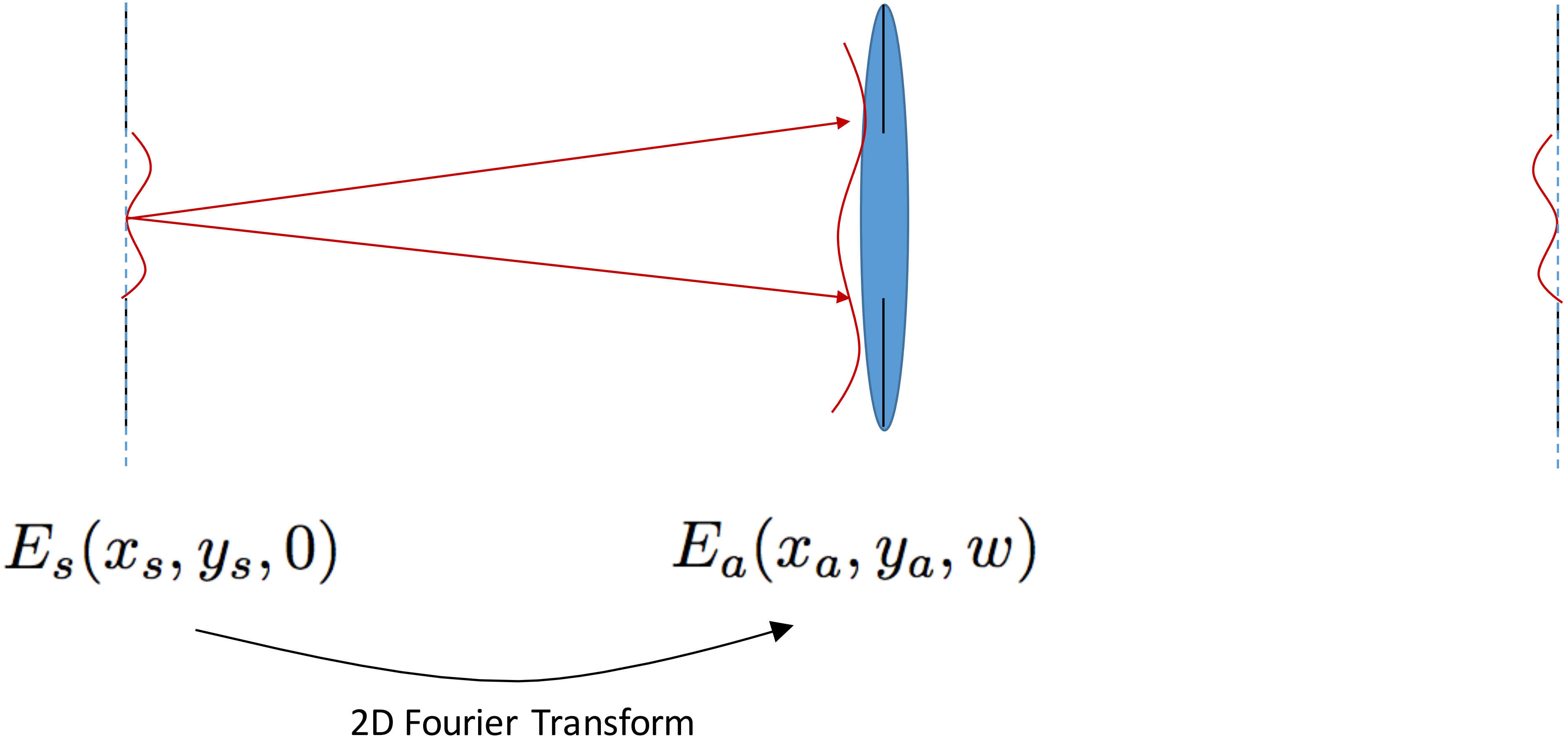
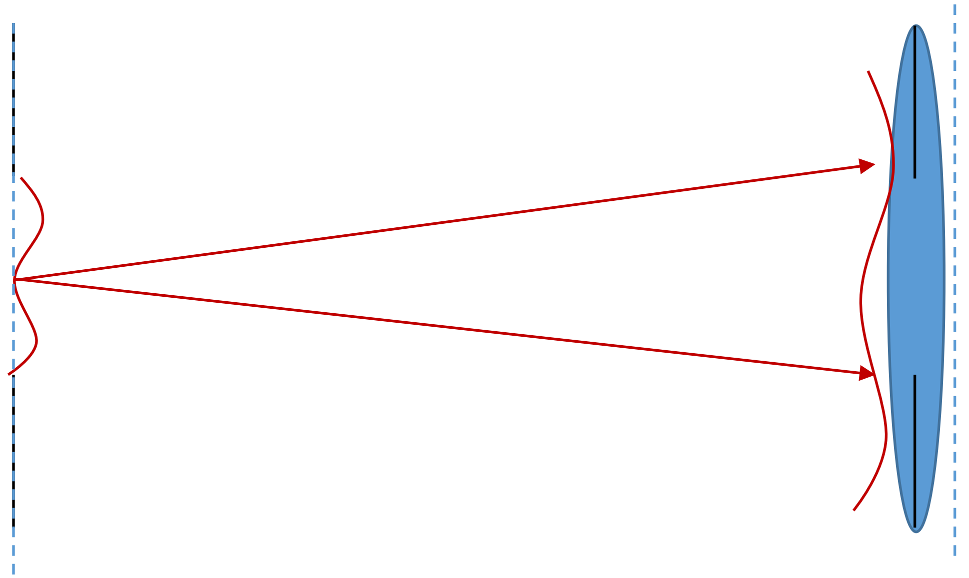

magnitude of cheetah
phase of cheetah
magnitude of zebra
phase of zebra

Model of a microscope (or camera) using Fourier transforms:



Model of a microscope (or camera) using Fourier transforms:

$$E'_a(x_a, y_a, w)$$



$$E_s(x_s, y_s, 0) \qquad E_a(x_a, y_a, w)$$

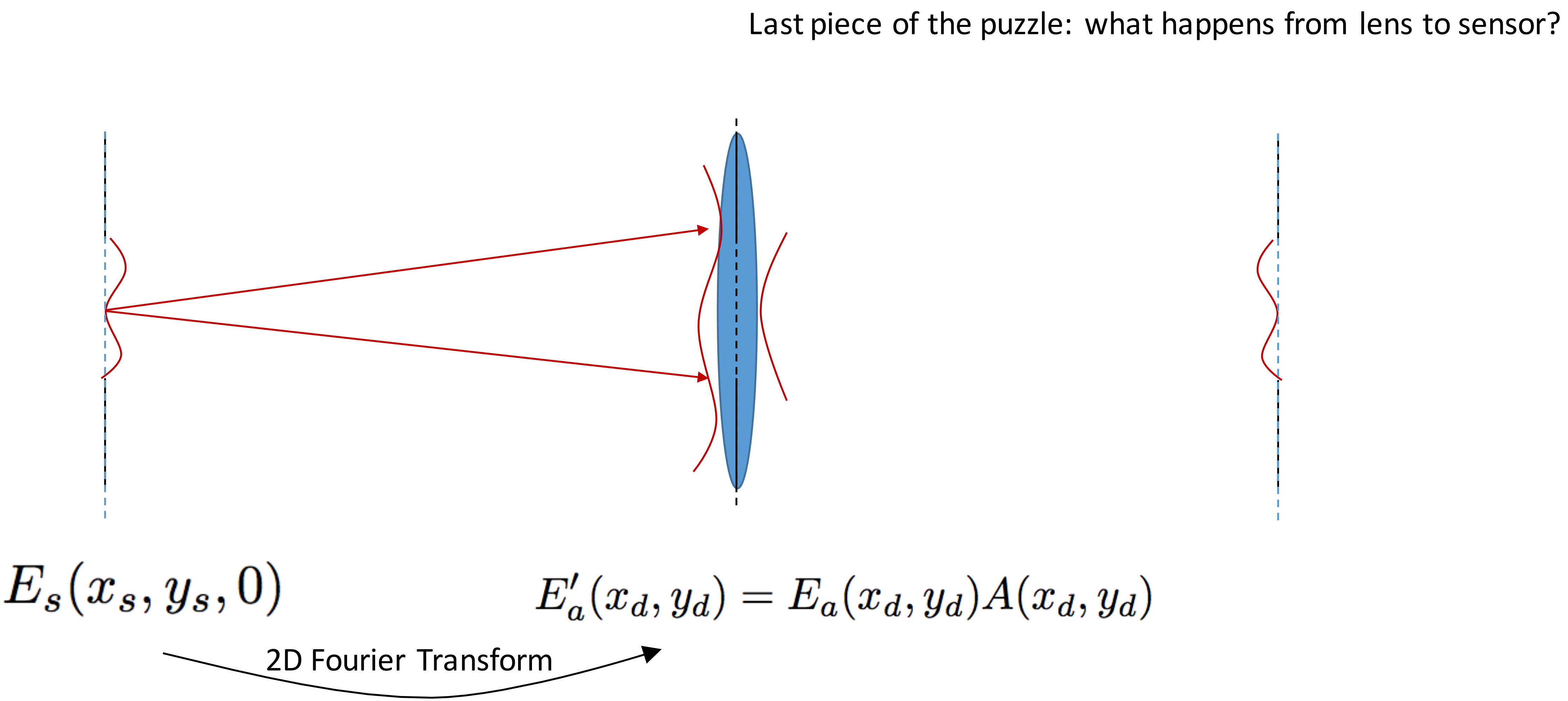
2D Fourier Transform

Effect of the lens is to block light.

Use *thin object approximation* to determine distribution of light on the immediate other side of the lens stop:

$$E'_a(x_d, y_d) = E_a(x_d, y_d)A(x_d, y_d)$$

Model of a microscope (or camera) using Fourier transforms:



Model of a microscope (or camera) using Fourier transforms:

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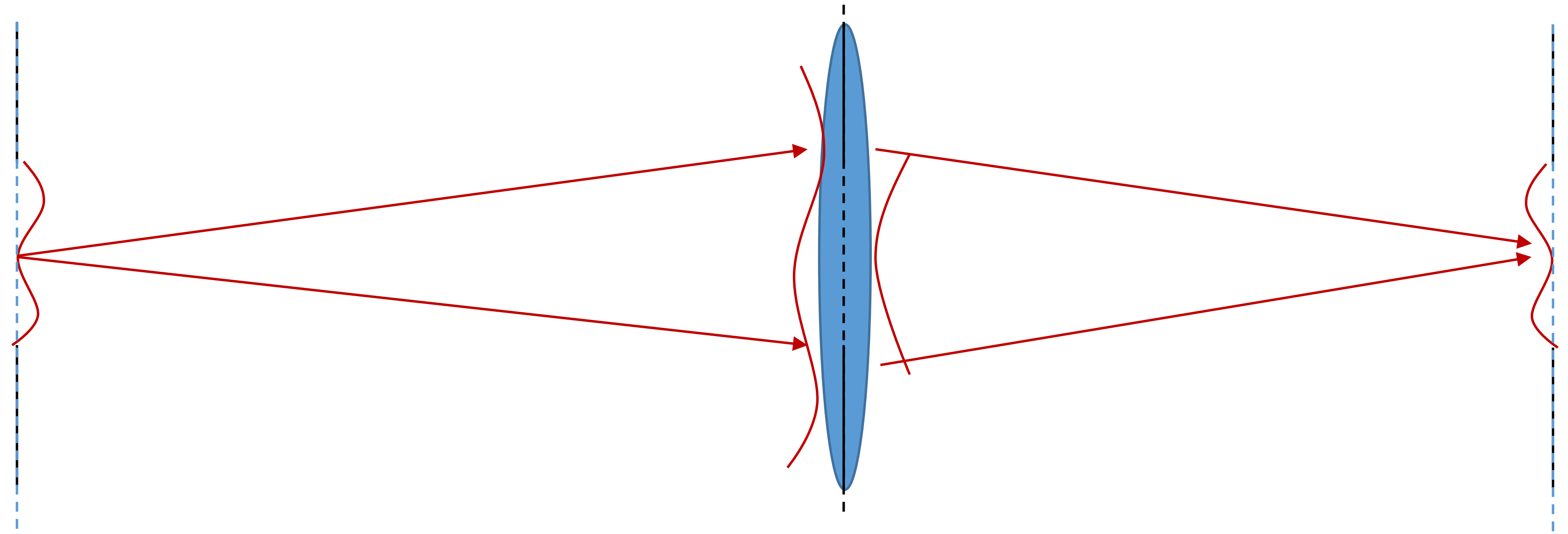
Last piece of the puzzle: what happens from lens to sensor?

inverse Fourier transform!



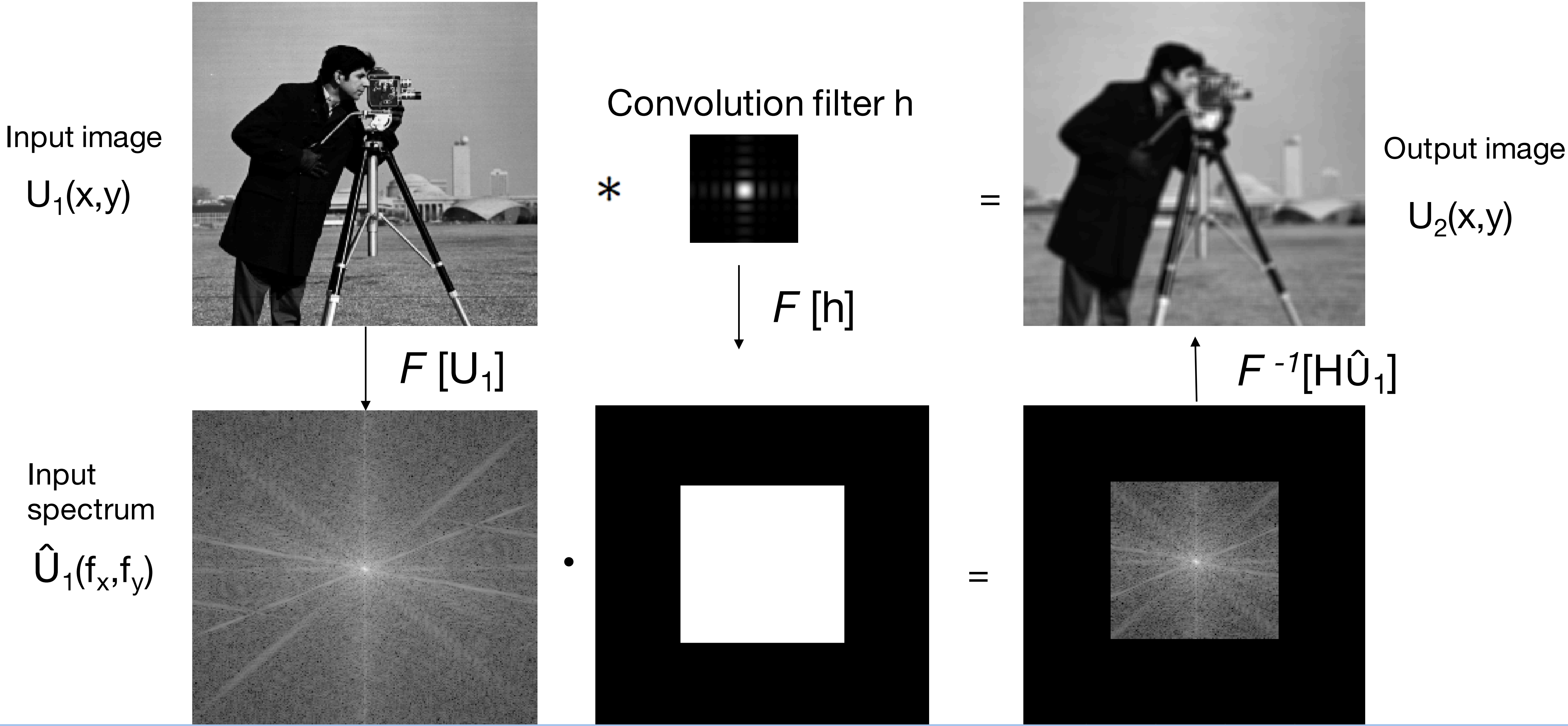
$$E_s(x_s, y_s, 0)$$

$$E'_a(x_d, y_d) = E_a(x_d, y_d)A(x_d, y_d)$$

2D Fourier Transform

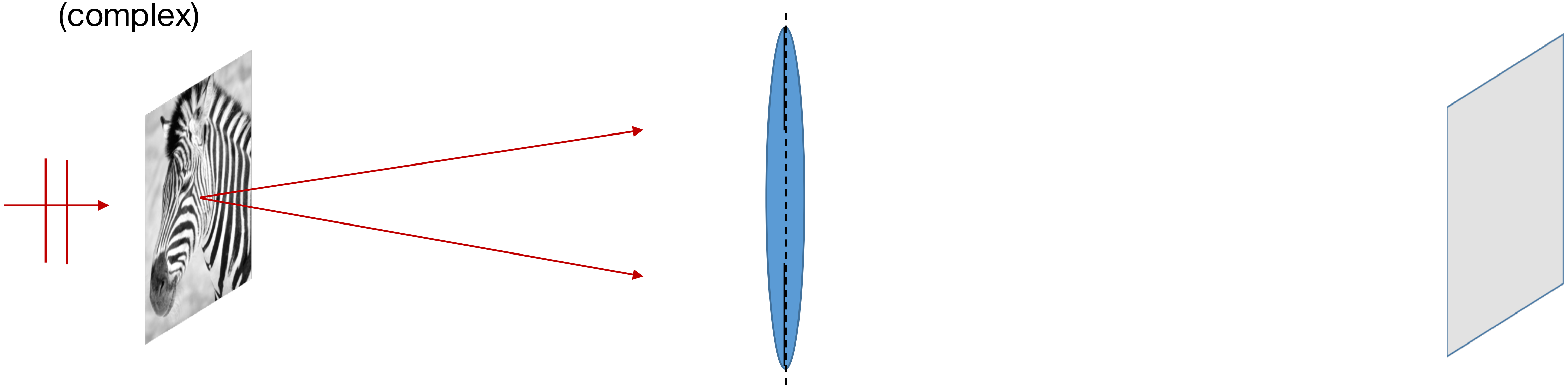
2D inverse Fourier Transform

This process should sound familiar….



Model of image formation for wave optics (coherent light):

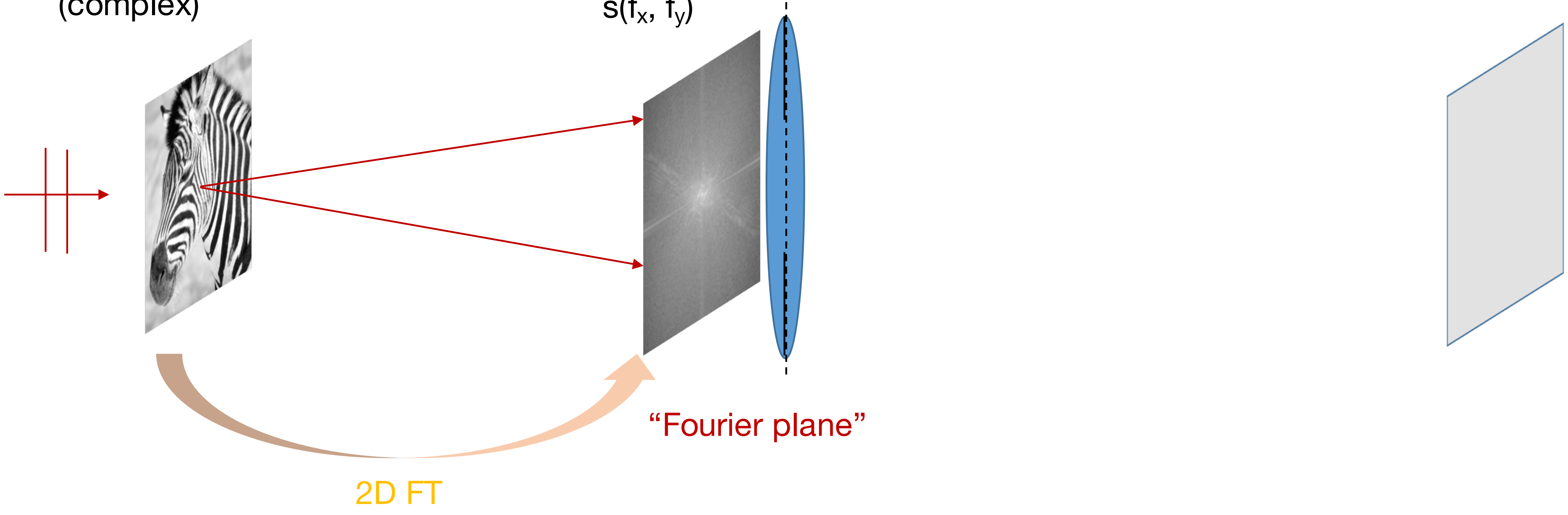
1. Discrete sample function s(x,y) (complex)



Model of image formation for wave optics (coherent light):

1. Discrete sample function s(x,y) (complex)

2. Compute its 2D Fourier transform $\hat{s}(f_x, f_y)$



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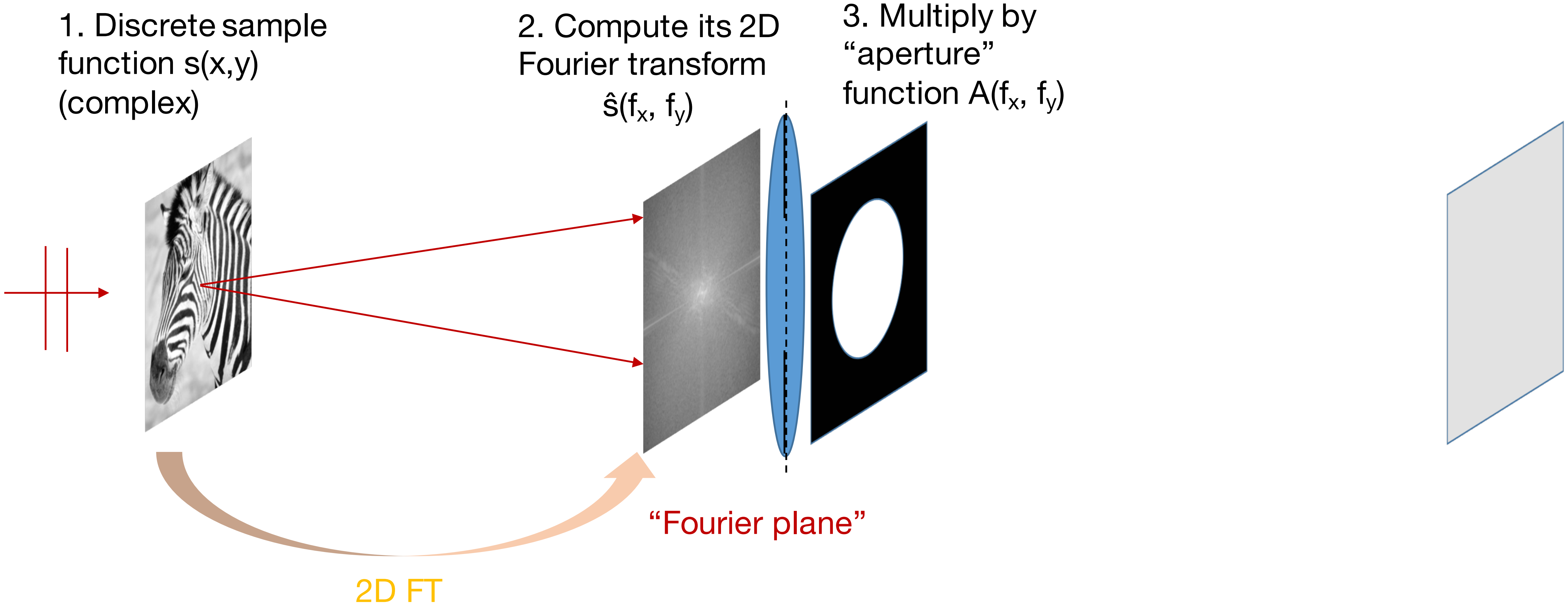

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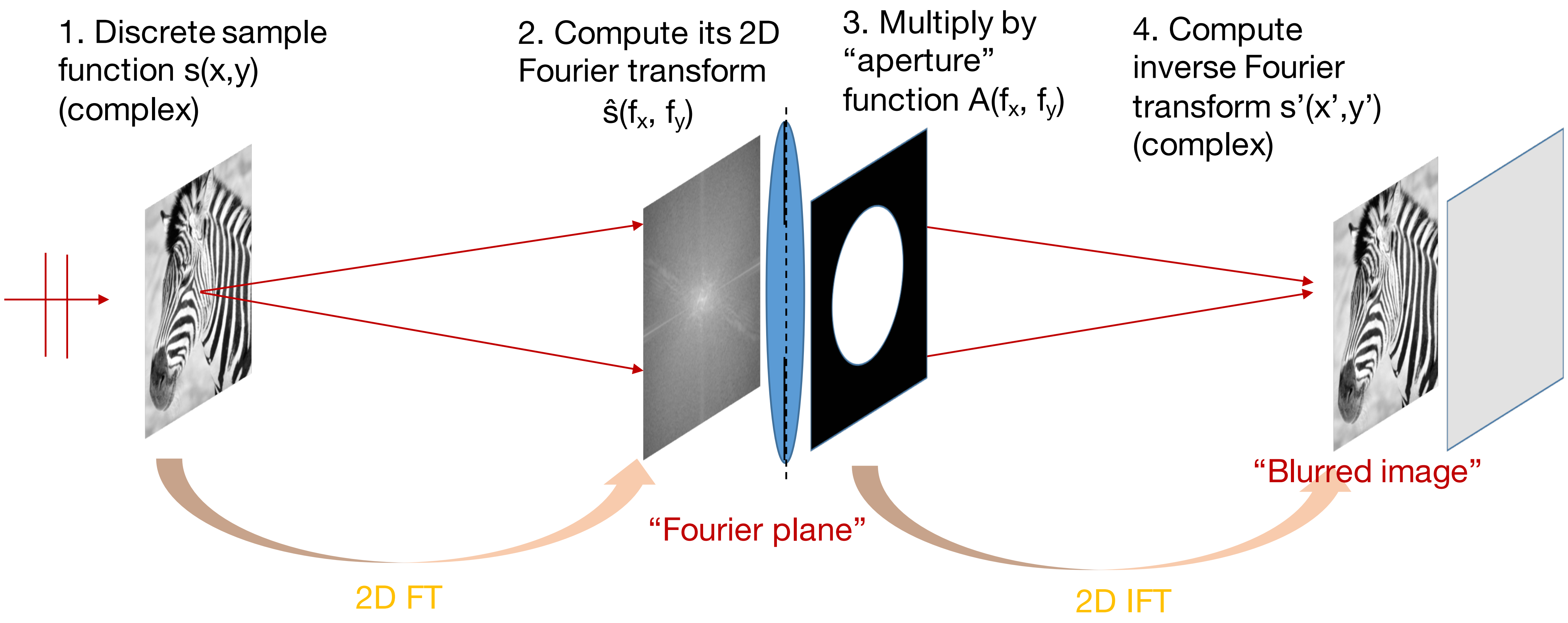

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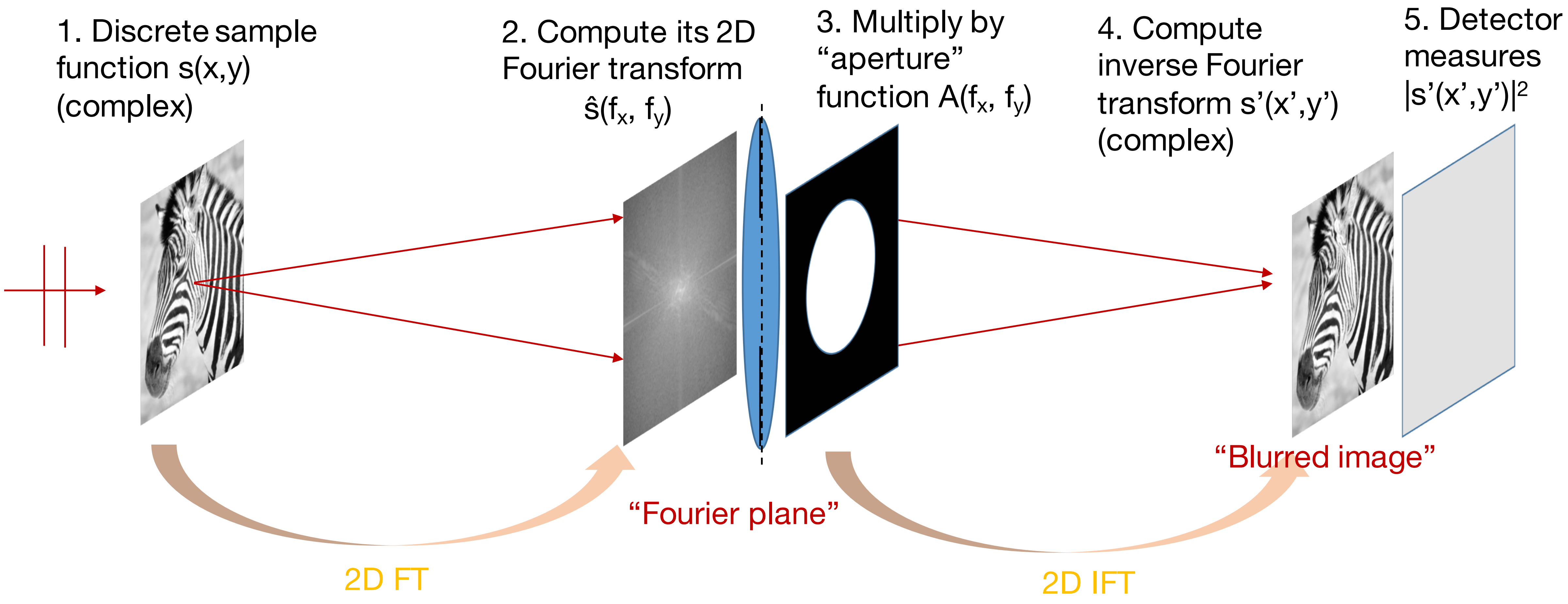


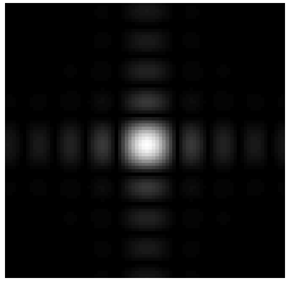
Can also model this using the Convolution Theorem

Aperture function (lens shape)

Camera blur function (IFT of lens shape)

$A(f_x, f_y)$

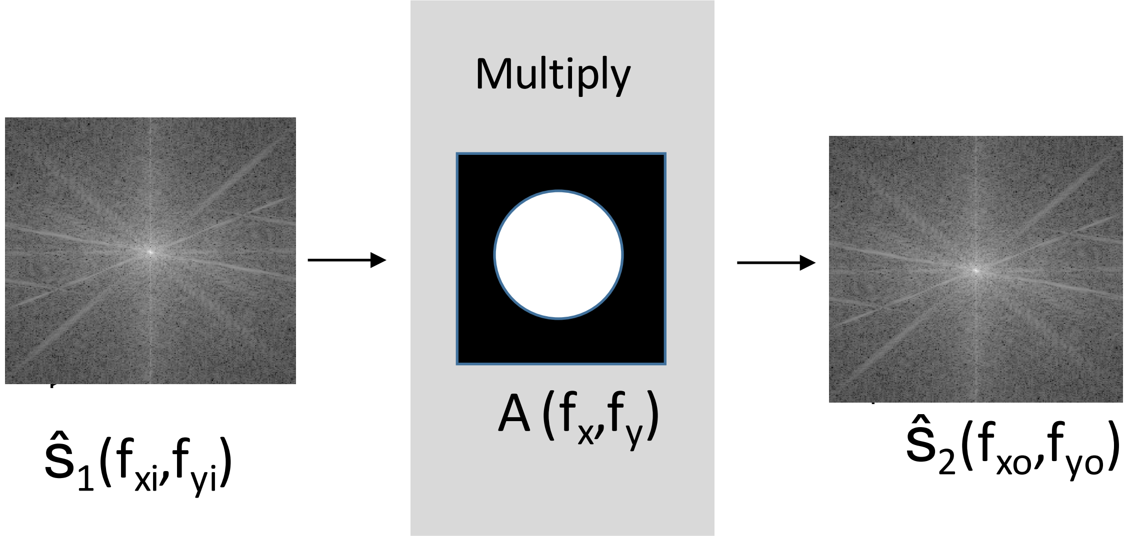
$h(x, y)$



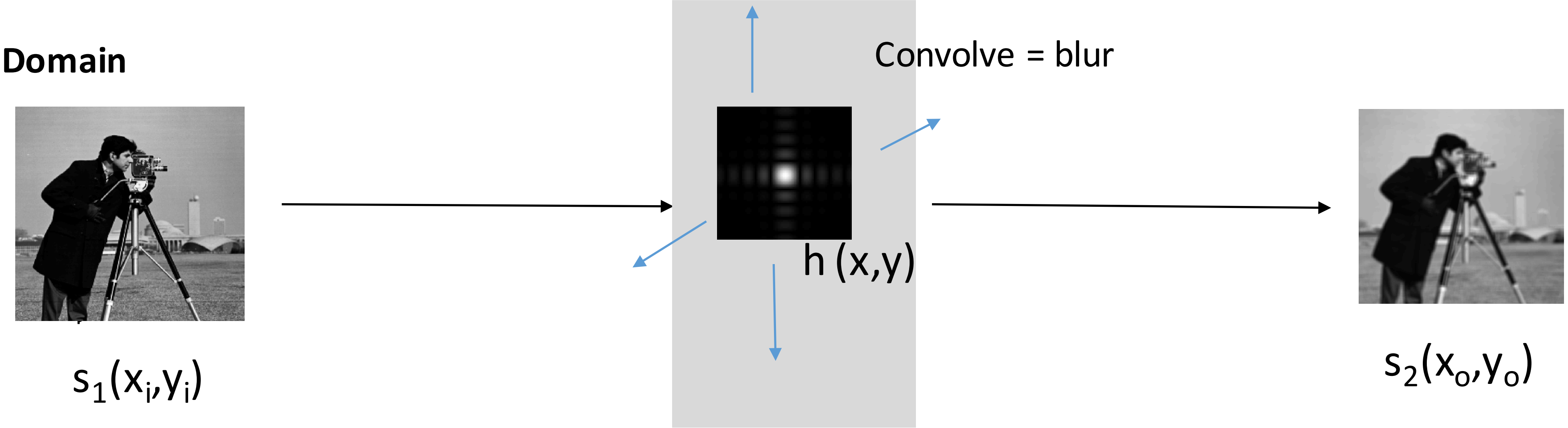
2D IFT

Two modeling choices for the camera:

Spatial Frequency Domain



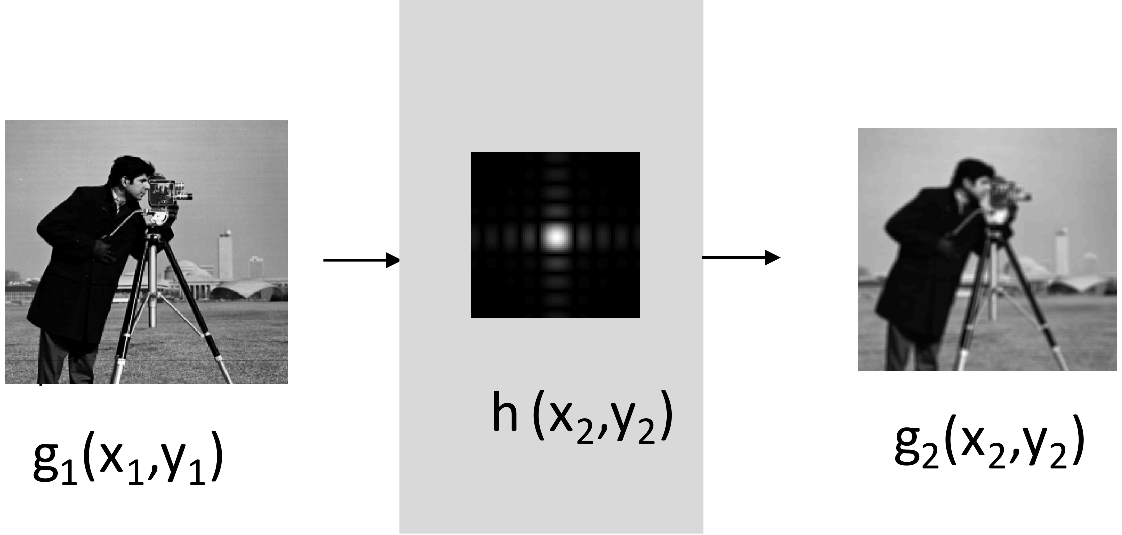
Spatial Domain



Linear systems and the black box

The optical black box system and the point-spread function:

Light $g_1(x_i,y_i)$ entering “black box” optical system modified by system point-spread function

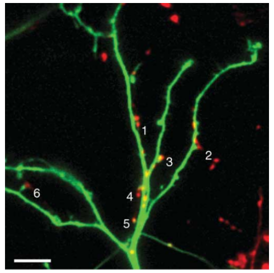


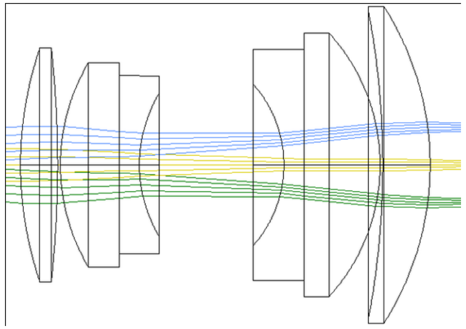
$$g_2(x_2, y_2) = \iint_{-\infty}^{\infty} g_1(x_1, y_1) h(x_2 - x_1, y_2 - y_1) dx_1 dy_1$$

Assume shift invariance: This is the system point-spread function

Summary of two models for image formation

- **Interpretation #1: Radiation (*Incoherent*)**
- Model: Rays





- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

Mathematical model of for incoherent image formation

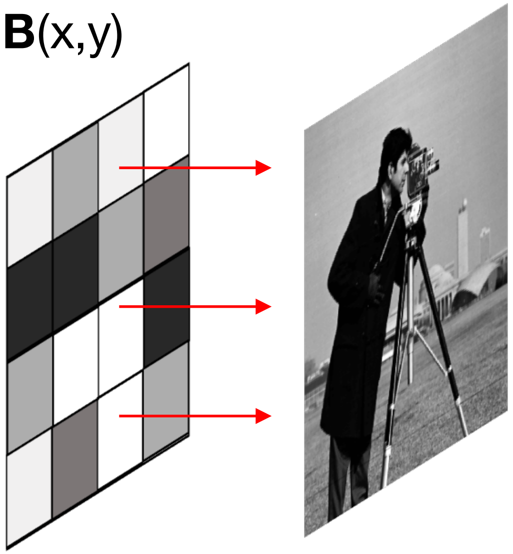
- All quantities are real, and non-negative

Object absorption:

$\mathbf{S_0}(x,y)$

Illumination brightness:

$\mathbf{B}(x,y)$



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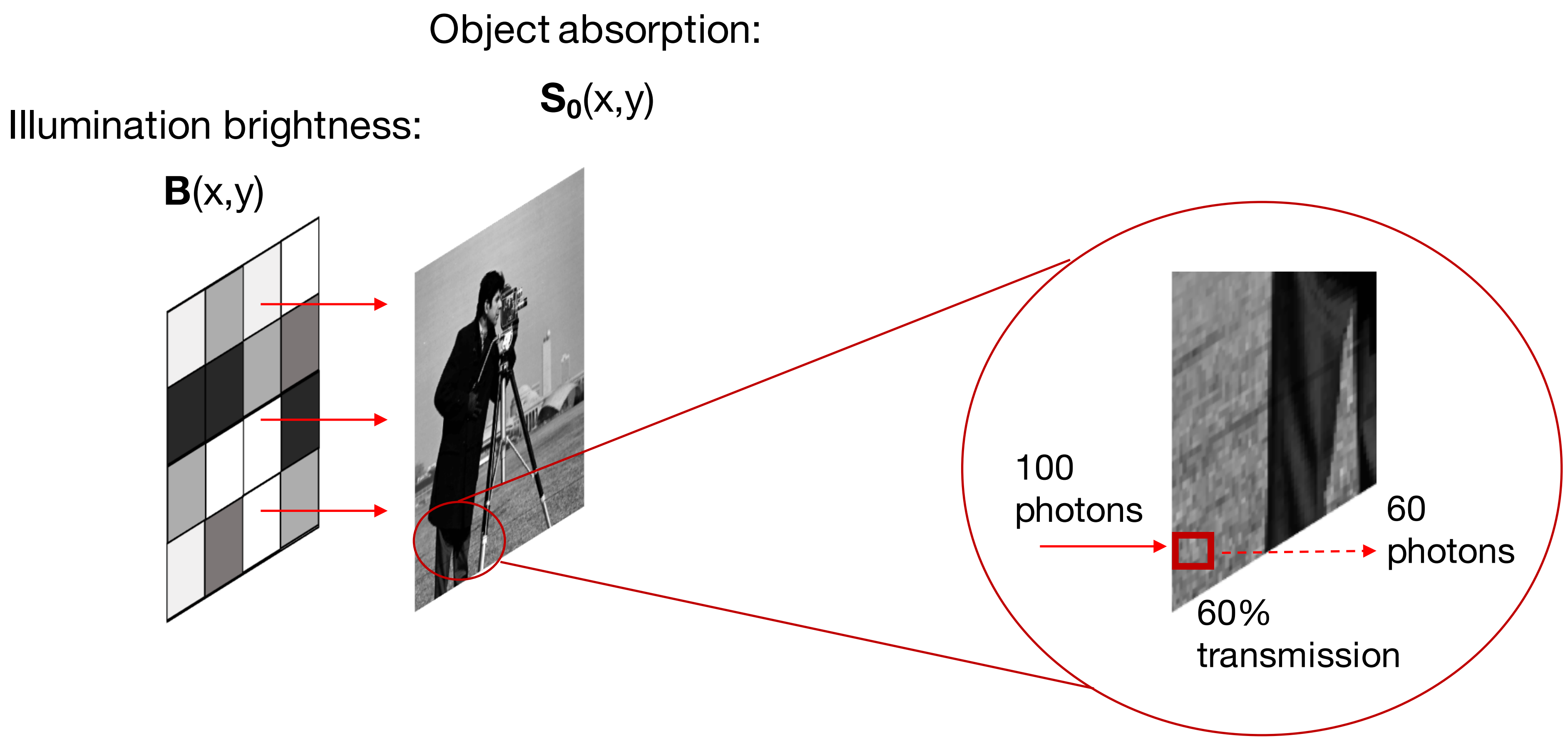

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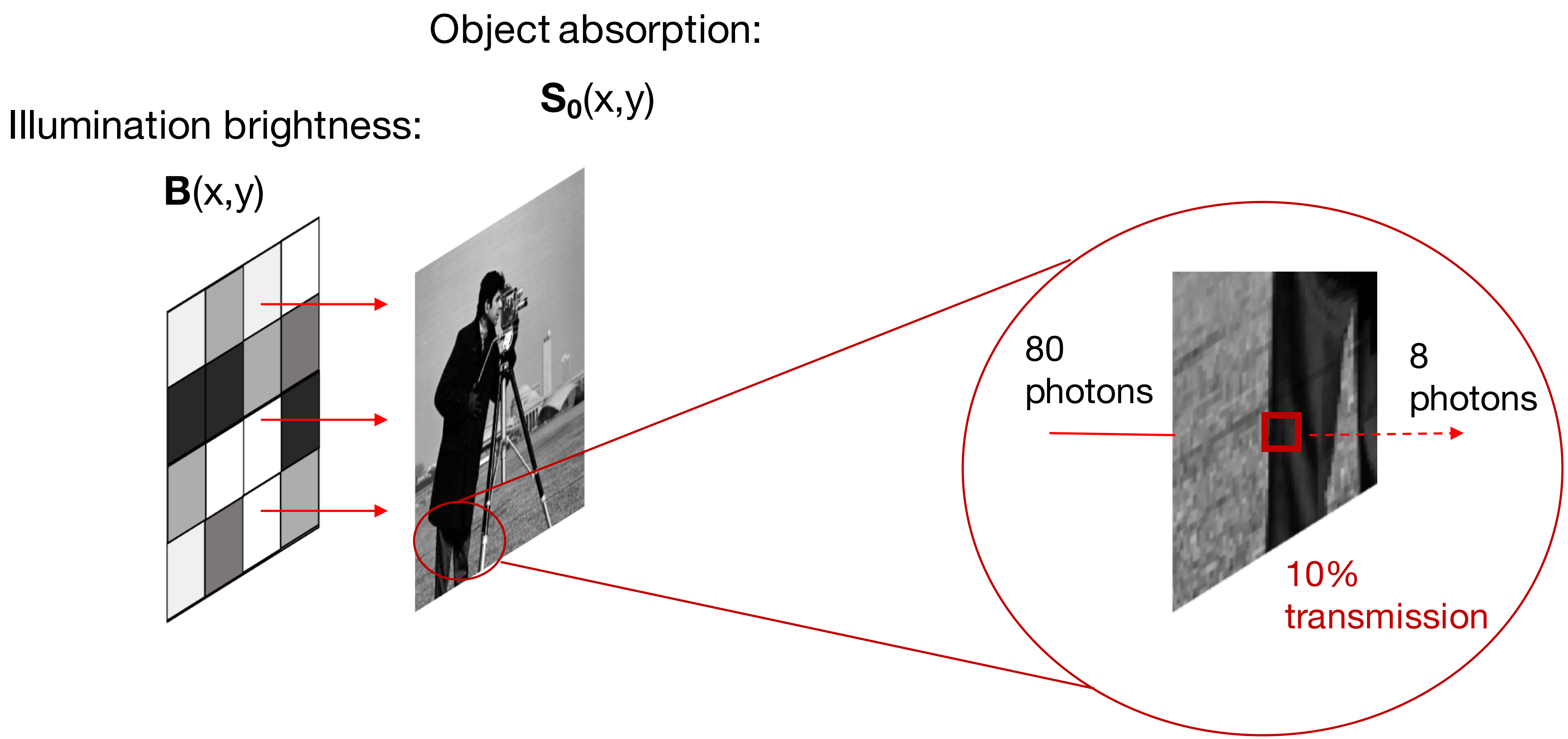

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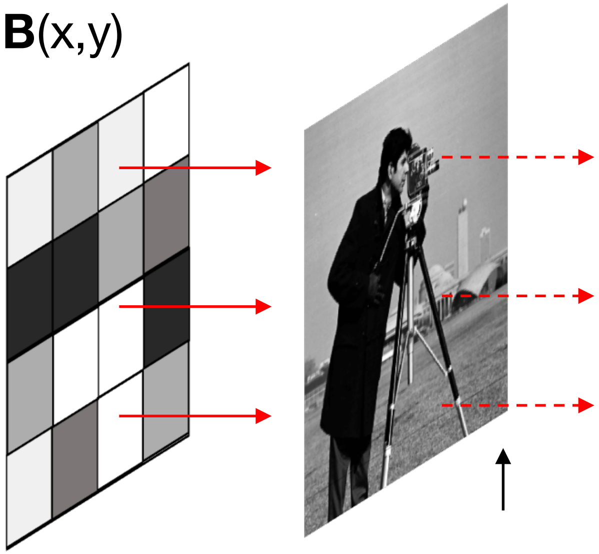
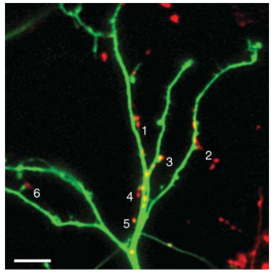
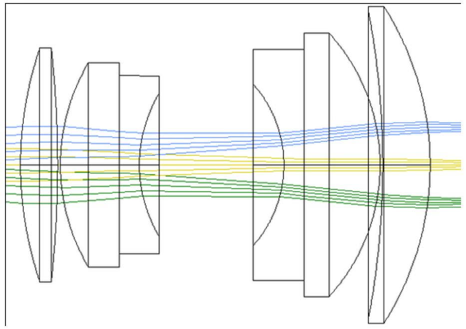


$\mathbf{B}\ \mathbf{S_0}$

multiplication

Summary of two models for image formation

- **Interpretation #1: Radiation (*Incoherent*)**
- Model: Rays

- Real, non-negative
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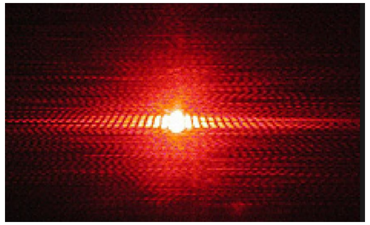
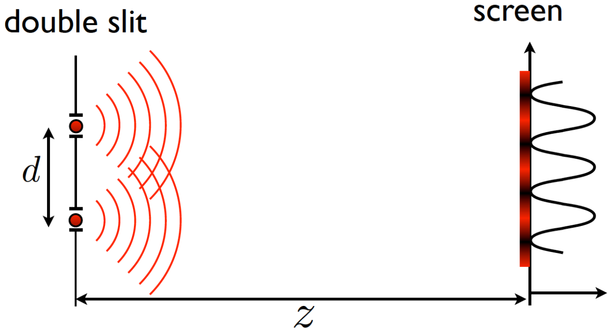
$\mathbf{I_s = B\ S_0}$

- **Interpretation #2: Electromagnetic wave (*Coherent*)**
- Model: Waves





- Complex field
- Models Interference

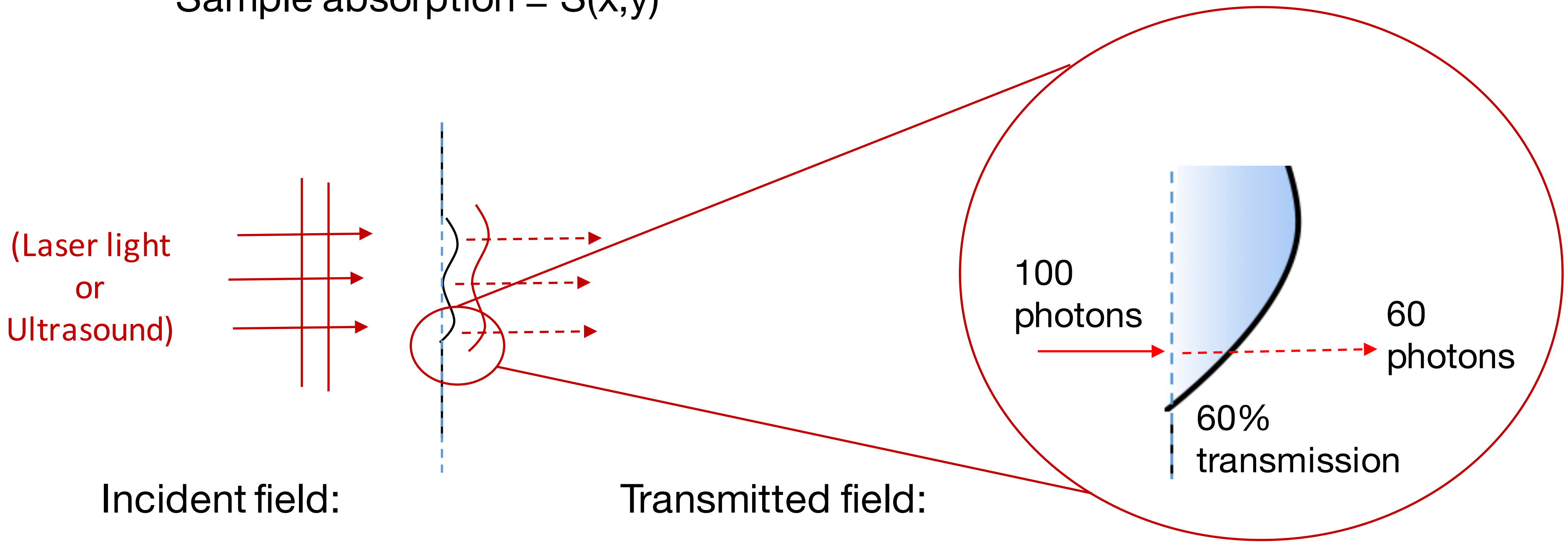
$E_{tot} = E_1 + E_2$

Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = $S(x,y)$

Point #1: Amplitudes behave just like before



$A_t(x,y) = A_i(x,y)\, S(x,y)$

Incident field:

$C(x,y) = A_i(x,y)$

Transmitted field:

$U(x,y) = A_t(x,y) = ?$

Mathematical model of for coherent image formation

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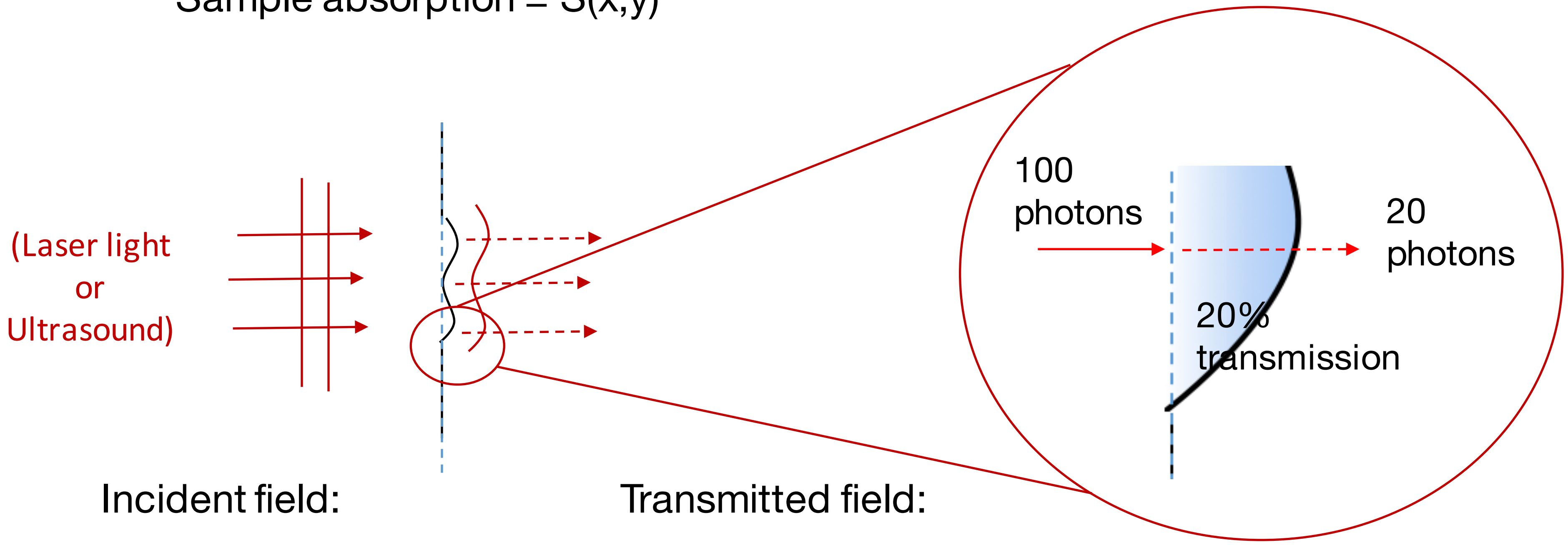

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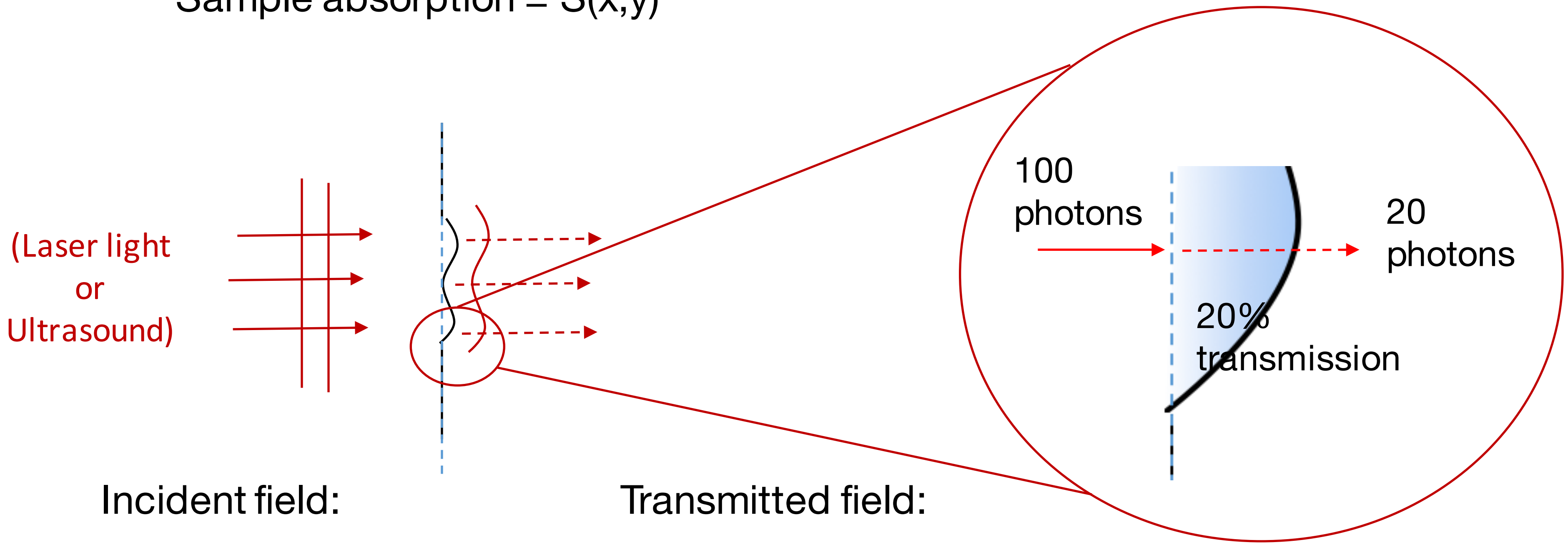

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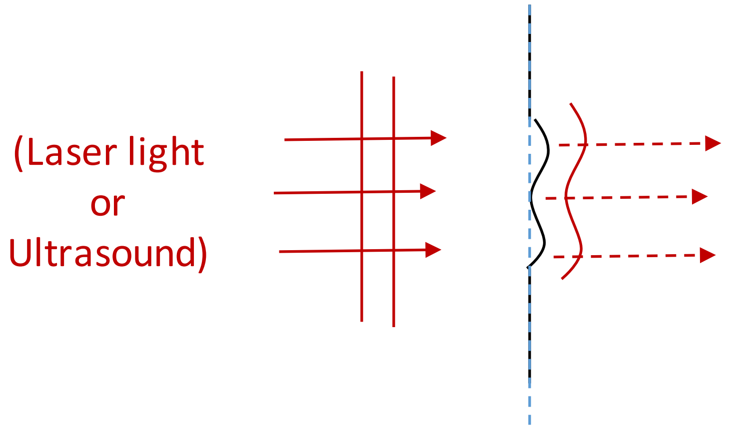
Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = $S(x,y)$

Sample phase delay = $\exp[ik\varphi(x,y)]$

New: complex phase delay
- Needed to represent wave
- Represents wave delay across space



Incident field:

$C(x,y) = A_i(x,y) \exp[ik\varphi_i(x,y)]$

Transmitted field:

$U(x,y) = A_i(x,y)\, S(x,y) \exp[ik\varphi_t(x,y)]$

Mathematical model of for coherent image formation

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Sample absorption = S(x,y)

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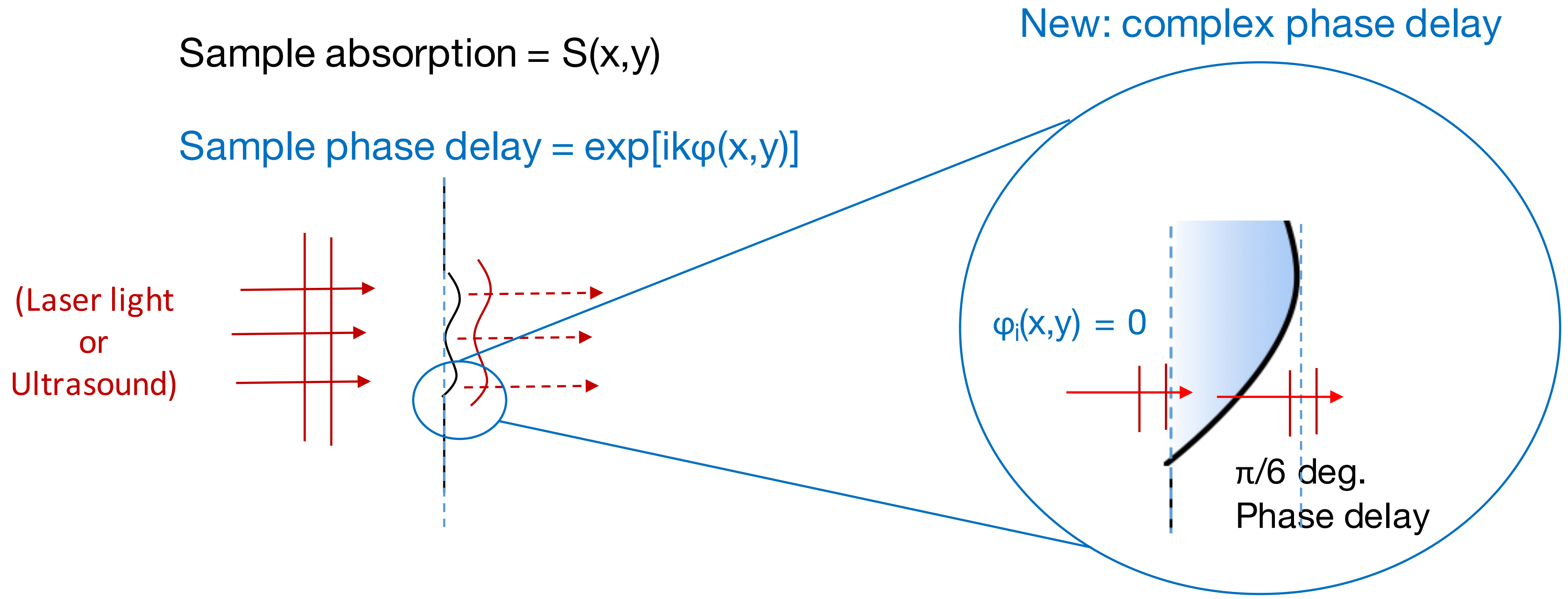

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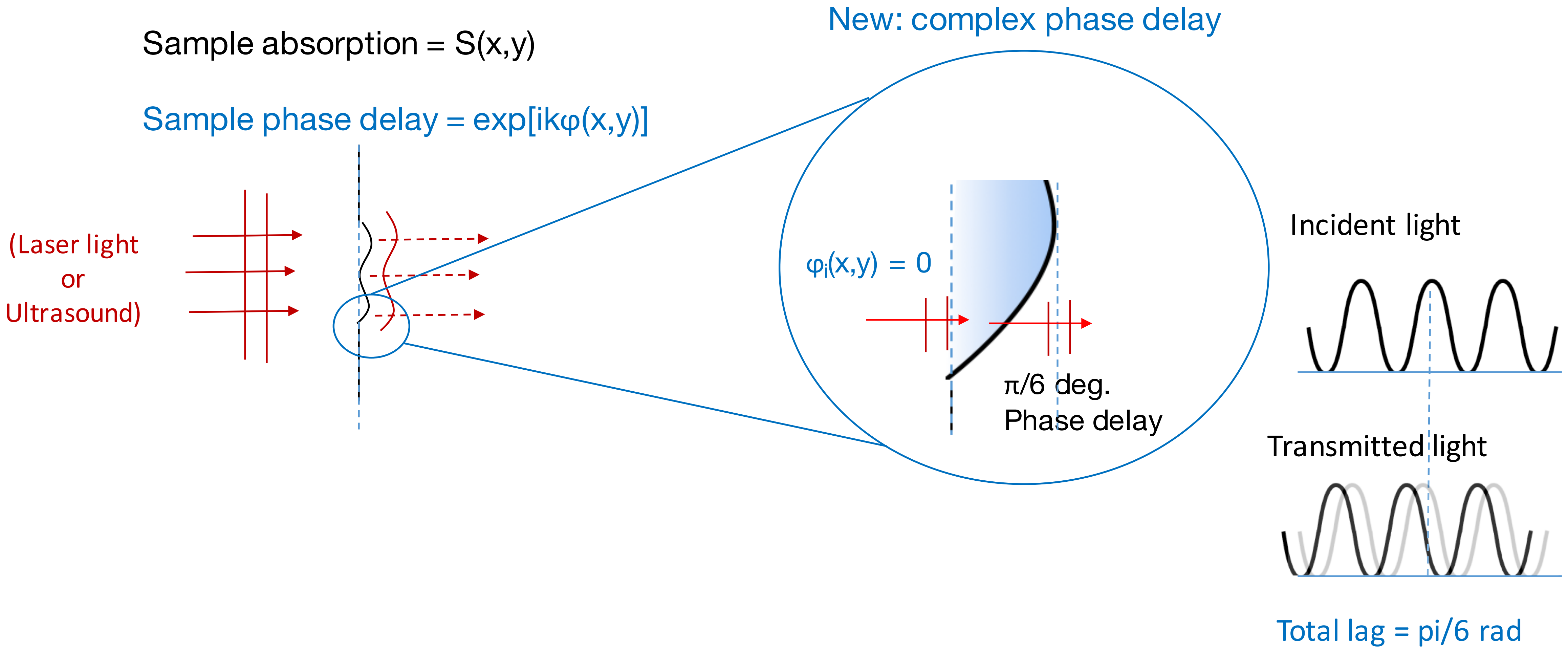

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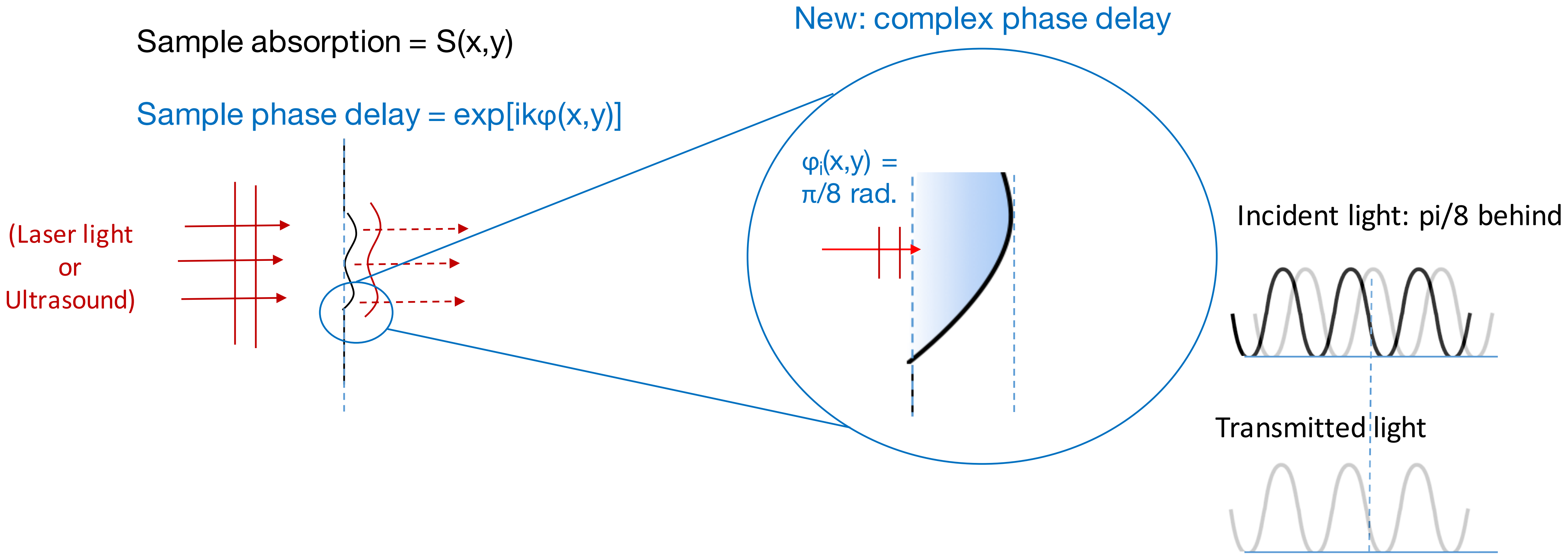

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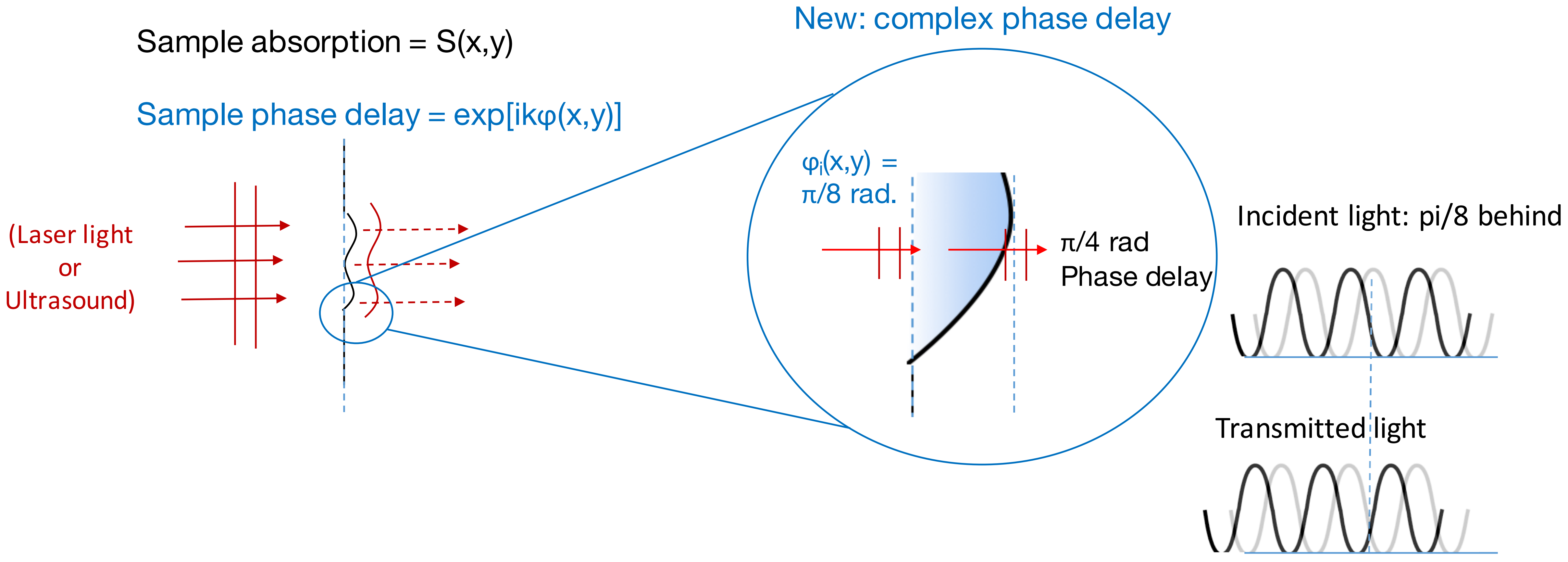

Total lag = π/8+ π/4= 3π/8 rad

Mathematical model of for coherent image formation

deep imaging

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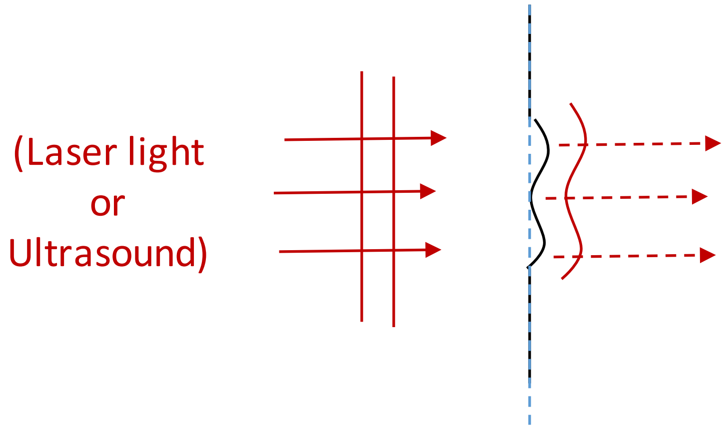
Sample absorption = S(x,y)

Sample phase delay = $\exp[ik\varphi(x,y)]$

Output phase is sum of phase delays, product of phasors



$$\varphi_t(x,y) = \varphi(x,y) + \varphi_i(x,y)$$

$$\exp[ik\varphi_t(x,y)] = \exp[ik\varphi_i(x,y)]\ \exp[ik\varphi(x,y)]$$

Incident field:

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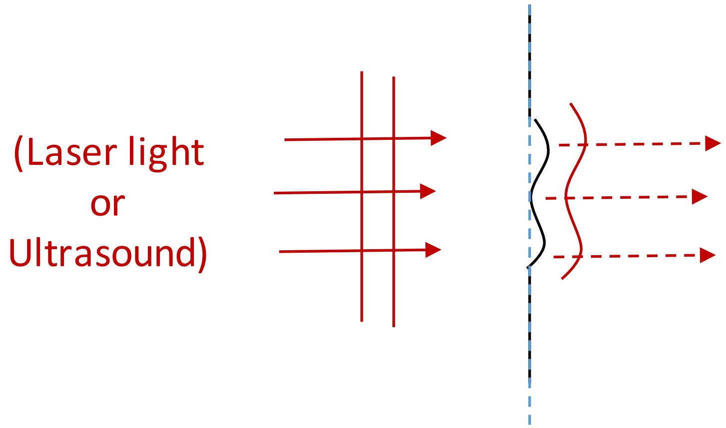

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Conclusion:

Transmitted field = incident field x complex sample :

$U(x,y) = C(x,y)\ S(x,y) \exp[ik\varphi(x,y)]$

Summary of two models for image formation

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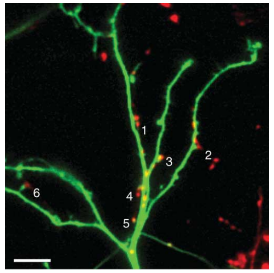

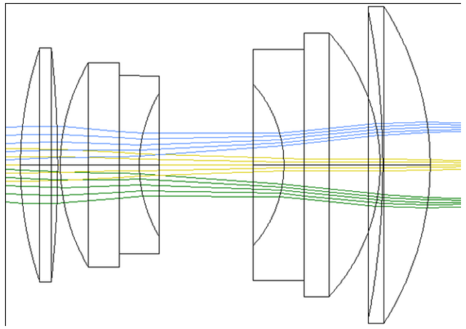

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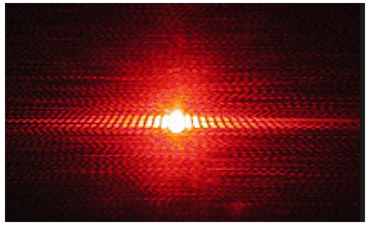

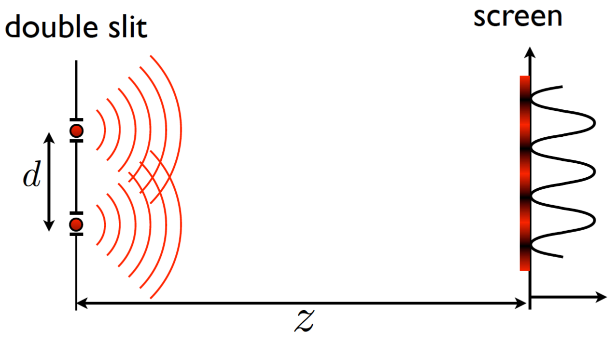

- Complex field
- Models Interference

$$E_{tot} = E_1 + E_2$$

$$\mathbf{U = C\ S_0}$$

U, C and S are complex!

Additional Information about sample index of refraction, spatial frequency and Fourier optics

Microscope illumination and sample index of refraction

So far: illuminate the sample and create a field that is equivalent to the sample's absorption and phase
– why is this true???

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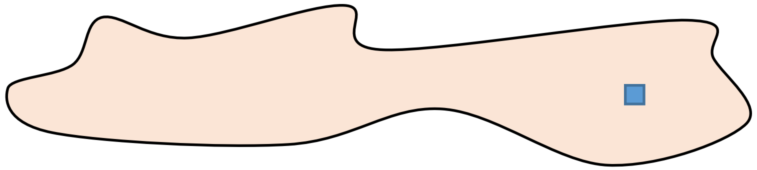

$a(x=1) = .2$

$\varphi(x=1) = .1$

*For more information, see D. Paganin, Coherent X-Ray Optics, Section 2.2

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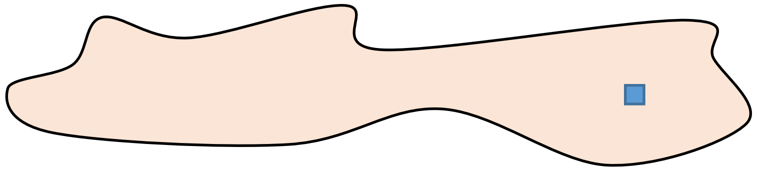

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Sample's effect on light is multiplication with $\exp[-ik * n(x,y)]$

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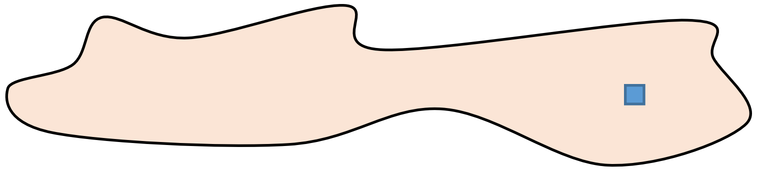

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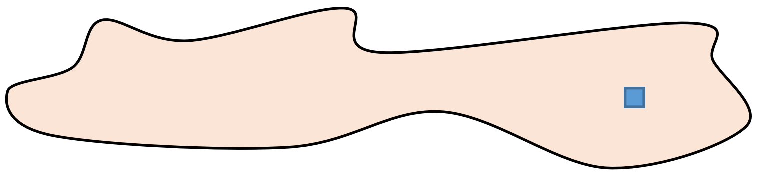

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$$U(x) = U_i(x) * \exp[-ik\, n(x)] = U_i(x)\, A(x)\, \exp[ik\varphi(x)]$$

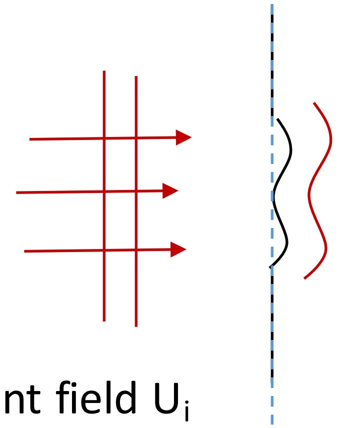
$A(x) = \exp[k\, a(x)]$

absorption

phase shift: new term for laser

Microscope illumination and sample index of refraction

Sample absorption = A(x)
Sample phase = exp[ikφ(x)]



Emerging field U = incident field $U_i(x)$ * sample function s(x)

Incident field U_i

Microscope illumination and sample index of refraction

Q: When is the emerging field equal to the absorption and phase?

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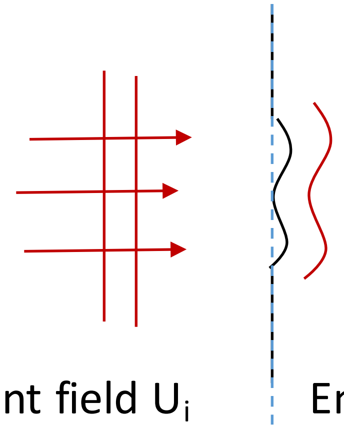

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Emerging field U

Microscope illumination and sample index of refraction

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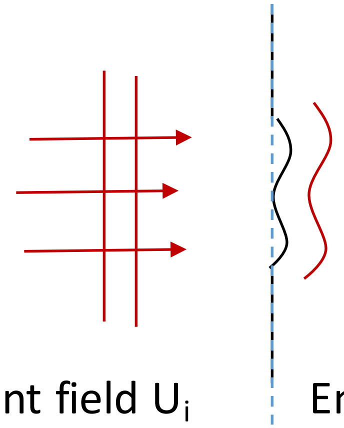
A: When the incident wave = 1, means uniform in amplitude and phase:

$$U_i(x) = 1 \longrightarrow U(x)=A(x)\ \exp[ik\varphi(x)]$$



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Microscope illumination and sample index of refraction

deep imaging

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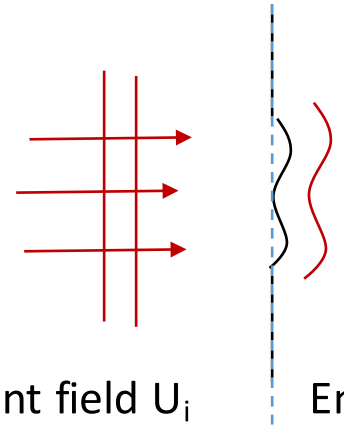
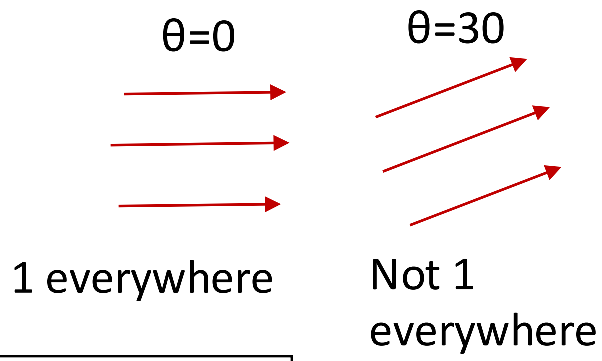


Incident field U_i

Emerging field U

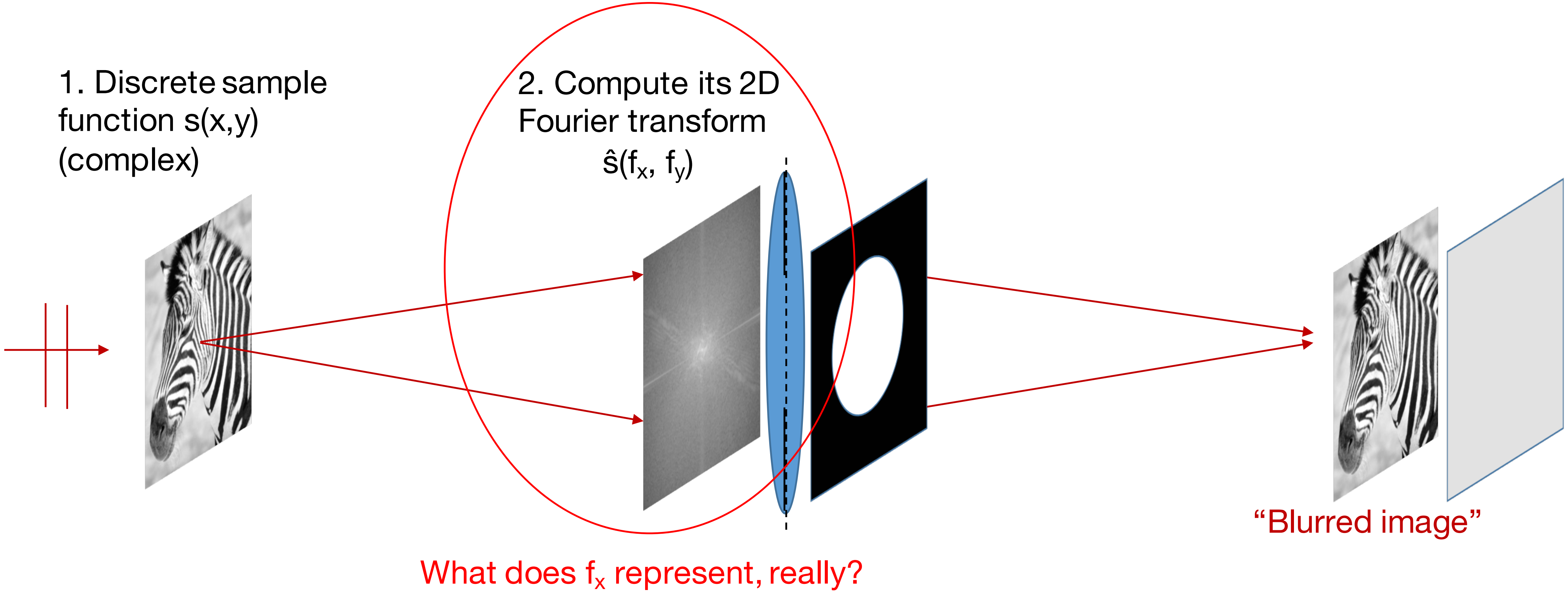
Plane wave $U_i(x) = 1 * \exp(ik \bullet x)$

$$U_i(x) = \exp(ikx \sin(\theta))$$

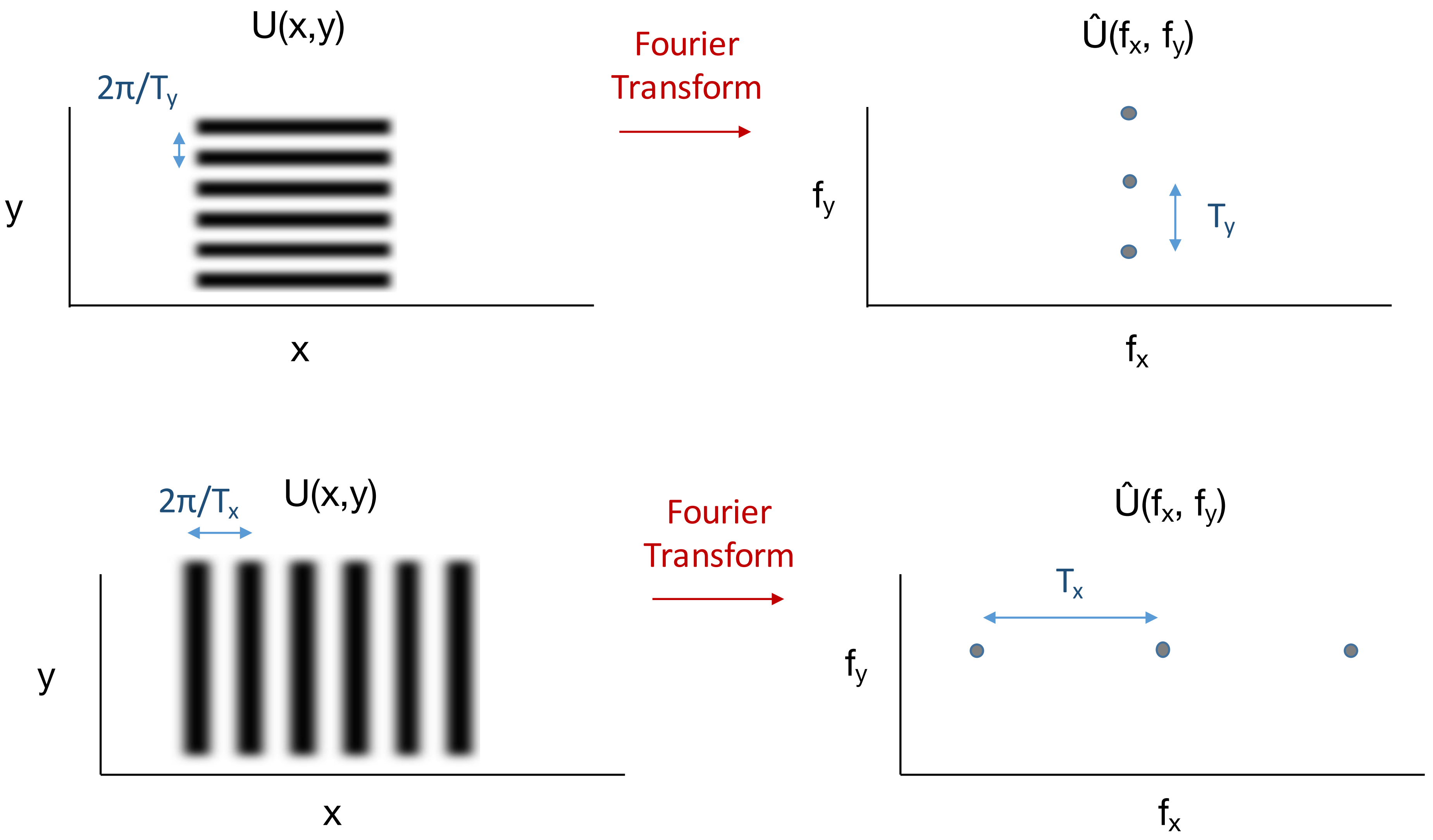


This is when incident wave hits the sample with θ=0!

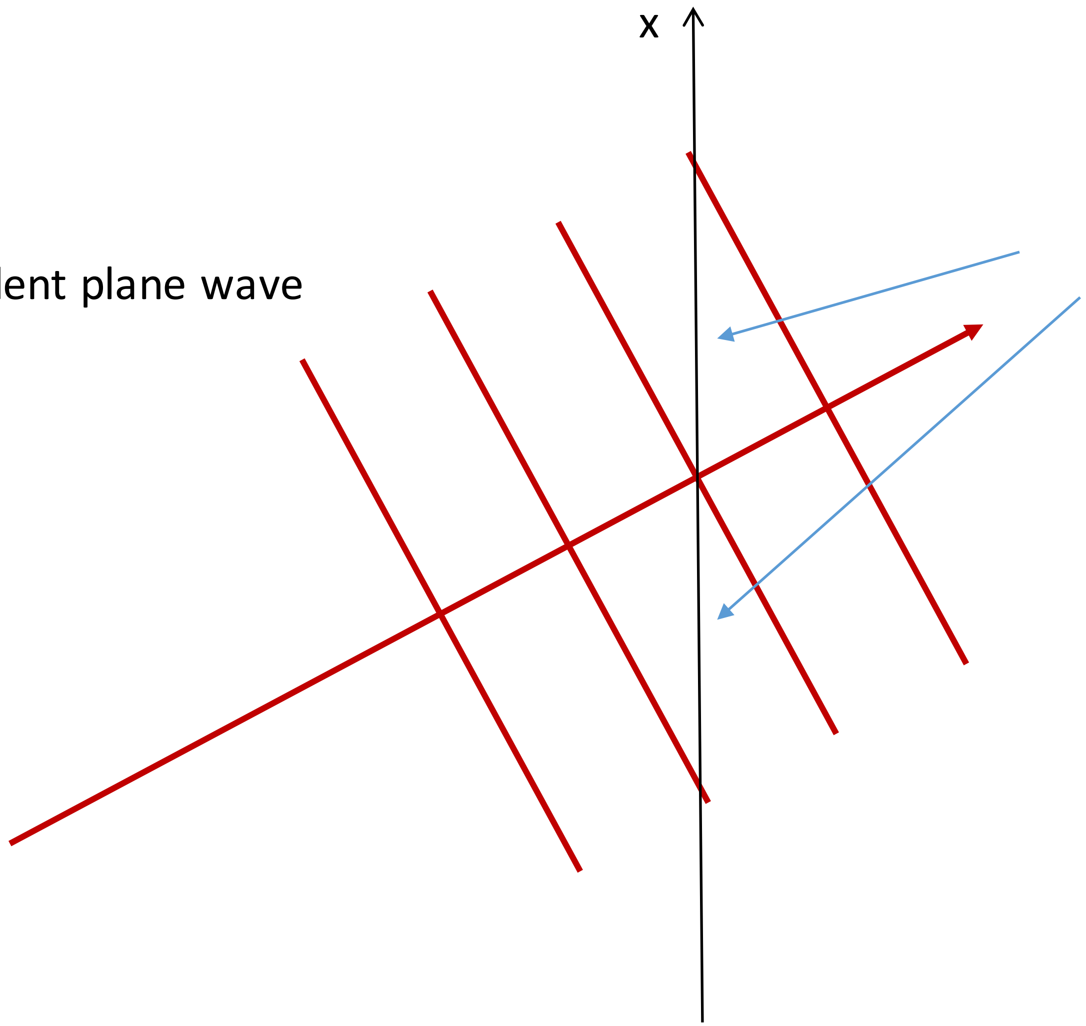
Model of image formation for wave optics (coherent light):



From before: Spatial frequencies = “stripes” within each image



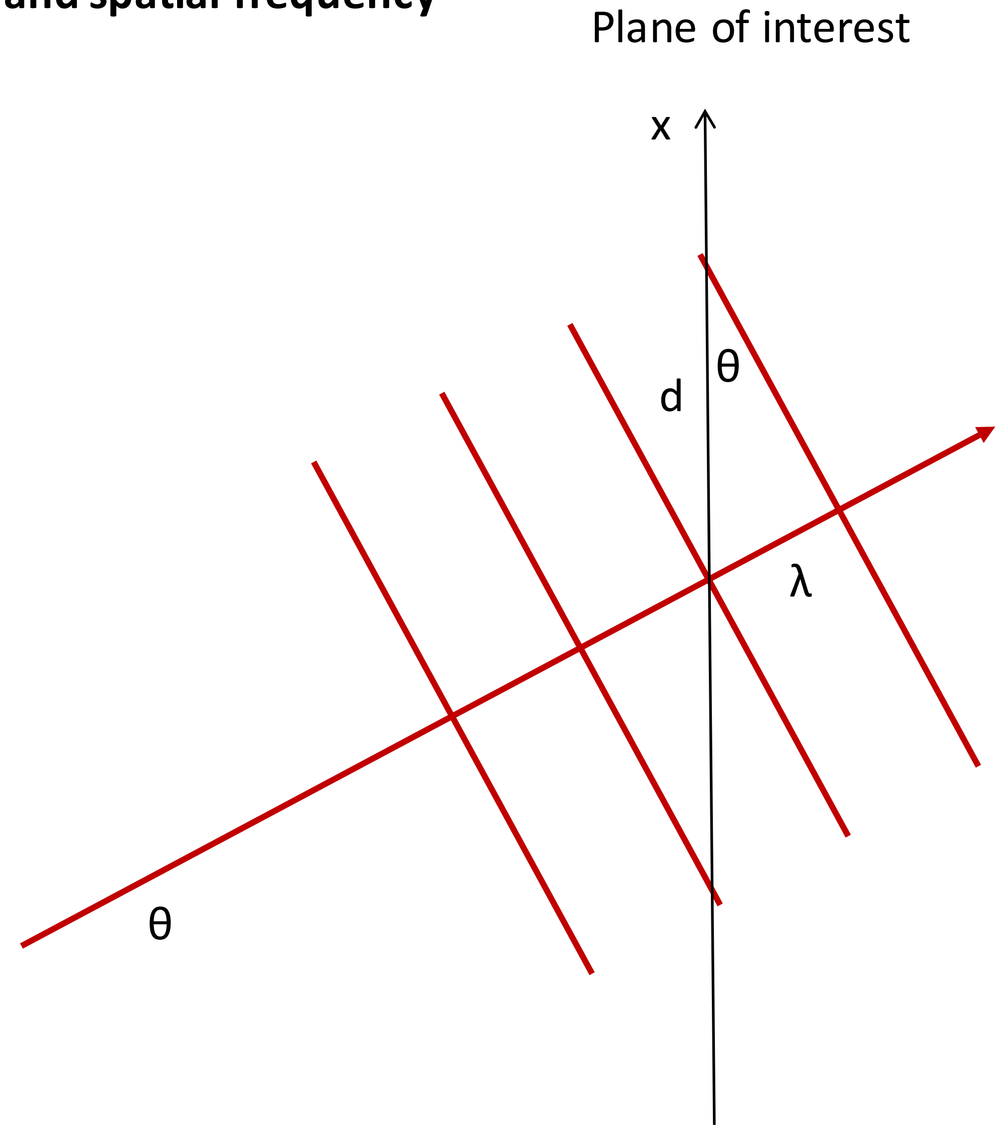
Ray angle and spatial frequency



Stripes are for complex fields!

Proportional to distance between subsequent peaks of wave along plane of interest

Ray angle and spatial frequency

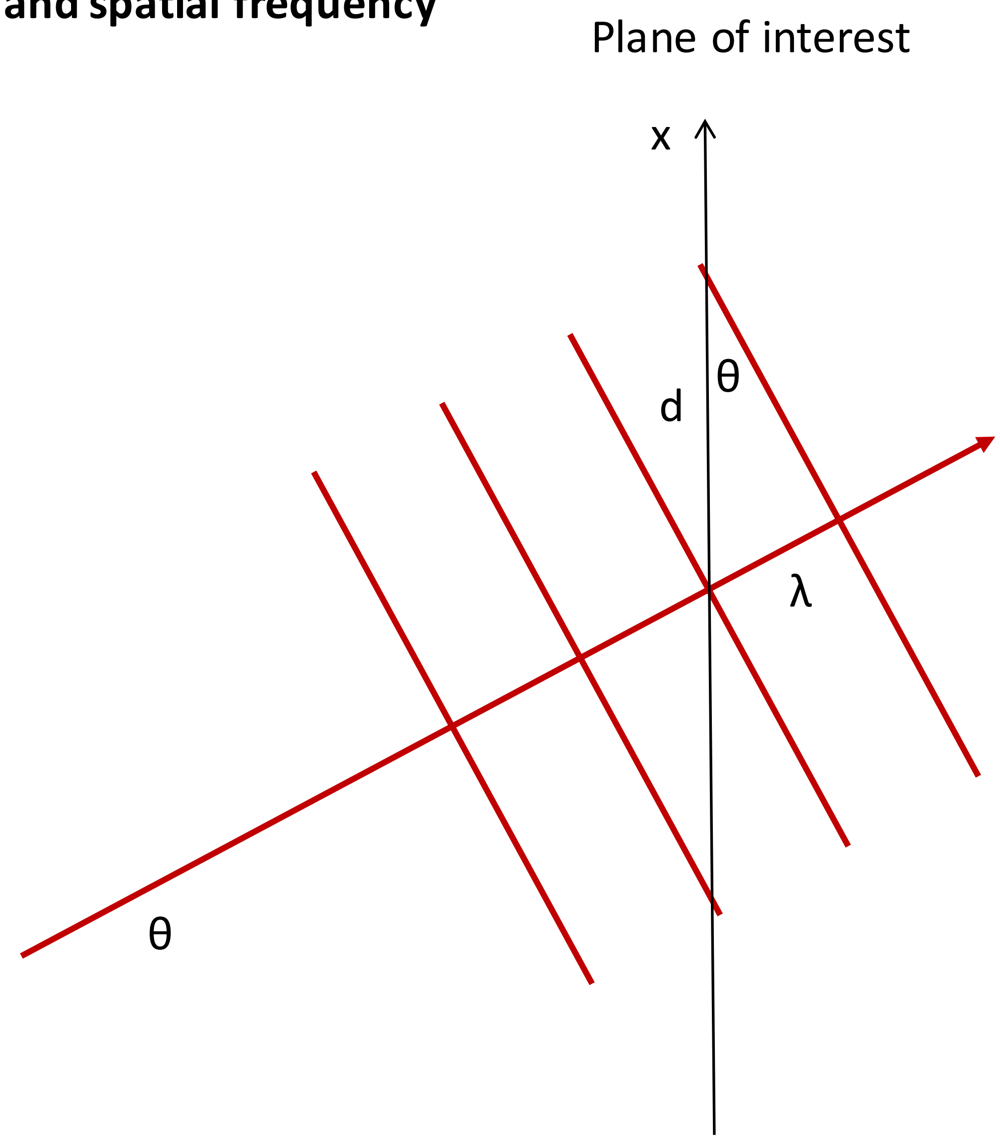


Distance to two crests = spatial period

$\text{Sin}(\theta) = \lambda/d$

$d = \lambda/ \text{Sin}(\theta)$

Ray angle and spatial frequency



Distance to two crests = spatial period

$\mathrm{Sin}(\theta) = \lambda/d$

$d = \lambda/\mathrm{Sin}(\theta)$

Spatial frequency = 1/spatial period
(number of periods per unit length)

$f_x = 1/d = \mathrm{Sin}(\theta)/\lambda$

Equivalent coordinates in the Fourier domain and at the Fourier plane

$f_x = \mathrm{Sin}(\theta)/\ \lambda$

$k_x = 2\pi f_x = 2\pi/\ \lambda\ \mathrm{Sin}(\theta)$

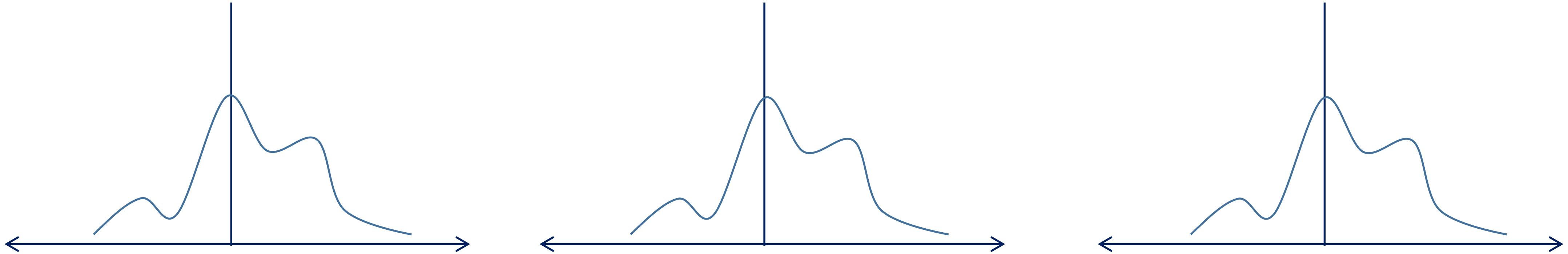


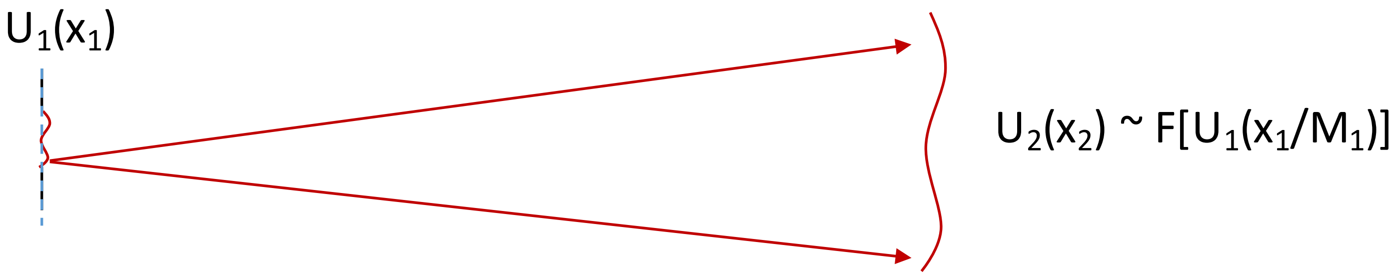
Image plane ray angle θ

Image plane spatial frequency f_x

Image plane wavevectors k_x

General rules for applying the Fourier transform in optics

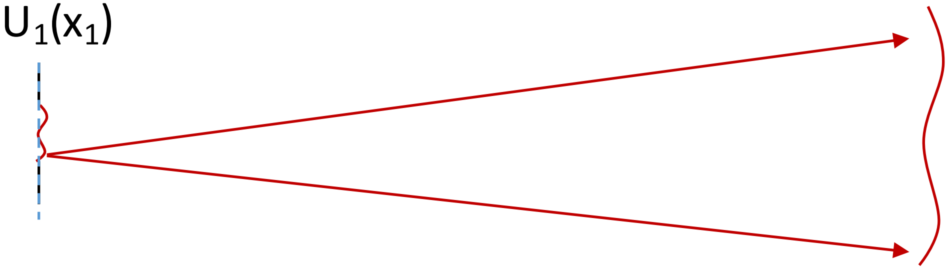
Situation 1: From an object to a plane "really far away"



General rules for applying the Fourier transform in optics

Situation 1: From an object to a plane "really far away"

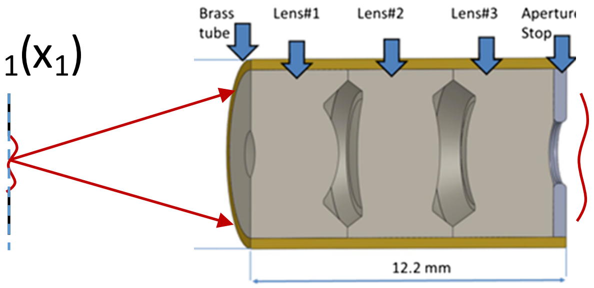
$U_1(x_1)$



$U_2(x_2) \sim F[U_1(x_1/M_1)]$

Situation 2: From an object to the back focal plane of the microscope objective lens

$U_1(x_1)$



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General rules for applying the Fourier transform in optics

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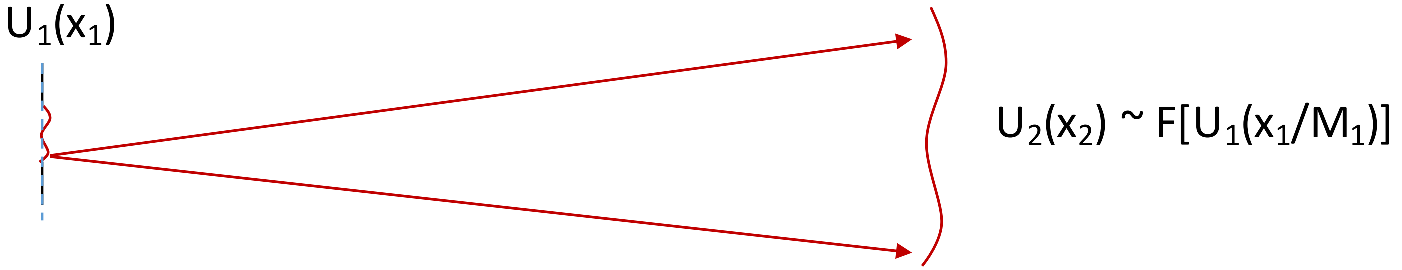

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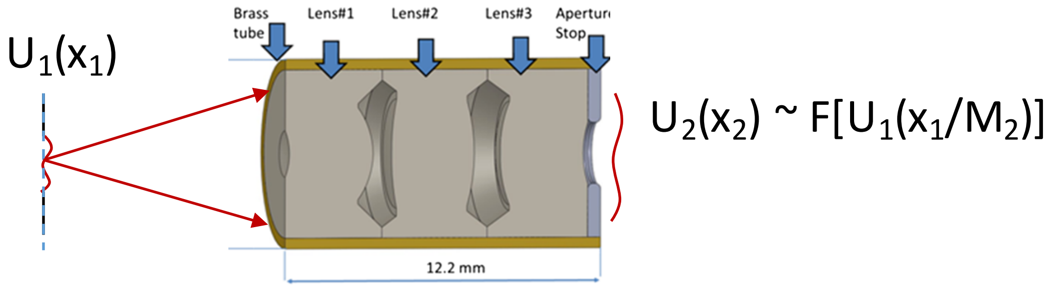
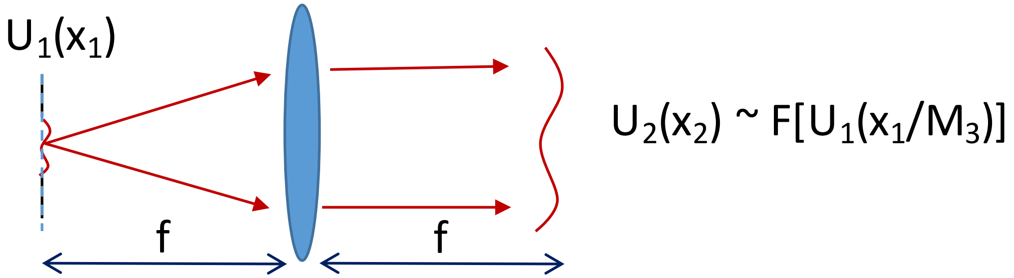
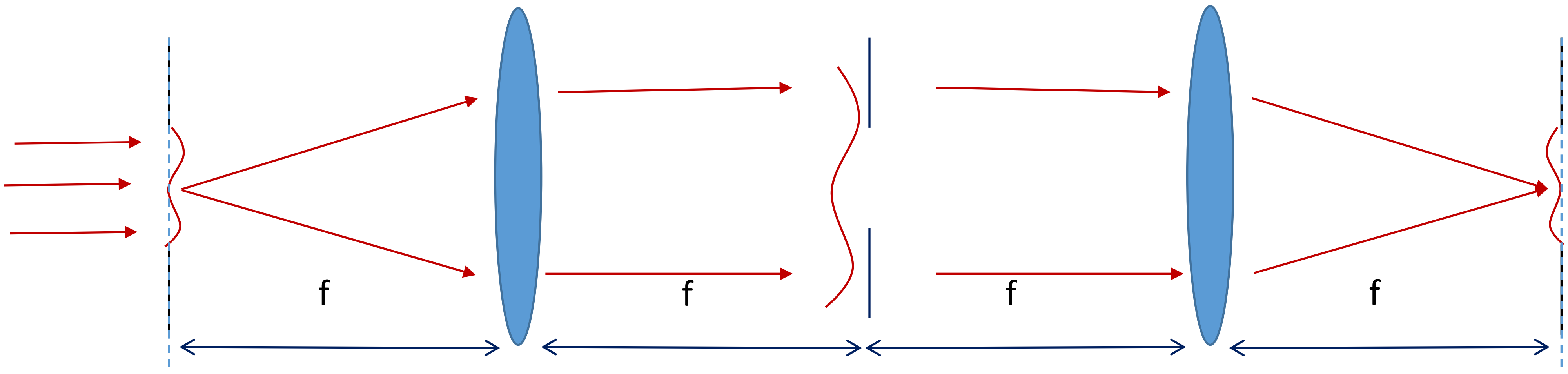


Situation 3: From an object to a plane 1 focal length away from a lens (1f-1f system)



A more exact model: the 4f optical system

deep imaging



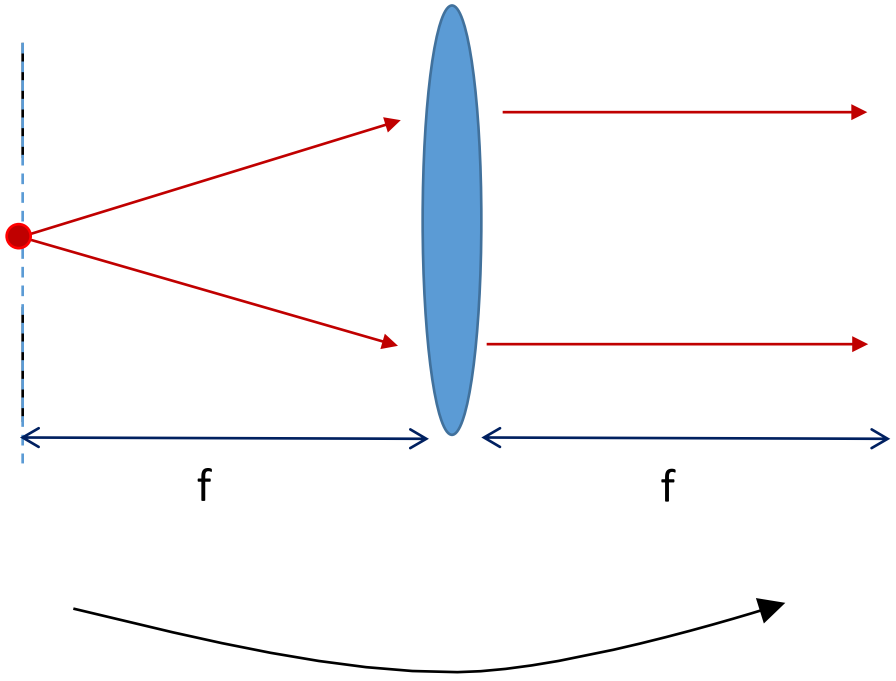
$$E_s(x_s, y_s, 0)$$

$$E'_a(x_d, y_d) = E_a(x_d, y_d)A(x_d, y_d)$$

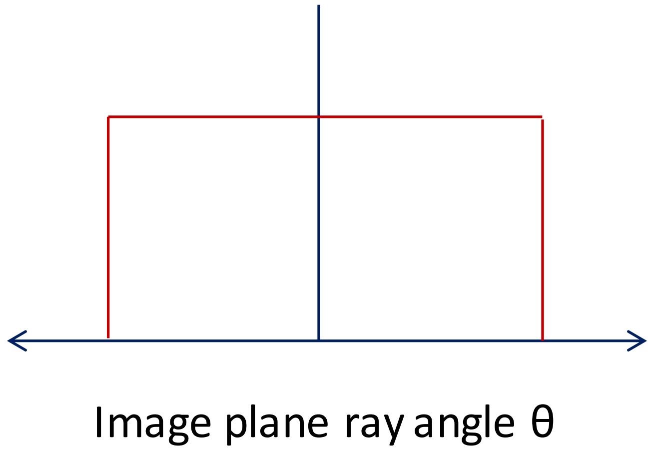
2D Fourier Transform

2D inverse Fourier Transform

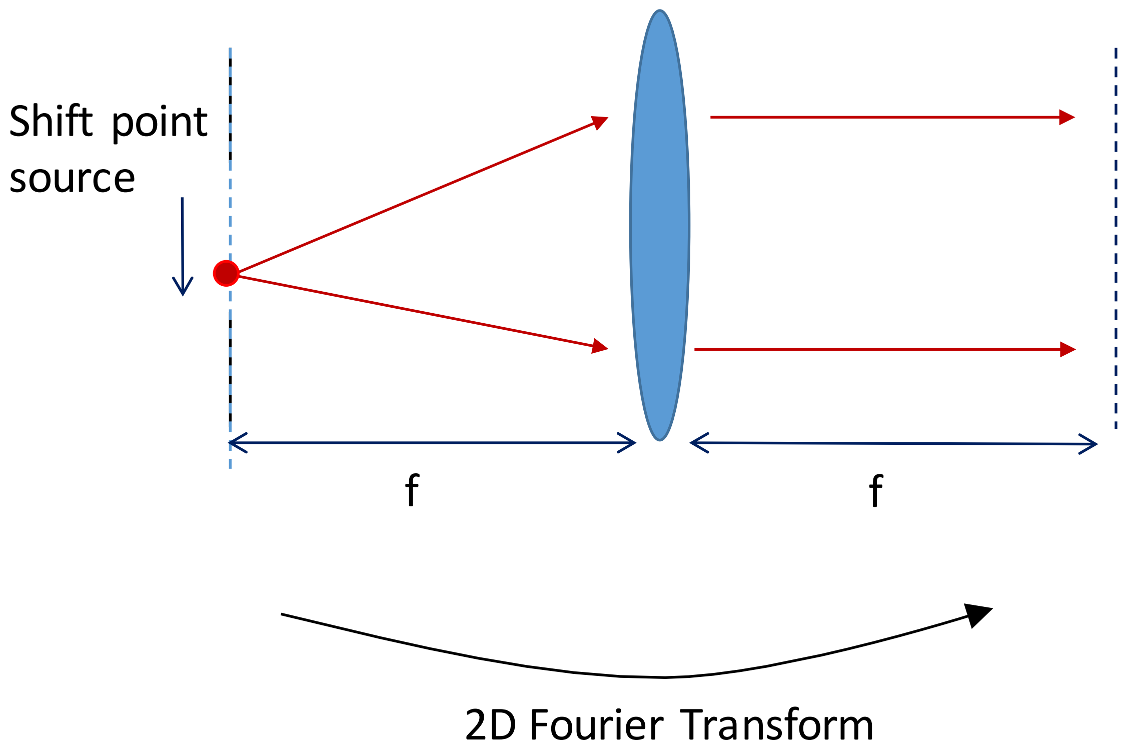
A more exact model: the 4f optical system



The Fourier plane provides a measure of the **ray angles at the image plane**

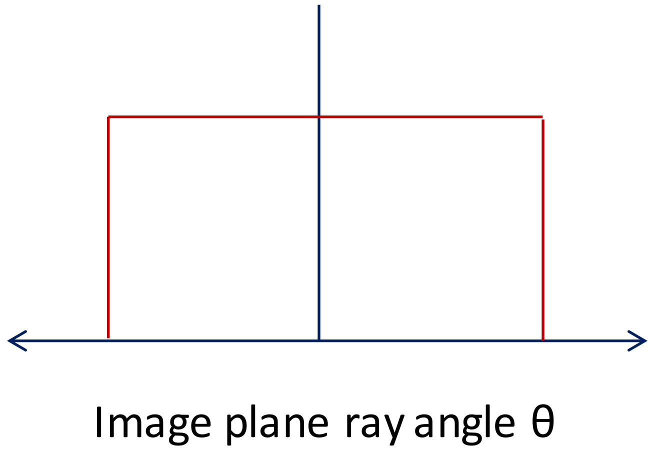


A more exact model: the 4f optical system

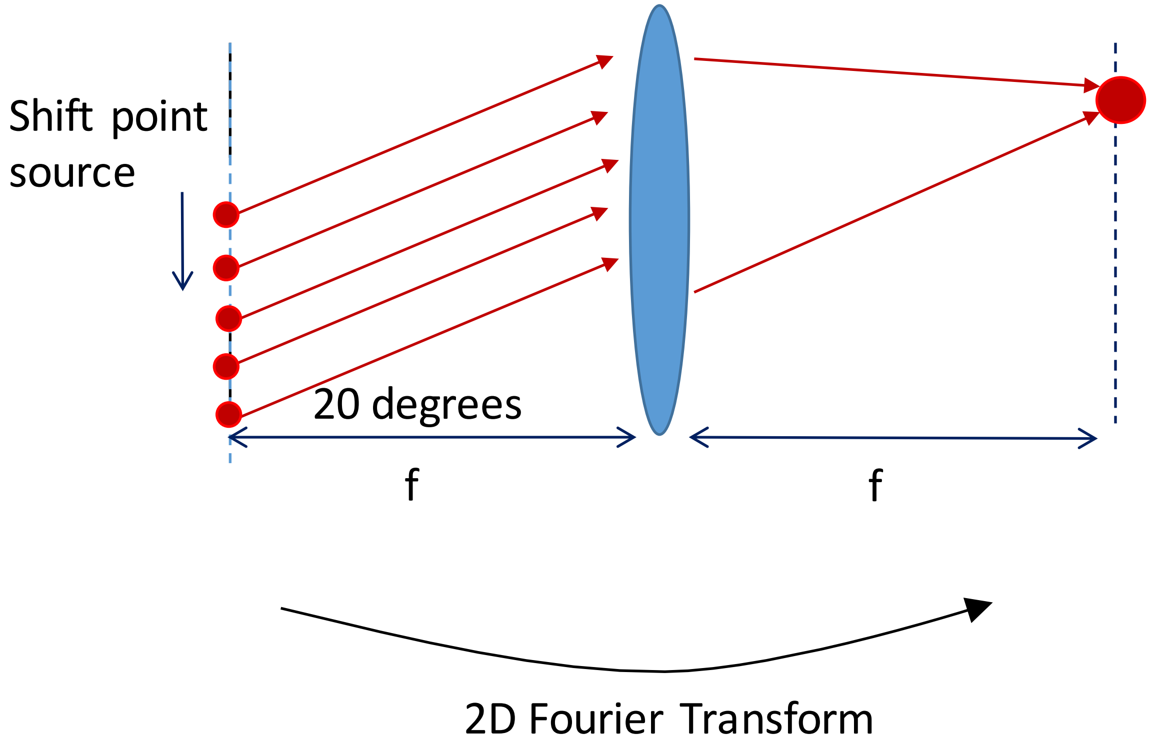


The Fourier plane provides a measure of the **ray angles at the image plane**

Doesn't contain info about spatial distribution light

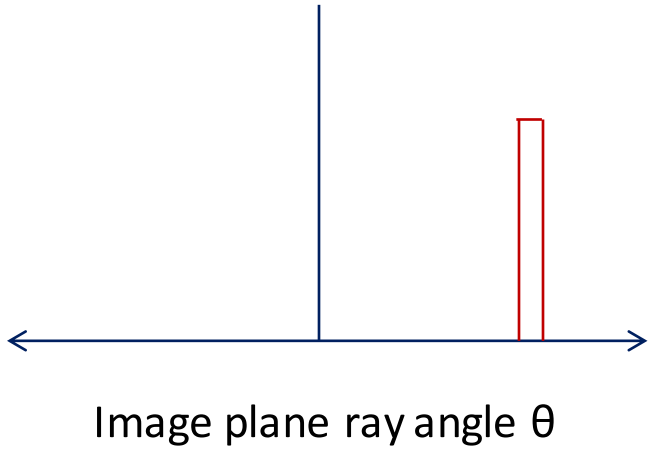


A more exact model: the 4f optical system

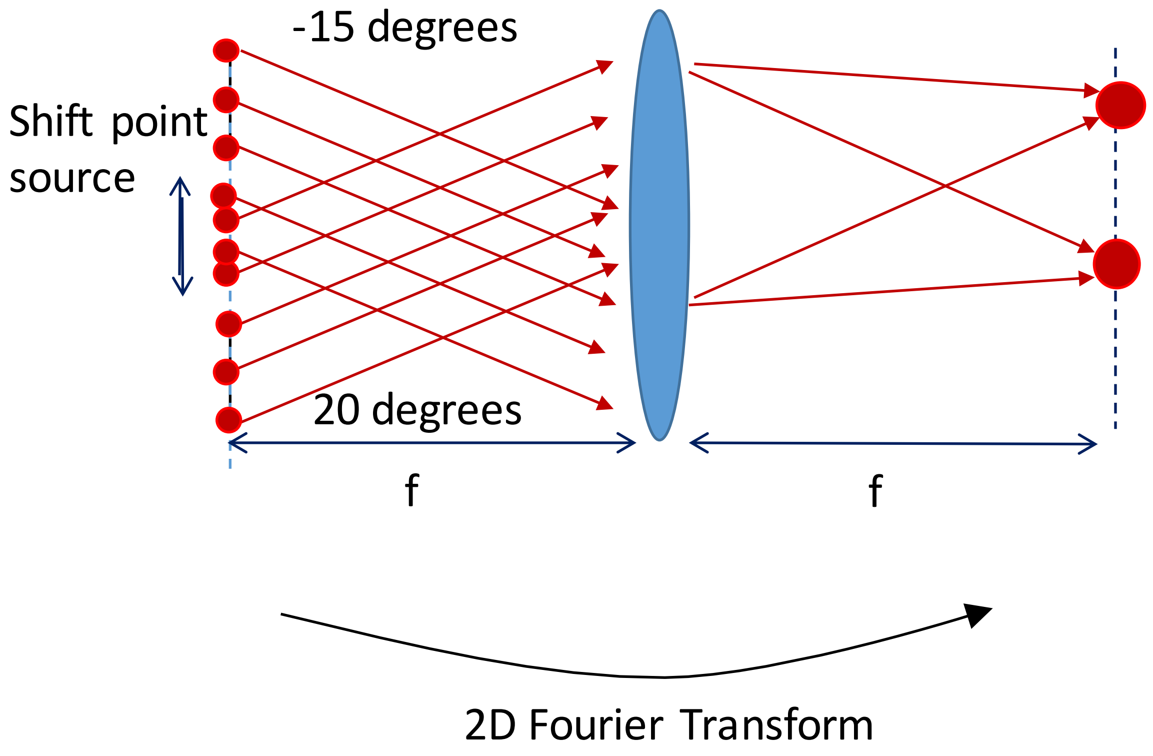


The Fourier plane provides a measure of the **ray angles at the image plane**

Rays are leaving image plane at +20 degrees

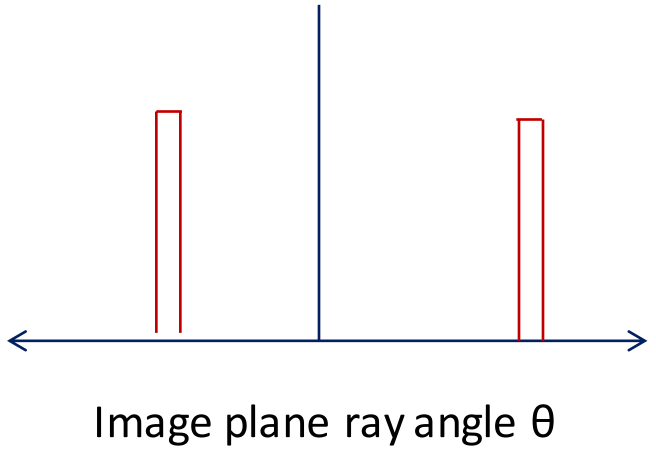


A more exact model: the 4f optical system



The Fourier plane provides a measure of the **ray angles at the image plane**

Rays are coming in at +20 degrees and -15 degrees



You typically go between 4 functions to describe one imaging system: