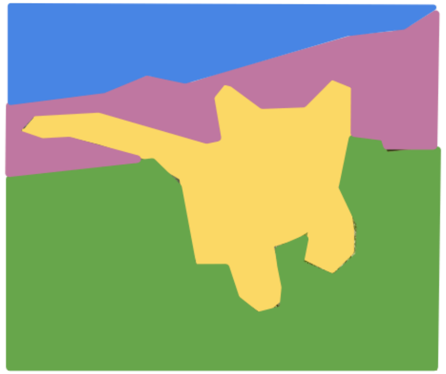
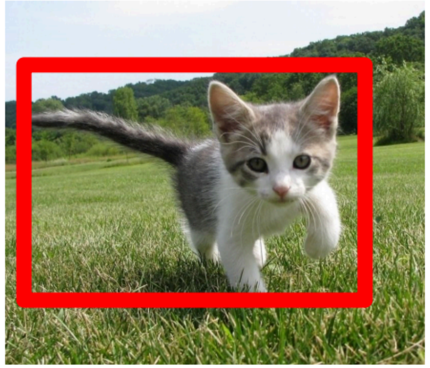
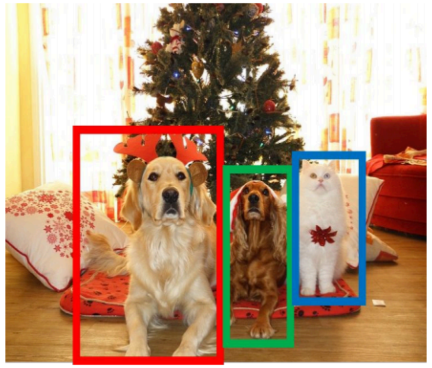
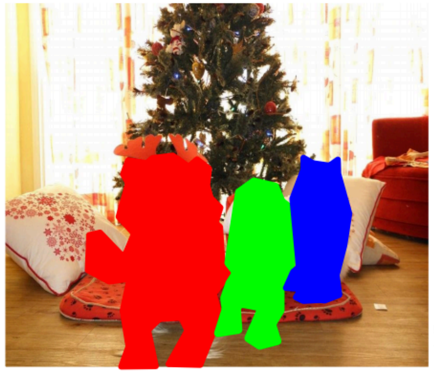
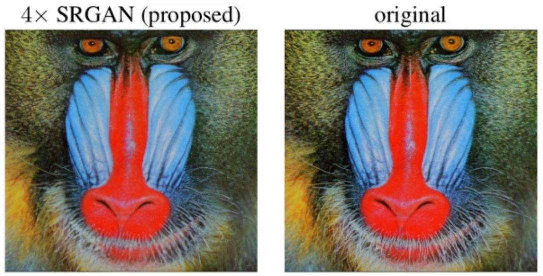
deep imaging

Lecture 15: Introduction to Physical Layers in Machine Learning

Machine Learning and Imaging

BME 548L
Roarke Horstmeyer

Other Computer Vision Tasks - **We'll pick this back up next class**

Semantic Segmentation	Classification + Localization	Object Detection	Instance Segmentation	Super-resolution
				
GRASS, CAT, TREE, SKY	CAT	DOG, DOG, CAT	DOG, DOG, CAT	Figure 1: Super-resolved image (left) is almost indistinguishable from original (right). [4× upscaling]
No objects, just pixels	Single Object	Multiple Object	Multiple Object	
Balanced equation	**Over-determined**	**Over-determined**	**Over-determined**	**Under-determined**

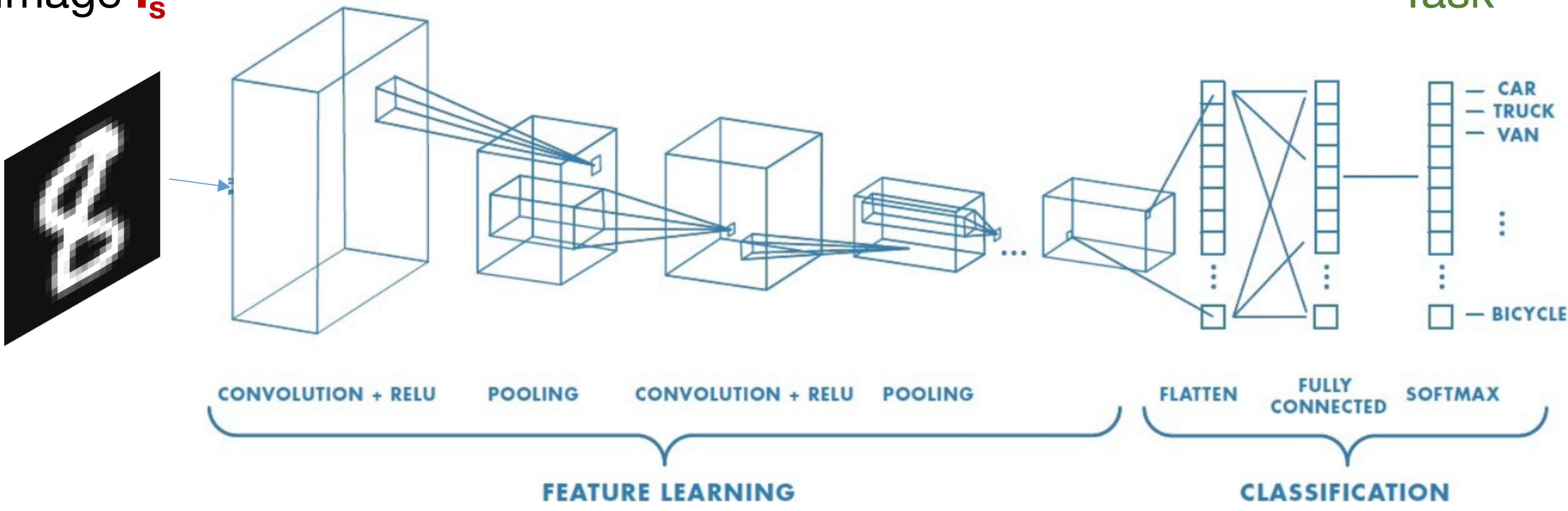
This image is CC0 public domain

Stanford CS231n - http://cs231n.stanford.edu

Bringing together physical and digital image representations

Image $\mathbf{I_s}$

Task

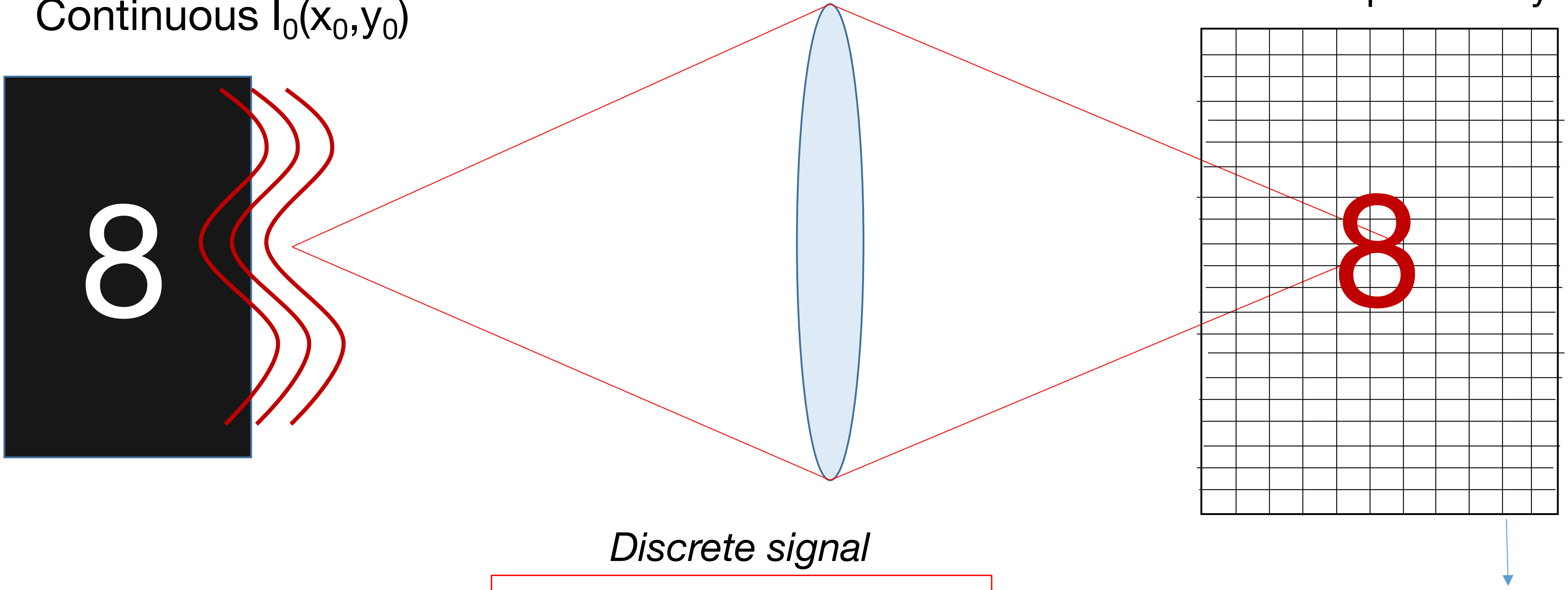


$$\text{Task} = \mathbf{W_n} \ldots \text{ReLU}[\mathbf{W_1}\ \text{ReLU}[\mathbf{W_0}\ \mathbf{I_s}]\ldots]$$

Simple model of image formation

Continuous $I_0(x_0,y_0)$

n x *m* pixel array



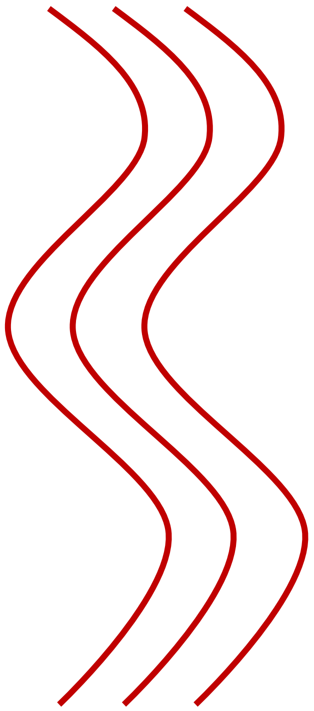
Discrete signal

$$\boxed{I_s(x,y), (x,y) \in Z^{n \times m}}$$

← Digitazation ← Photons to electrons

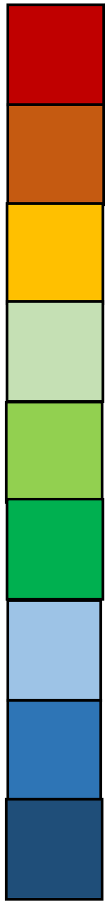
What does the Sampling Theorem mean for us?

Continuous functions



*conditions

Discretize vectors
(and matrices)



17
20
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25
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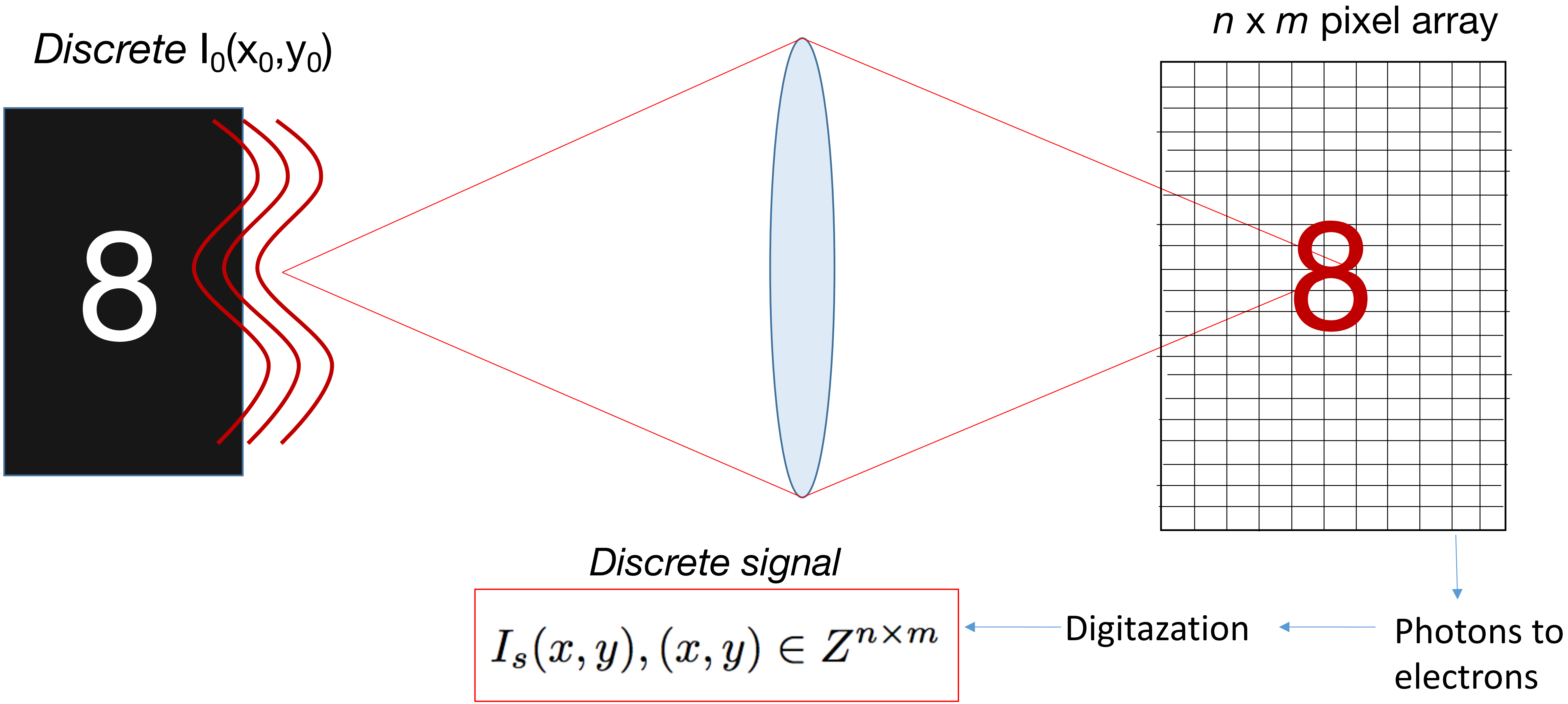
26
29

Simple model of image formation



Bringing together physical and digital image representations

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Physical Layers

Digital Layers

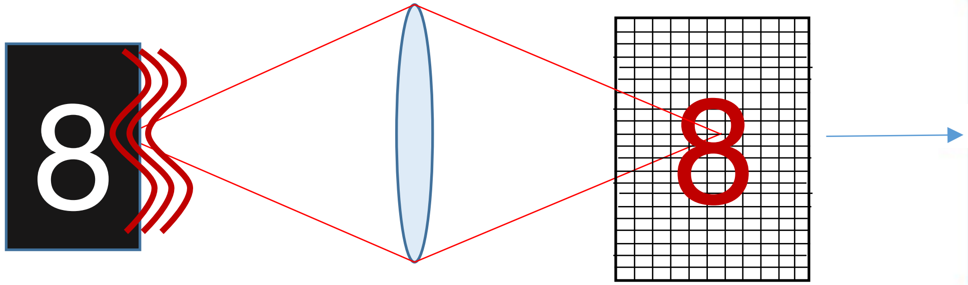
Physical world

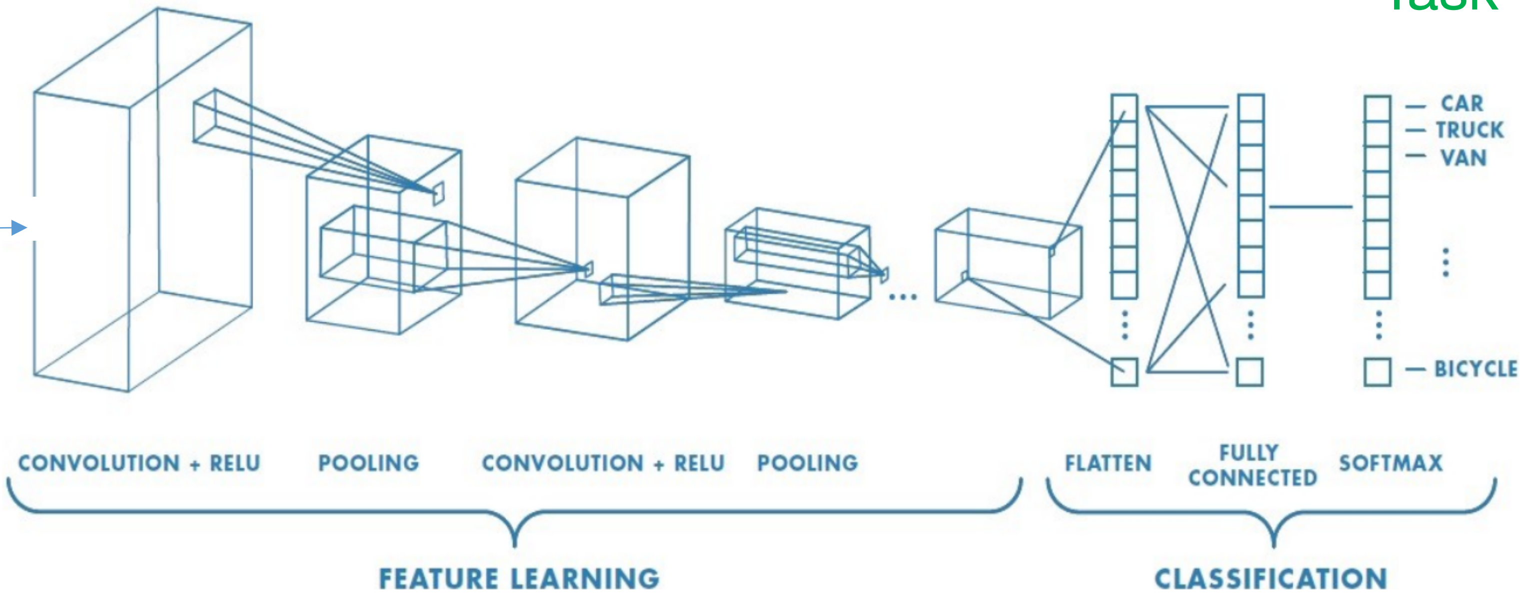
$I_0(x_0,y_0)$

Digital Image

$I_s(x,y)$

Task





$$\mathbf{I_s} = f[\mathbf{I_0}]$$

Digital layers

Physical layers

$$\text{Task} = \mathbf{W_n} \ldots \text{ReLU}[\mathbf{W_1}\ \text{ReLU}[\mathbf{W_0}\ f[\mathbf{I_0}]] \ldots]$$

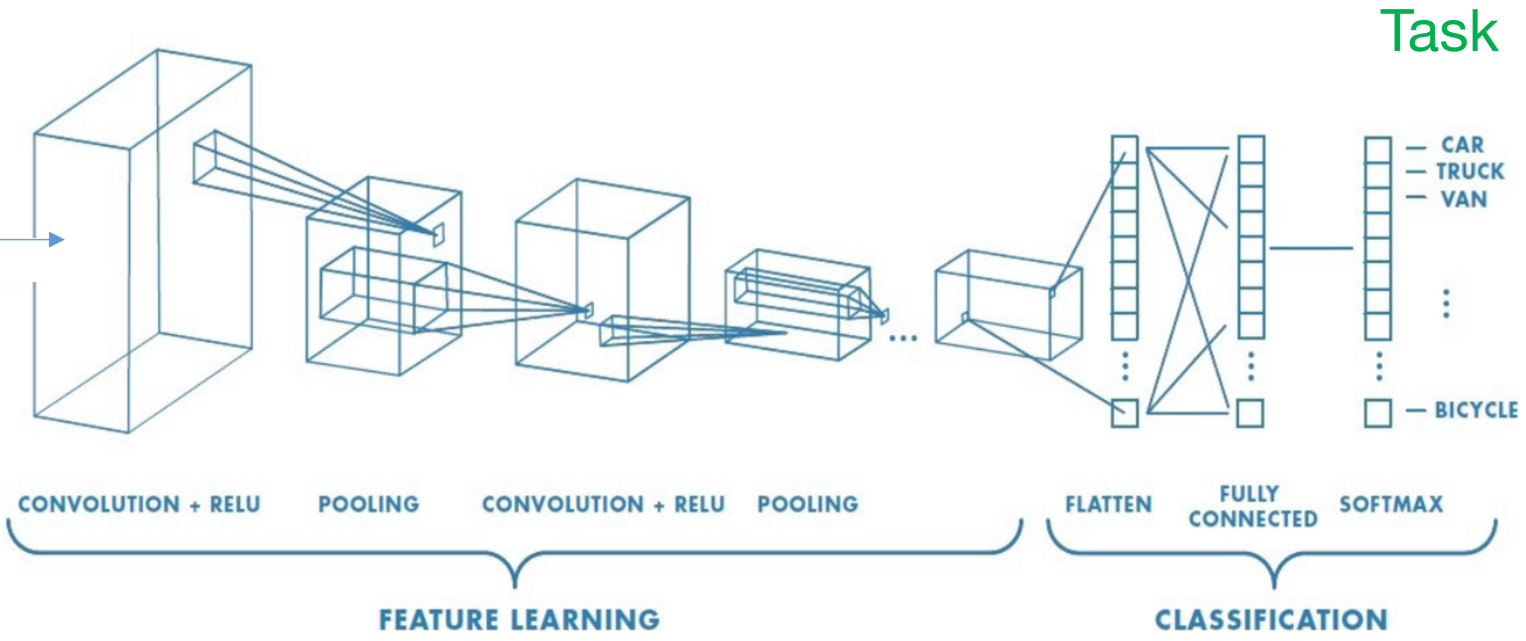
Bringing together physical and digital image representations

deep imaging

Physical Layers

Digital Layers

Physical Function $\mathbf{I}_0$ → f[ ] → Digitized $\mathbf{I}_s$ →



$$\mathbf{I}_s = f[\mathbf{I}_0]$$

Digital layers

Physical layers

$$\text{Task} = \mathbf{W}_n \ldots \text{ReLU}[\mathbf{W}_1 \,\text{ReLU}[\mathbf{W}_0 \, f[\mathbf{I}_0]]\ldots]$$

Required properties of physical mapping f[ ] for DNN optimization?

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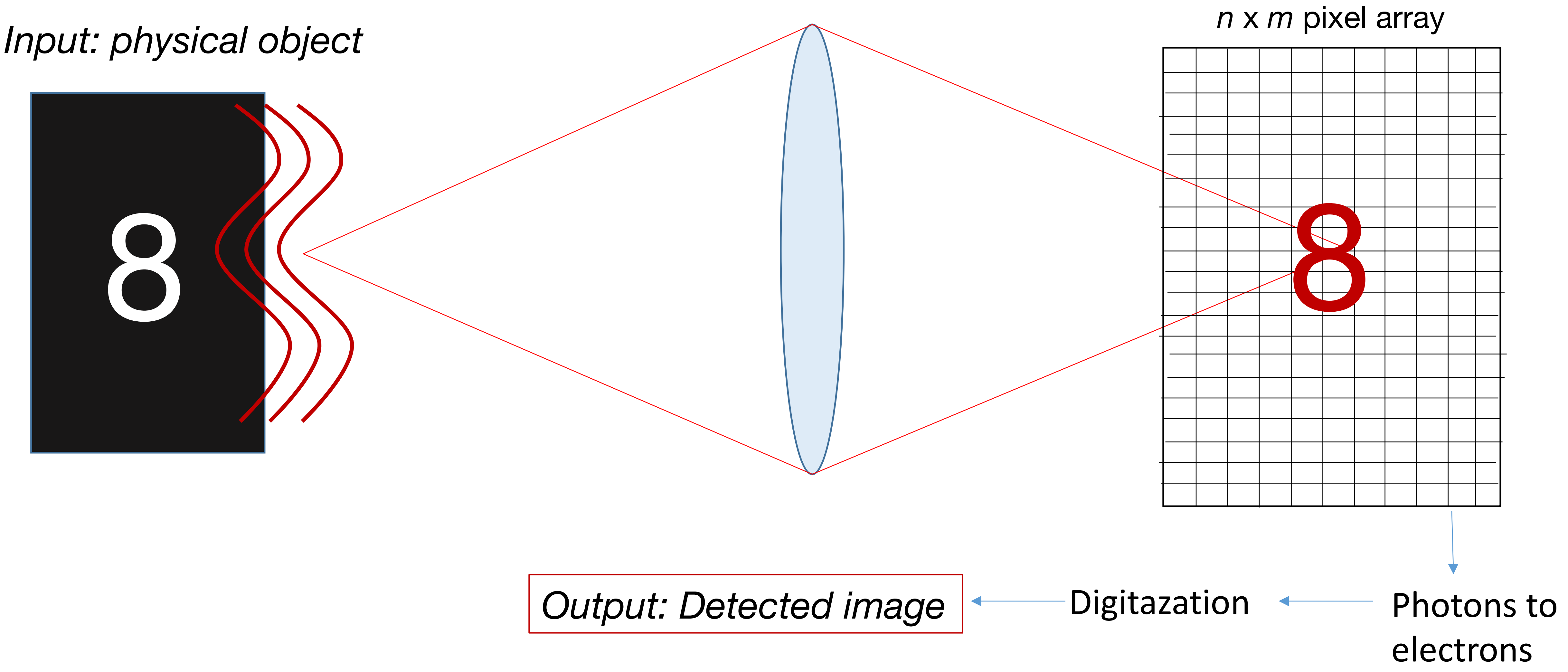
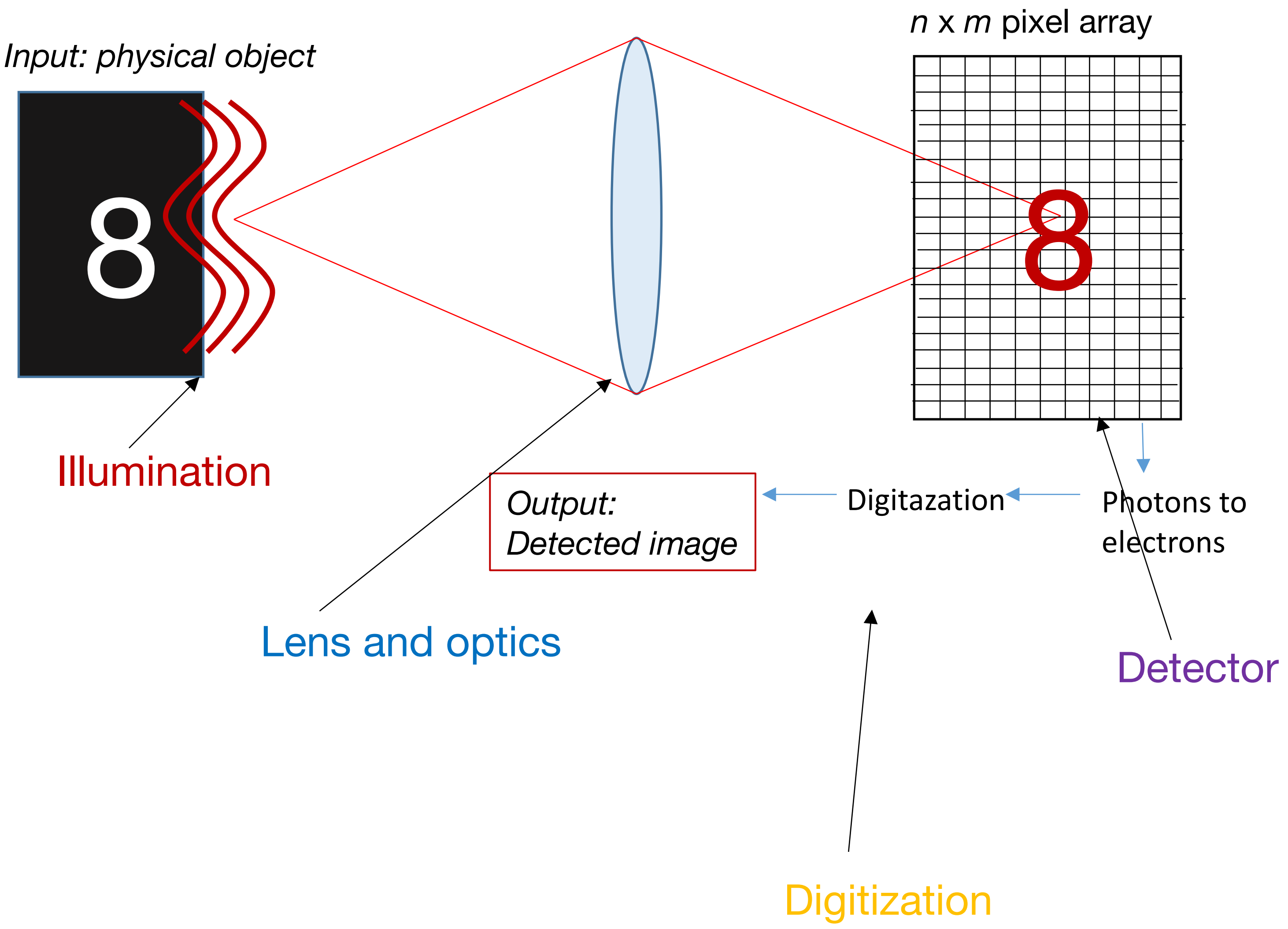
- Finite
- Non-zero gradients
- Differentiable*
- Known structure (for now…)
- Anything else?

What physical parameters effect image formation?

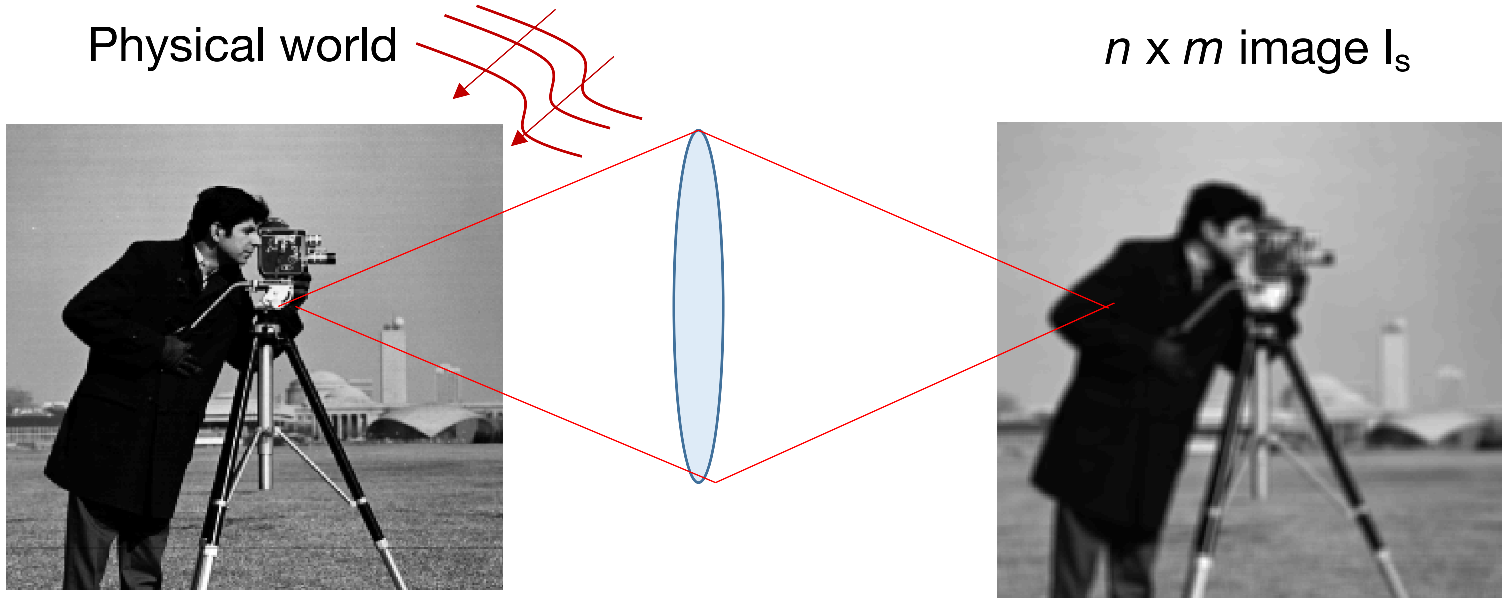


What physical parameters effect image formation?

- Illumination
  - Spatial pattern
  - Angle of incidence
  - Color, polarization
- Lens and optics
  - Position/orientation
  - Shape
  - Focus
  - Transparency
- Detector
  - Pixel size
  - Pixel shape & fill factor
  - Color filters
  - Other filters
- Digitization
  - E to P curves
  - Digitization schemes/thresholds
  - Data transmission, multiplexing
- Physical object

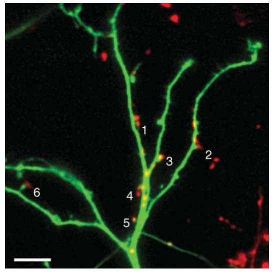


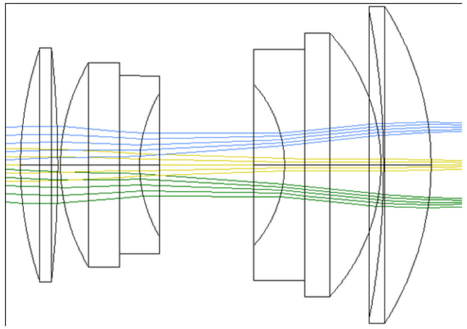
Simple mathematical model of image formation



First - what is light and how can we model it?

- Interpretation #1: Radiation (*Incoherent*)
- Model: Rays





- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

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First - what is light and how can we model it?

- Interpretation #1: Radiation (*Incoherent*)
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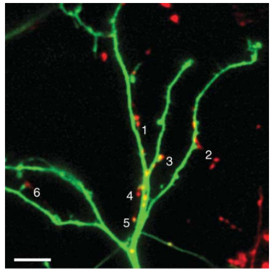
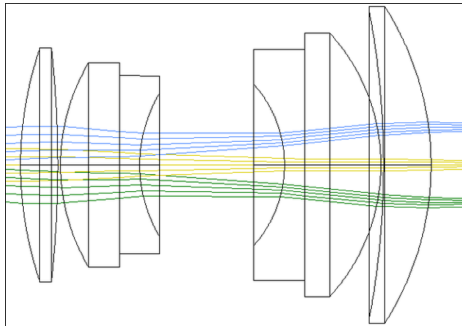
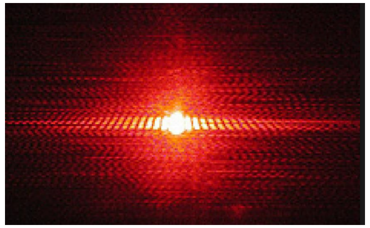


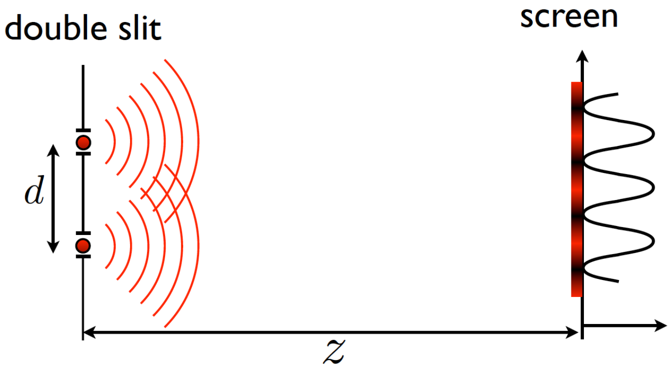


- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

- Interpretation #2: Electromagnetic wave (*Coherent*)
- Model: Waves



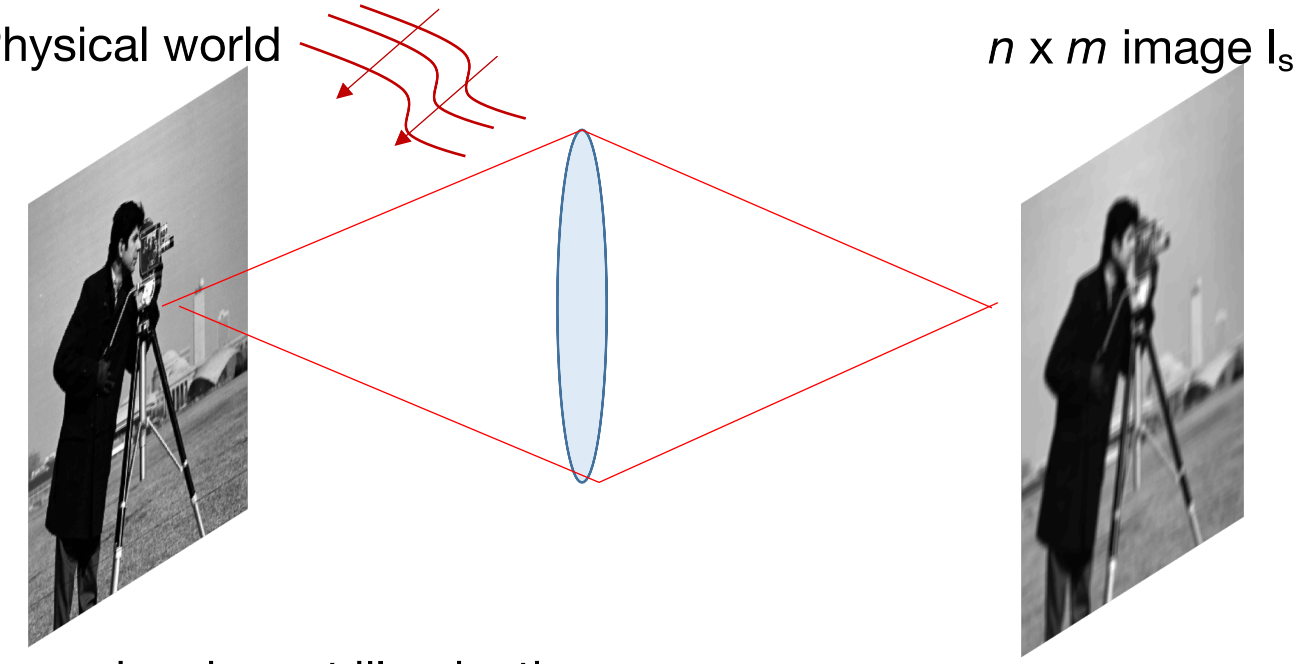


- Complex field
- Models Interference

$$E_{tot} = E_1 + E_2$$

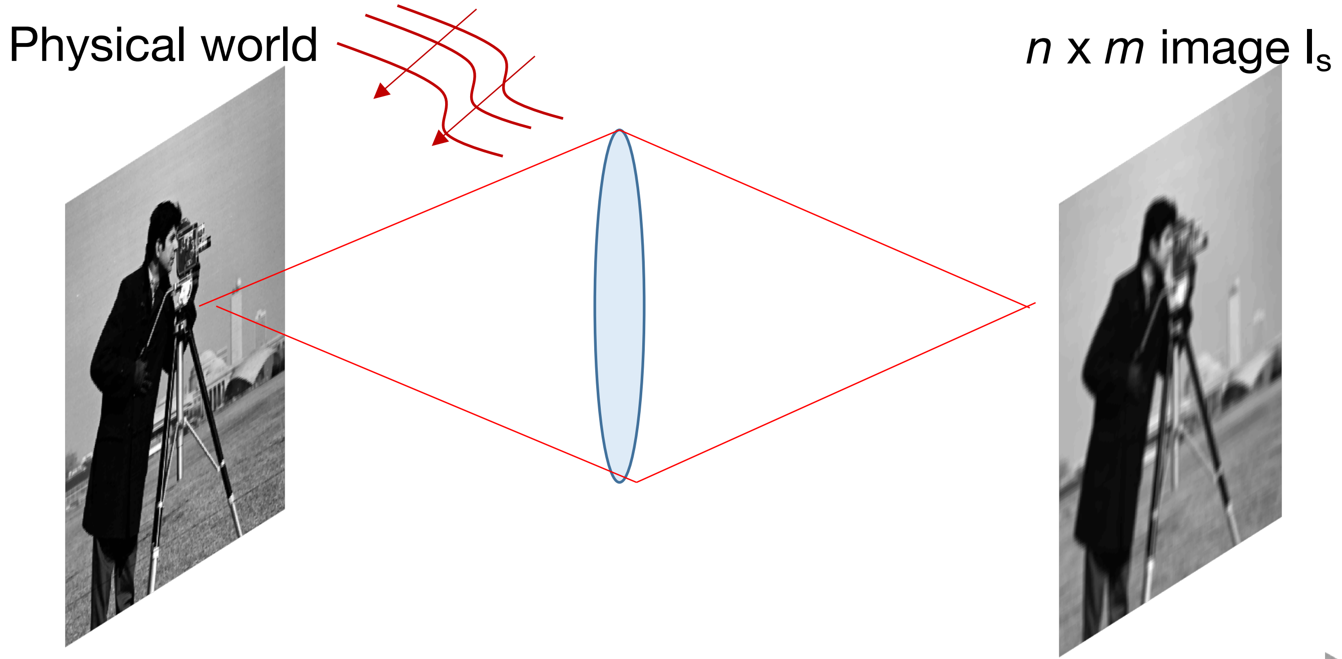
- Interpretation #3: Particle
- Model: Photons

Simple mathematical model of image formation



- Assume incoherent illumination
- Assume thin 2D object
- Object is real, non-negative map of absorption/reflectivity

Simple mathematical model of image formation



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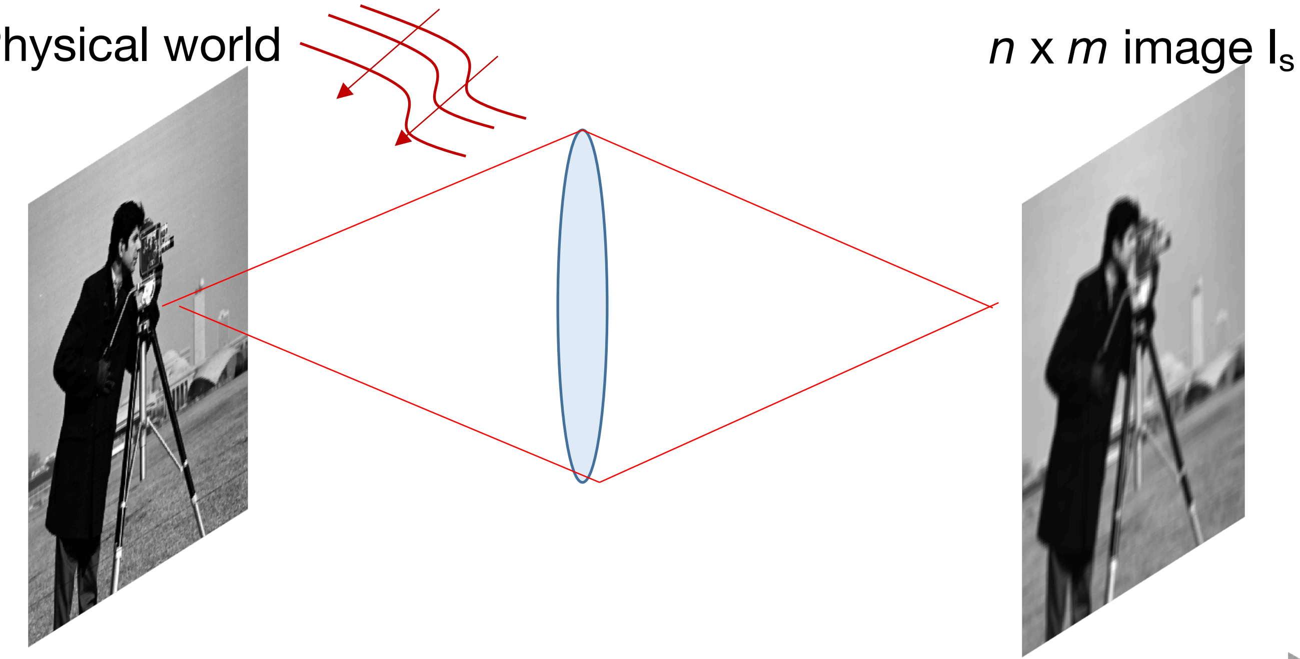
Simple mathematical model of image formation



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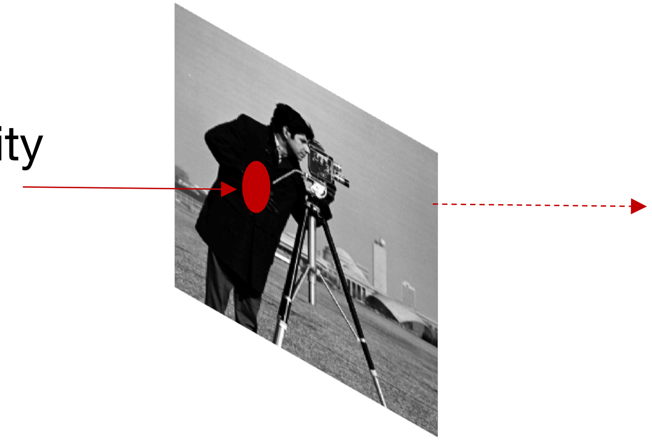
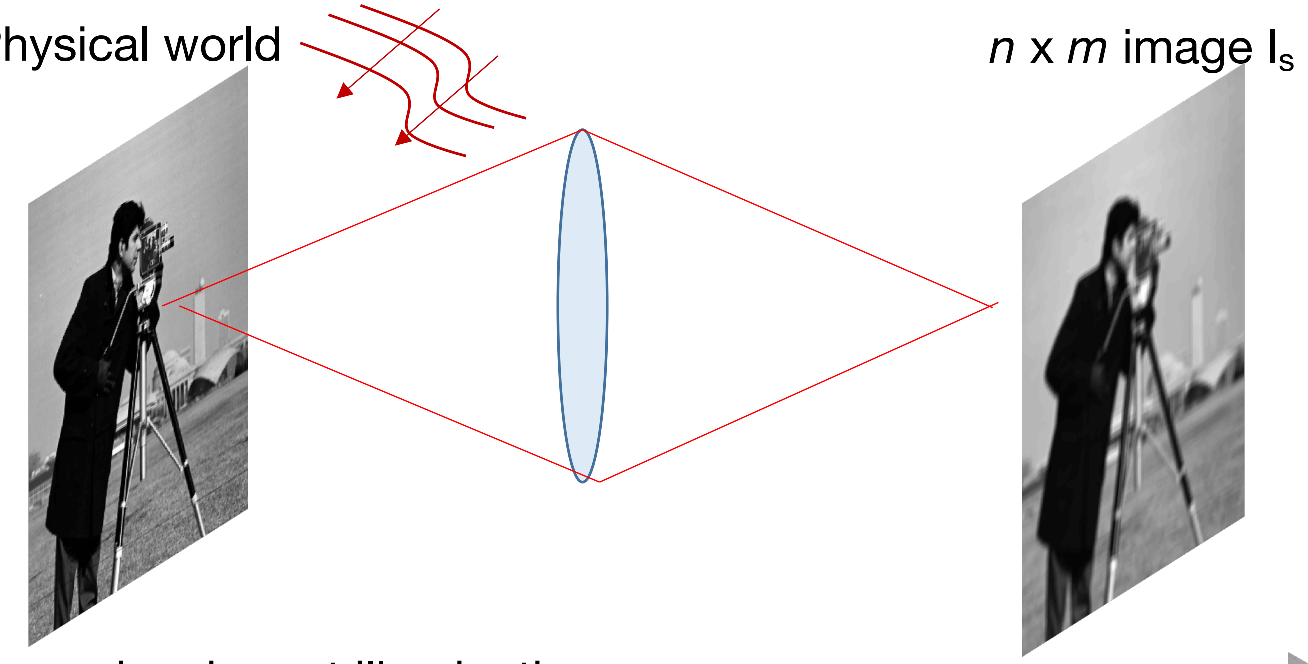


Simple mathematical model of image formation



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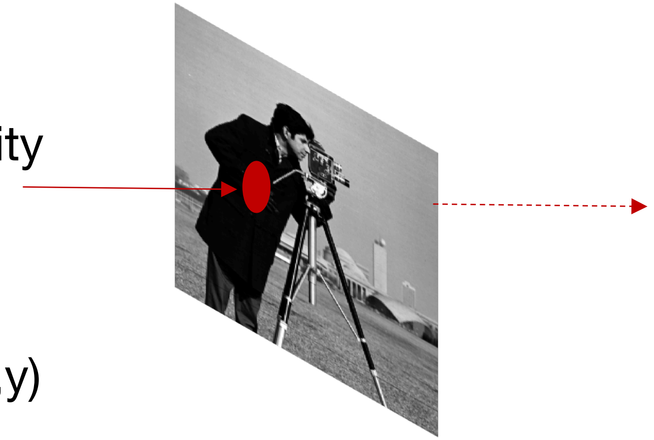
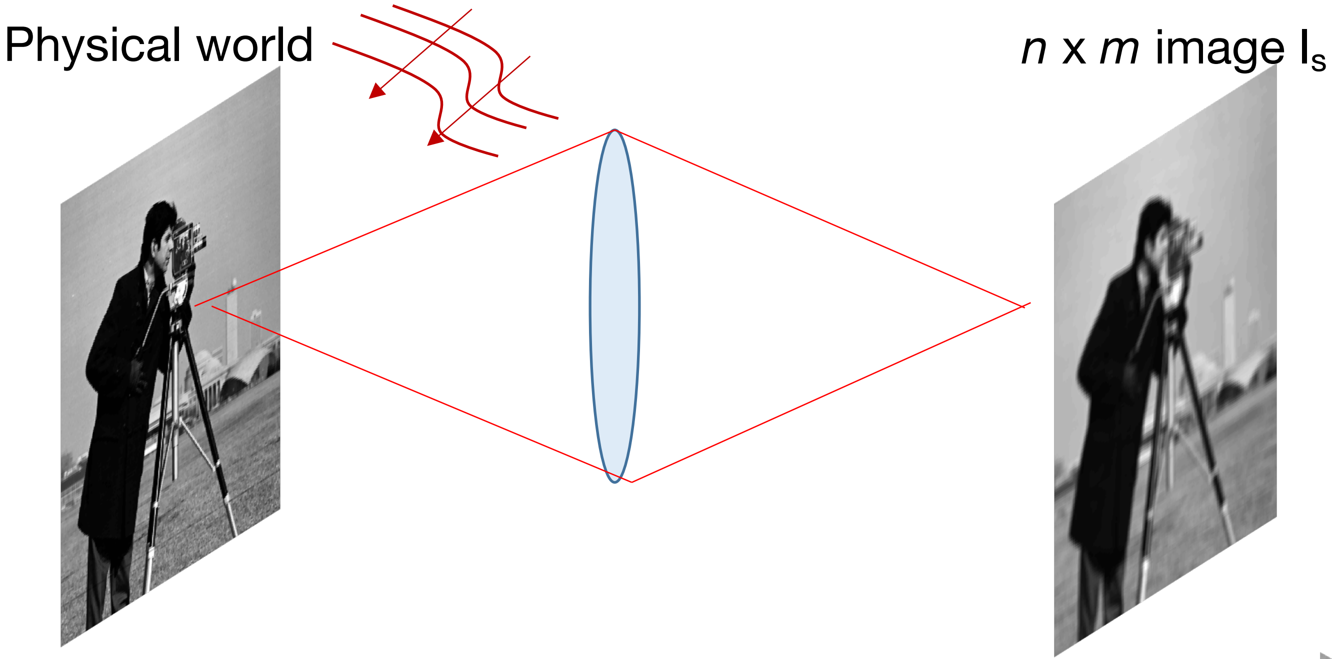


Object transmission: $\mathbf{I_0}(x,y)$
Illumination pattern: $\mathbf{s}(x,y)$
Light exiting object surface: $\mathbf{I_e}(x,y) = \mathbf{I_0}(x,y) \circ \mathbf{s}(x,y)$

Simple mathematical model of image formation



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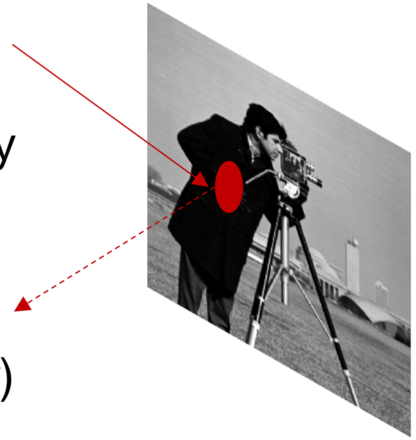
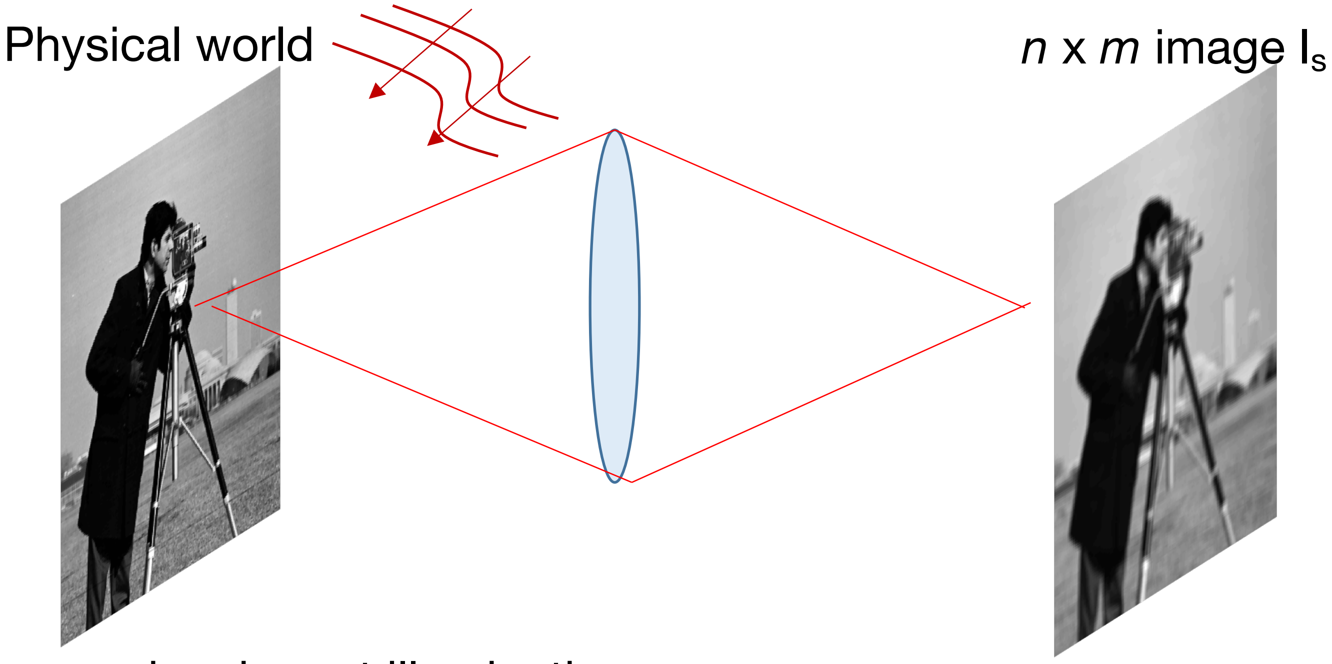


Object *reflectivity*: $\mathbf{I_0}(x,y)$
Illumination pattern: $\mathbf{s}(x,y)$
Light exiting object surface: $\mathbf{I_e}(x,y) = \mathbf{I_0}(x,y) \circ \mathbf{s}(x,y)$

Simple mathematical model of image formation



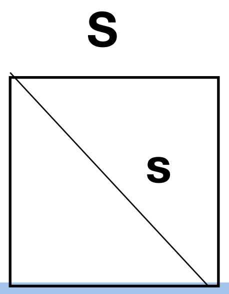
- Assume incoherent illumination
- Assume thin 2D object
- Object is real, non-negative map of absorption/reflectivity

Modeling incoherent illumination

$$\mathbf{I}_e(x,y) = \mathbf{I}_0(x,y) \circ \mathbf{s}(x,y)$$

$$\mathbf{I}_e = \mathbf{S}\,\mathbf{I}_0$$

diag(**S**) = **s**



Example #1: Optimized illumination pattern (one color)

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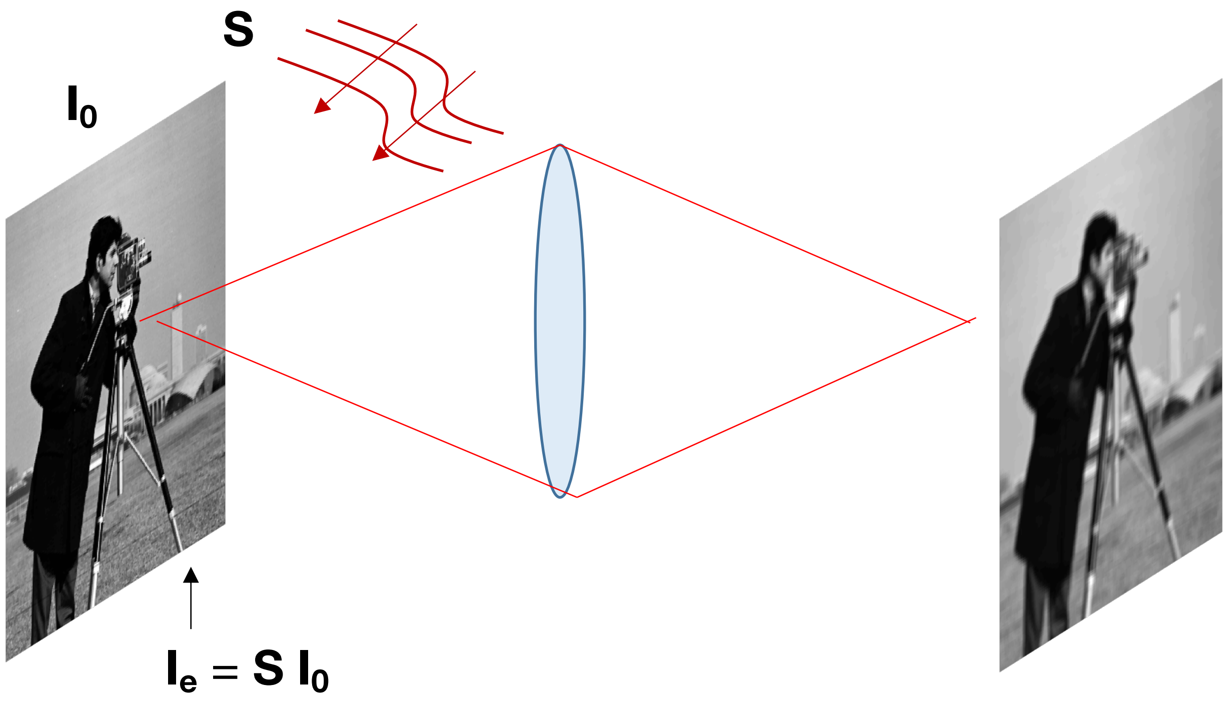
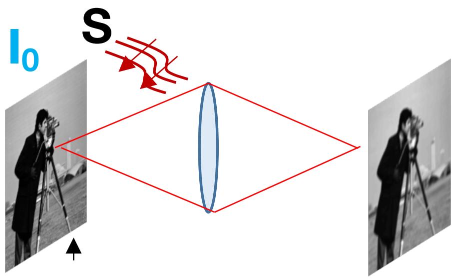


First, assume perfect camera:
intensity at image plane $\mathbf{I_p} = \mathbf{I_e} = \mathbf{S}\,\mathbf{I_0}$

Example #1: Optimized illumination pattern (one color)

Training data:
[$I_0(x, y)$, y]
I_0:100 x 100
Label y: 1x2

Example #1: Optimized illumination pattern (one color)



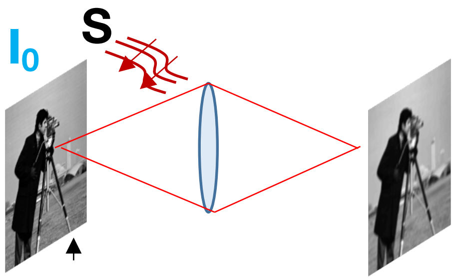
$[I_e(x, y), y]$

Training data:
$[I_0(x, y), y]$
I_0:100 x 100
Label y: 1x2

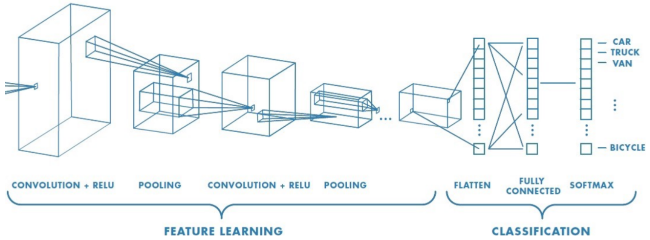
$\boxed{I_e(x,y) = \mathbf{S}\, I_0(x,y)}$

Physical Layer

Example #1: Optimized illumination pattern (one color)



$[I_e(x, y), y]$



Training data:
$[I_0(x, y), y]$
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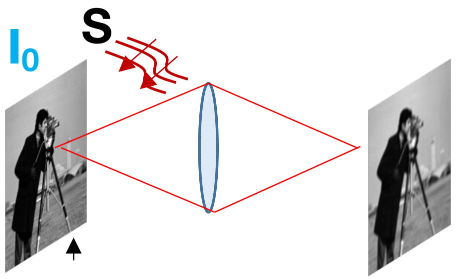
$\boxed{I_e(x,y) = \mathbf{S}\, I_0(x,y)}$

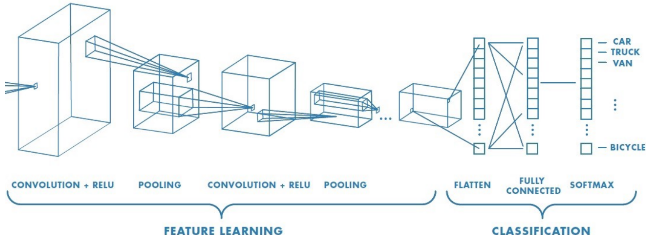
Physical Layer

Task = $\mathbf{W_n} \ldots \mathrm{ReLU}[\mathbf{W_1}\, \mathrm{ReLU}[\mathbf{W_0}\, I_e] \ldots]$

Example #1: Optimized illumination pattern (one color)



$[I_e(x, y), y]$



Training data:
$[I_0(x, y), y]$
I_0:100 x 100
Label y: 1x2

$\boxed{I_e(x,y) = \mathbf{S}\, I_0(x,y)}$

Physical Layer

Task = $\mathbf{W_n} \ldots \mathrm{ReLU}[\mathbf{W_1}\, \mathrm{ReLU}[\mathbf{W_0}\, I_e] \ldots]$

Tensorflow 1.0 code

```
training_images = tf.placeholder(tf.float32, [image_size, None])
training_labels = tf.placeholder(tf.float32, [None, 3])
illumination_pattern = tf.truncated_normal([image_size, 1], stddev = 0.1)
illumination_matrix = tf.linalg.diag(illumination_pattern)
illumianted_images = tf.matmul(illumination_matrix, training_images)

#Now, train CNN of your choice using [illumianted_images, training_labels]
#Output of train/test will be CNN weights, classification performance AND illumination_pattern!
```

(Will show CoLab Notebook for implementation in Tensorflow 2.0)

Option 1: tf.linalg.matmul

1.0 code

```
training_images = tf.placeholder(tf.float32, [image_size, None])
training_labels = tf.placeholder(tf.float32, [None, 3])
illumination_pattern = tf.truncated_normal([image_size, 1], stddev = 0.1)
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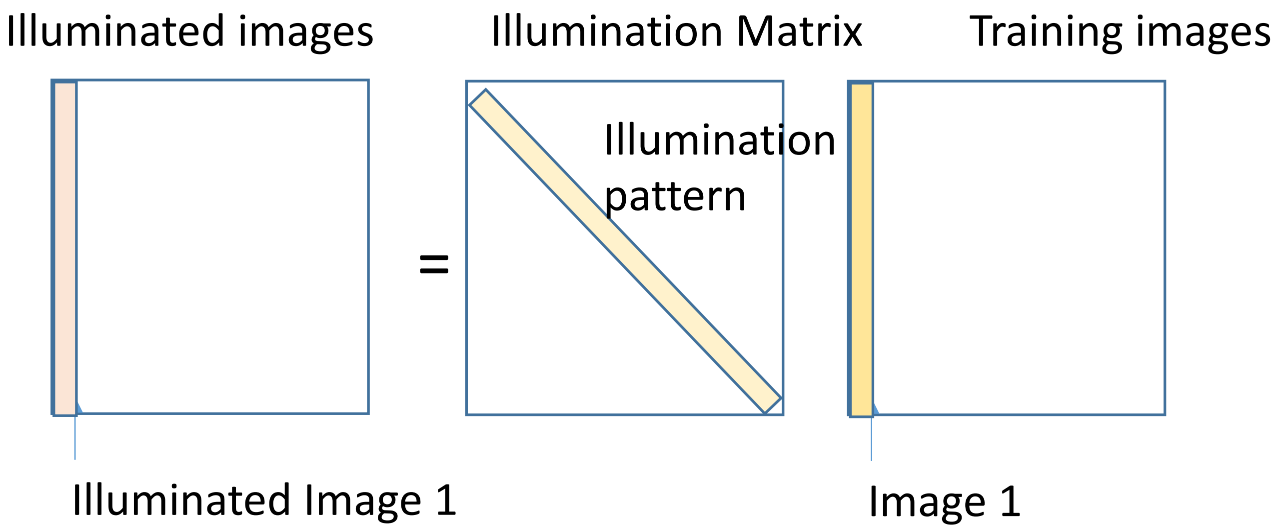
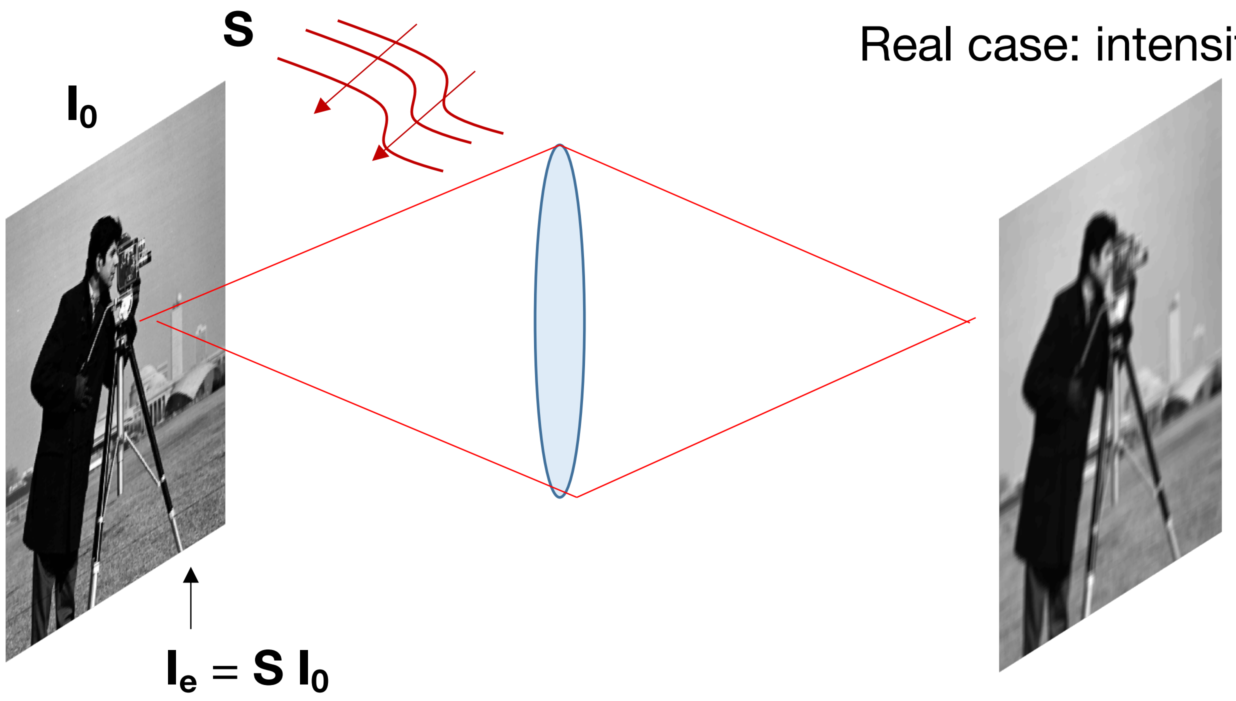


Option 2: tf.linalg.multiply (will show in CoLab Notebook)

Simple mathematical model of image formation

Real case: intensity at image plane $\mathbf{I_p}$ = **blurred**



Simple mathematical model of image formation

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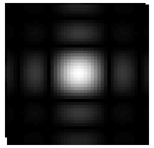
Lenses blur and rescale images:
(We'll learn how exactly next few weeks)

Input intensity

$*$

Convolution filter h



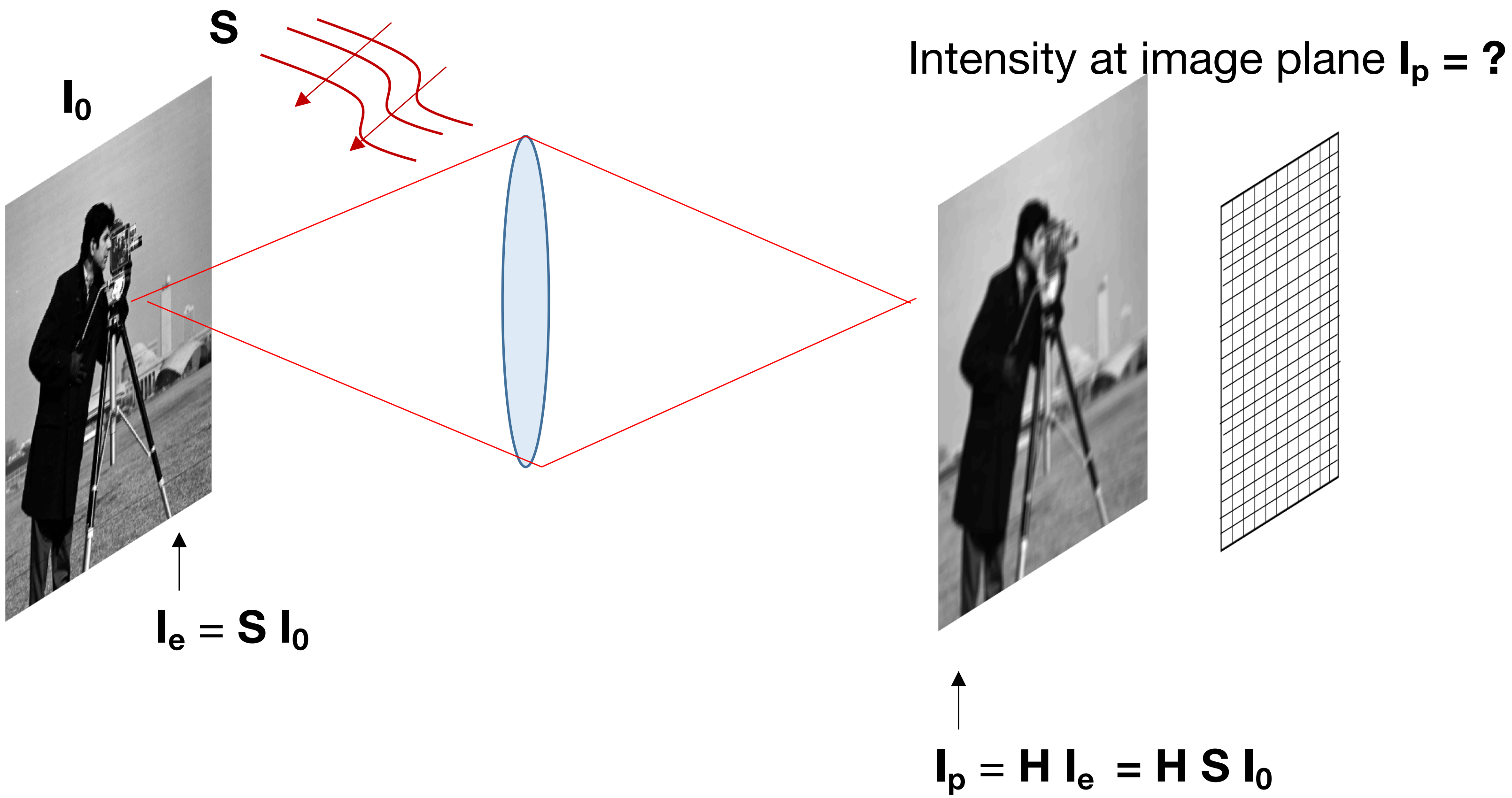
$=$

Output intensity

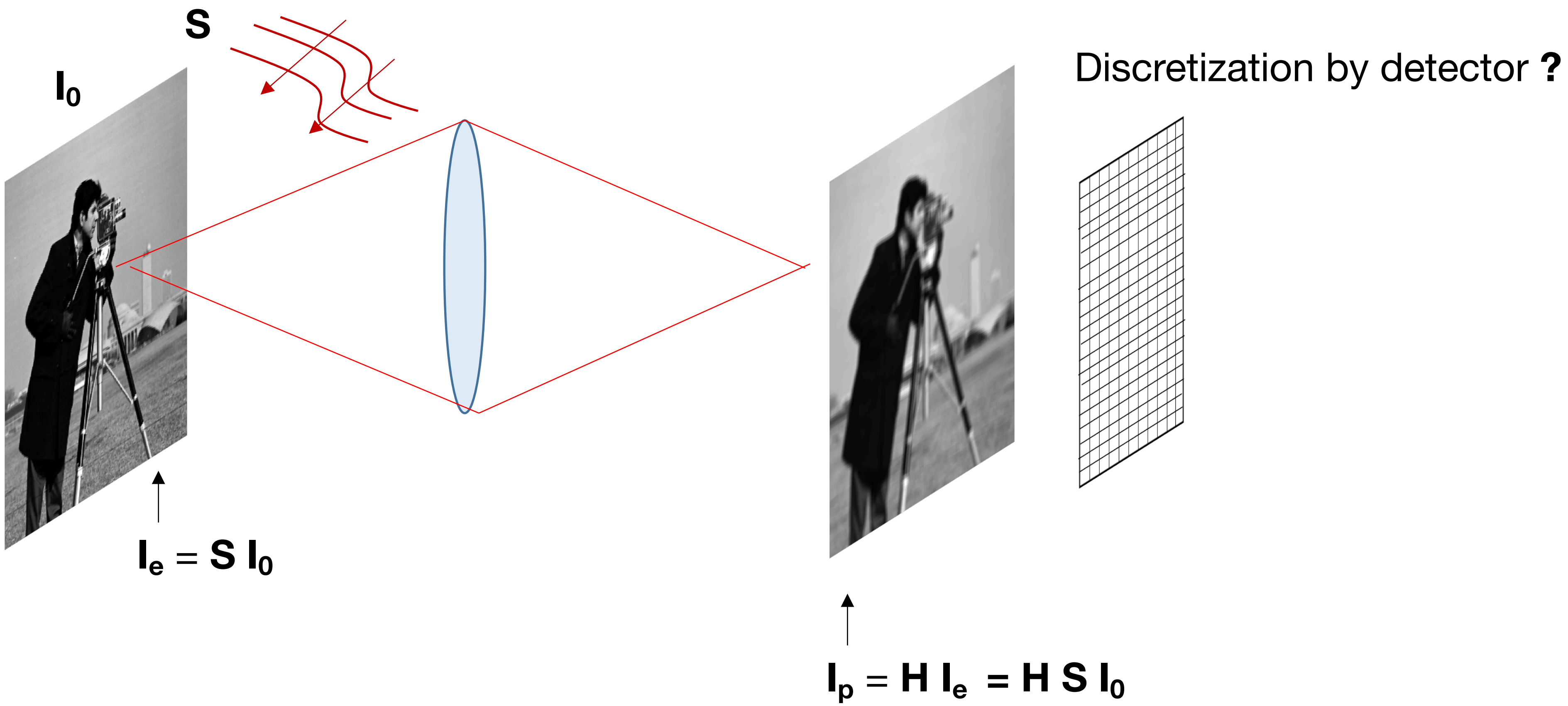
$$\mathbf{I_e}(x/M, y/M) \quad * \quad \mathbf{h}(x,y) \quad = \quad \mathbf{I_p}(x,y)$$

Assuming we've resized by M, $\mathbf{I_p} = \mathbf{I_e} * \mathbf{h} = \mathbf{H}\,\mathbf{I_e}$

Simple mathematical model of image formation



Simple mathematical model of image formation

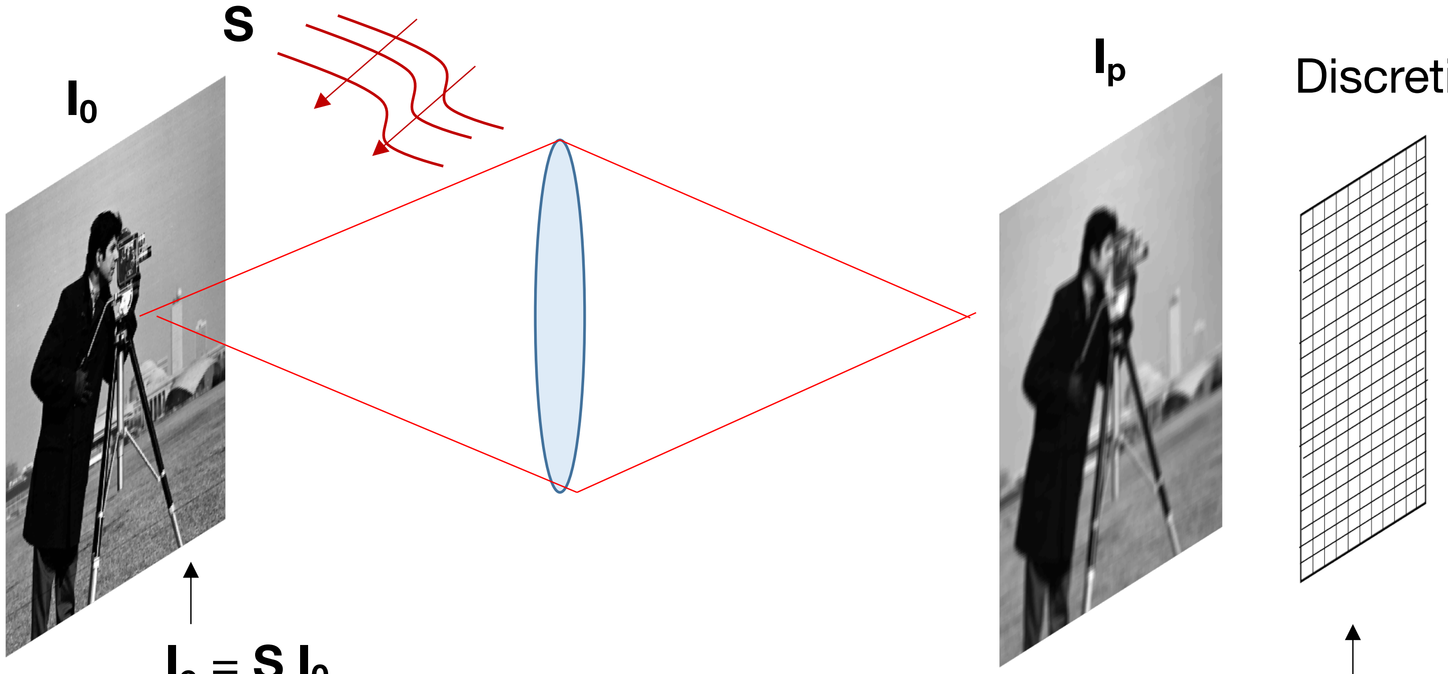


Use downsampling matrix (sum-pooling)

D =

$$\begin{matrix} 0.5 & 0.5 & 0 & 0 & 0 & \cdots & & \\ & & 0.5 & 0.5 & 0 & 0 & 0 & \cdots \\ & & & & 0.5 & 0.5 & 0 & 0 & 0 & \cdots \\ & \vdots & & & & \vdots \end{matrix}$$

Simple mathematical model of image formation



$I_e = S\,I_0$

Discretization by detector ?

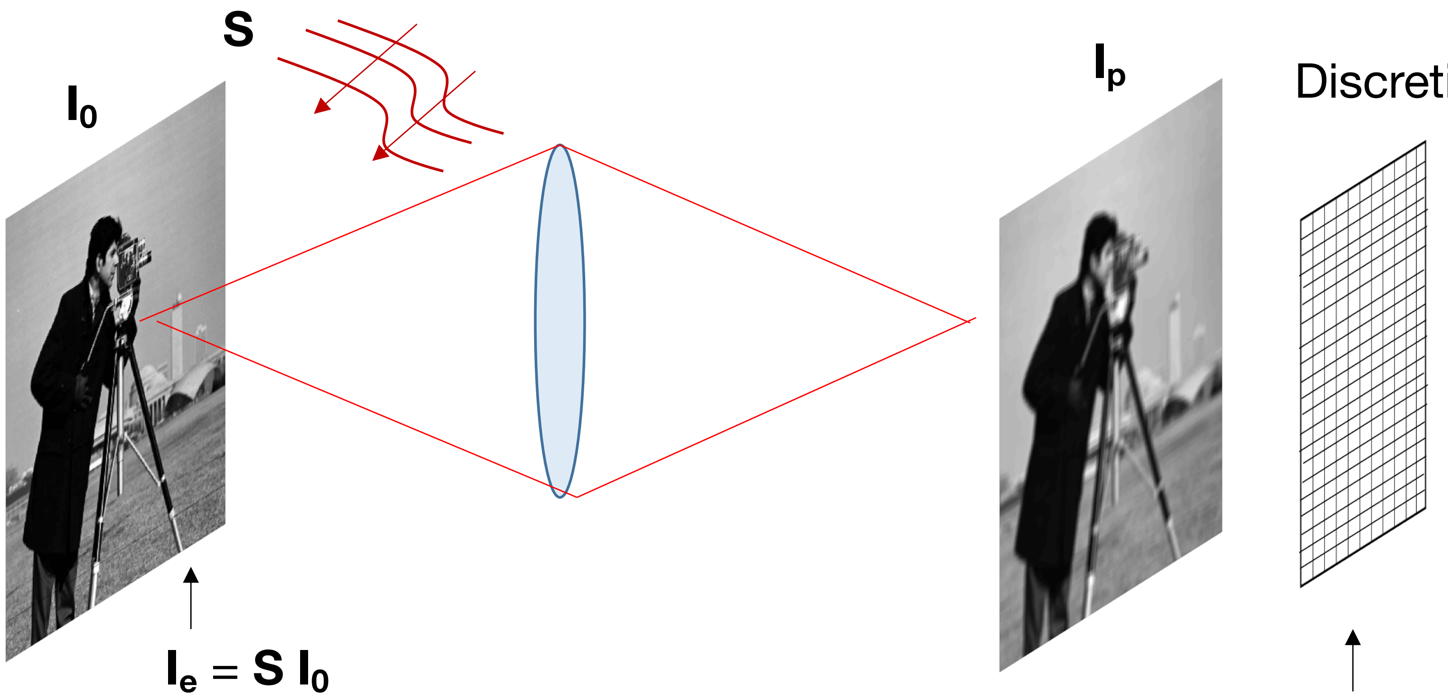
$$\boxed{I_s = D\,I_p = D\,H\,S\,I_0}$$

Use downsampling matrix (sum-pooling)

$$D = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 & \dots & & & & \\ & & 0.5 & 0.5 & 0 & 0 & 0 & \dots & & \\ & & & & 0.5 & 0.5 & 0 & 0 & 0 & \dots \\ & & \vdots & & & & \vdots & & & \end{bmatrix}$$

Simple mathematical model of image formation

deep imaging



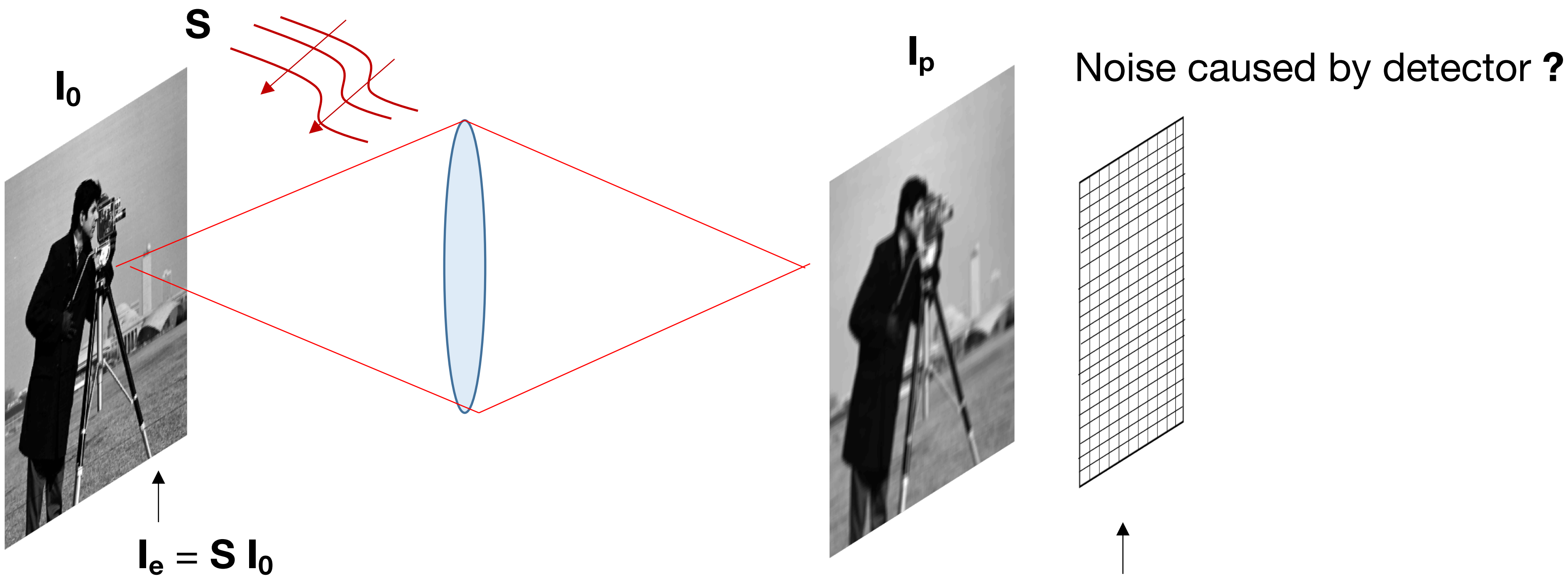
$$\boxed{I_s = D\, I_p = D\, H\, S\, I_0}$$

D =

w1 w2 0 0 0 ….
w3 w4 0 0 0 ….
w5 e6 0 0 0 ….
⋮ ⋮

*Can make these learnable weights

Simple mathematical model of image formation



$$I_s = D\ I_p + N$$

Can also add in detector-dependent noise N = k * np.random.randn(dx, dy)

(zero-mean Gaussian noise, for example)

Pause to take a look at:

physical_layers_example.ipynb

Physical Layers

Digital Layers

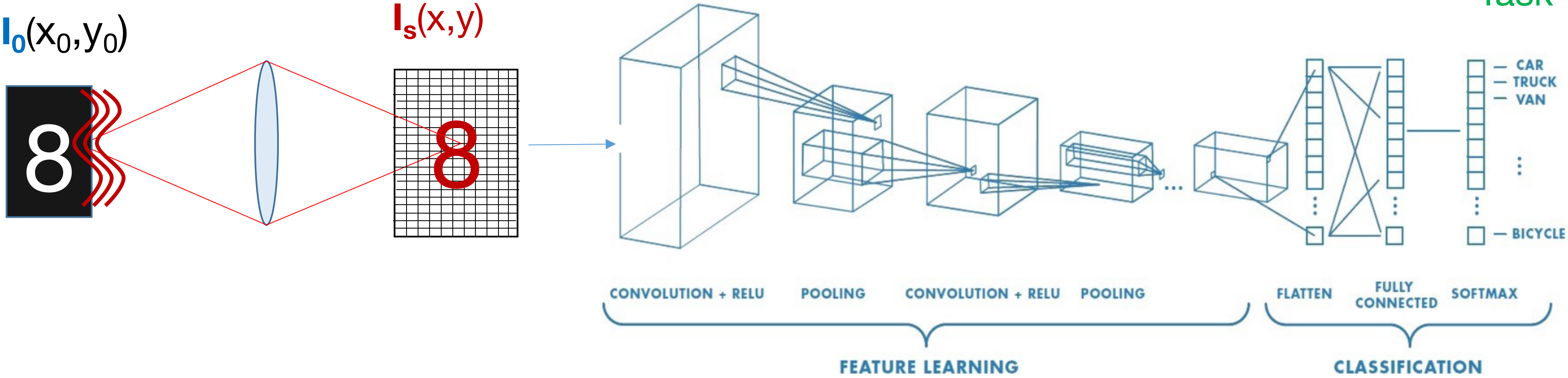
Physical world

$\mathbf{I_0}(x_0,y_0)$

Digital Image

$\mathbf{I_s}(x,y)$

Task



Digital layers

Physical layers

$$\text{Task} = \mathbf{W_n} \ldots \text{ReLU}[\mathbf{W_1}\ \text{ReLU}[\mathbf{W_0}\ f[\mathbf{I_0}]]\ldots]$$

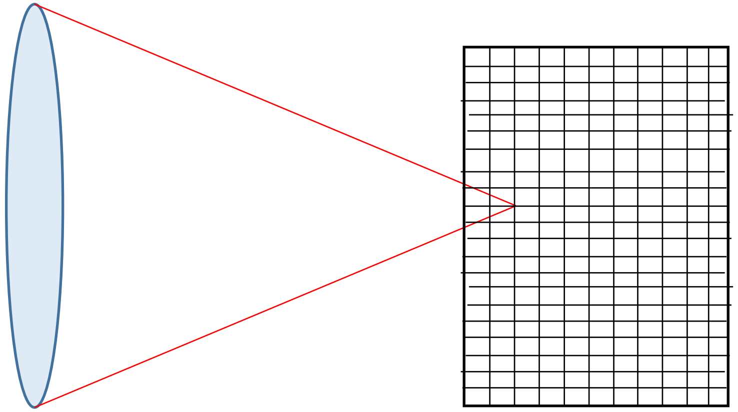
$$\text{Task} = \mathbf{W_n} \ldots \text{ReLU}[\mathbf{W_1}\ \text{ReLU}[\mathbf{W_0}\ \mathbf{D}\ \mathbf{H}\ \mathbf{S}\ \mathbf{I_0}]\ldots]$$

Example #2: Optimized color filter for a grayscale camera

“Ground truth” object:
$I_0(x, y, \lambda)$
100 x 100 pix. x 30 spectral channels

Example #2: Optimized color filter for a grayscale camera

deep imaging



Monochromatic camera sensor

“Ground truth” object:
$I_0(x, y, \lambda)$
100 x 100 pix. x 30 spectral channels

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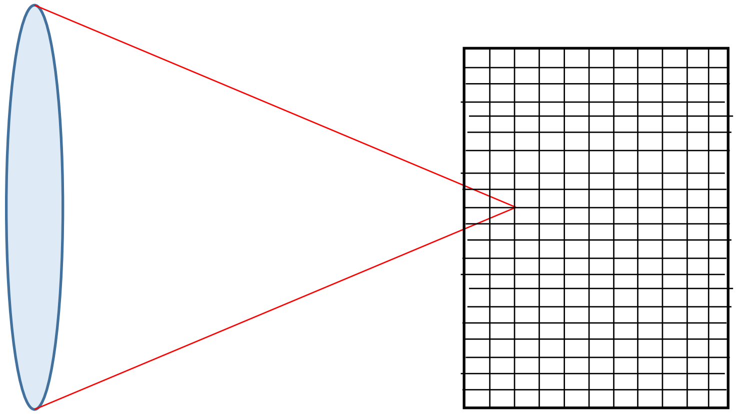

Monochromatic camera sensor

"Ground truth" object:

$I_0(x, y, \lambda)$

100 x 100 pix. x 30 spectral channels

$$I_s(x, y) = \sum_{\lambda} I_0(x, y, \lambda)$$

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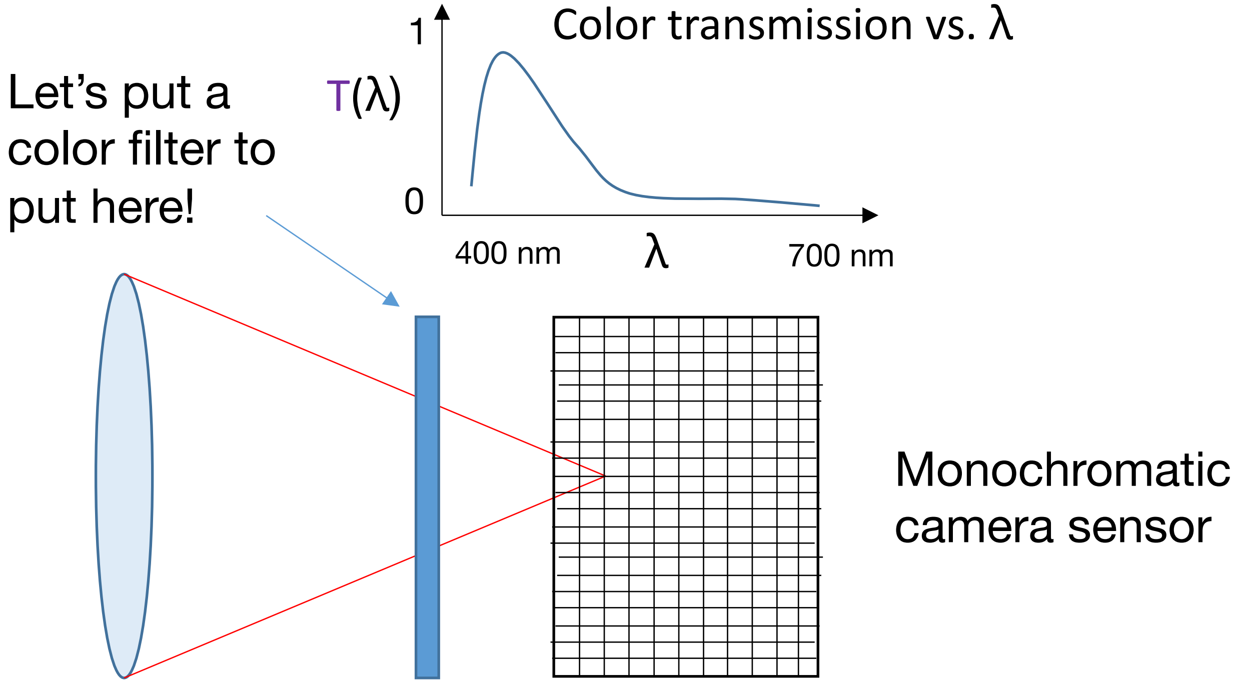

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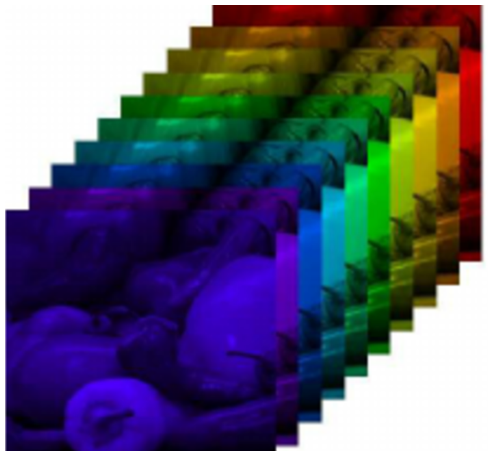
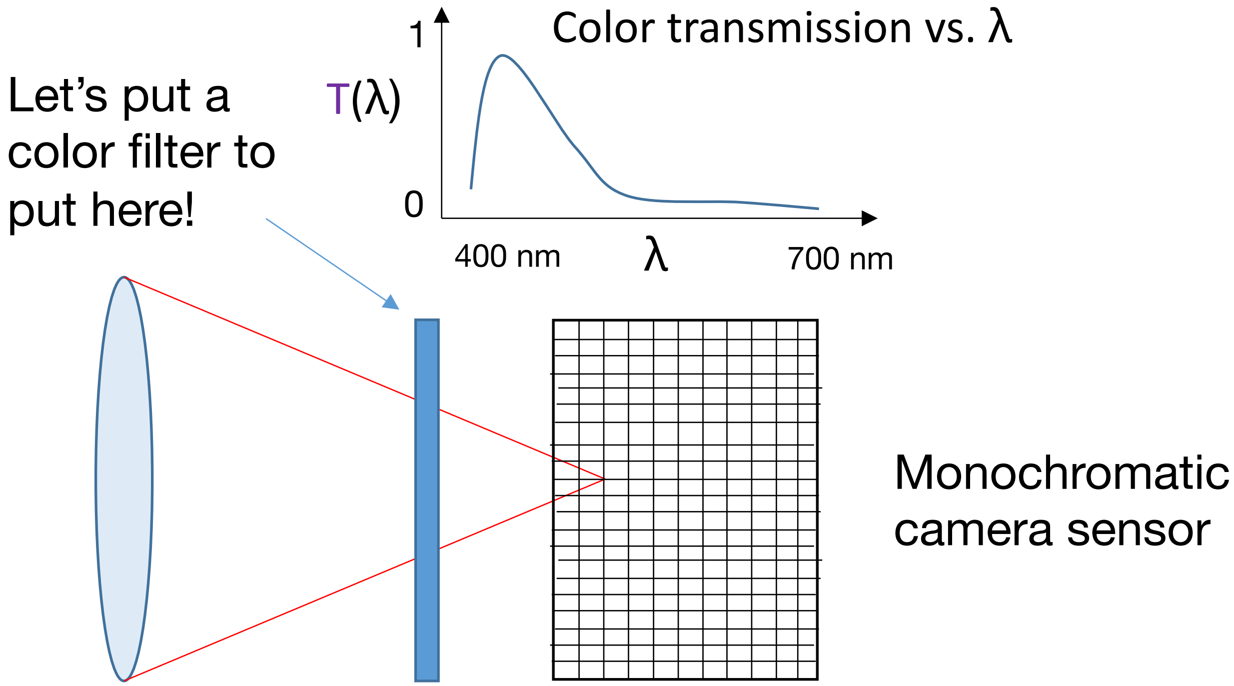


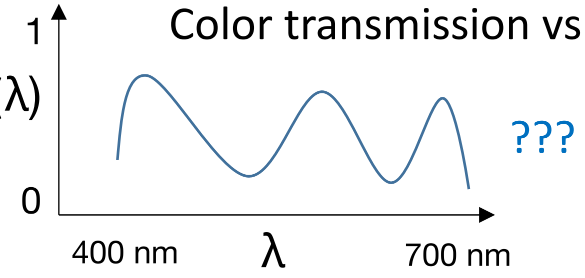


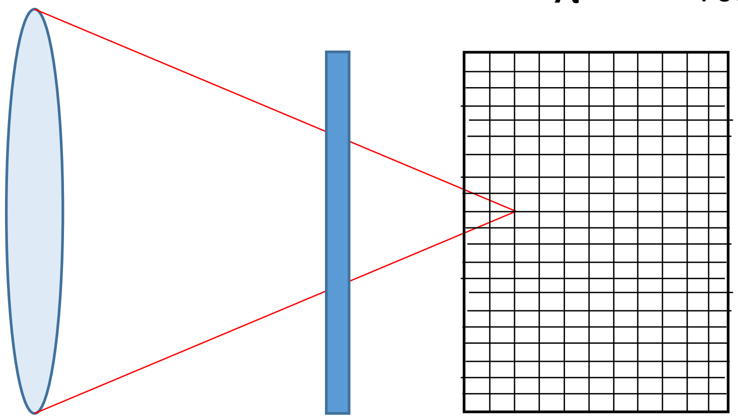
"Ground truth" object:
$I_0(x, y, \lambda)$
100 x 100 pix. x 30 spectral channels

$$\boxed{I_s(x, y) = \sum_{\lambda} T(\lambda)\, I_0(x, y, \lambda)}$$

Example #2: Optimized color filter for a grayscale camera

Design optimal color filter for classification:





Monochromatic camera sensor

Training data:
[$I_0(x, y, \lambda)$, y]
I_0:100 x 100 pix. x 30
Label y: 1x3 - pepper, broccoli, green beans

$$I_s(x, y) = \sum_{\lambda} W_0(\lambda)\, I_0(x, y, \lambda)$$

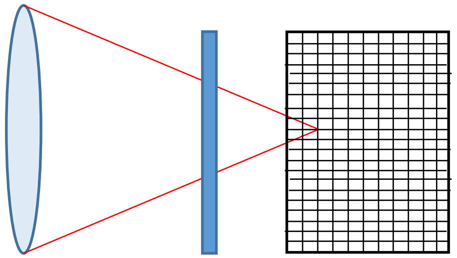
Physical Layer

Example #2: Optimized color filter for a grayscale camera

Training data:
[$\mathbf{I}_0(x, y, \lambda)$, y]
$\mathbf{I}_0$:100 x 100 x 30
Label y: 1x3

Example #2: Optimized color filter for a grayscale camera

deep imaging

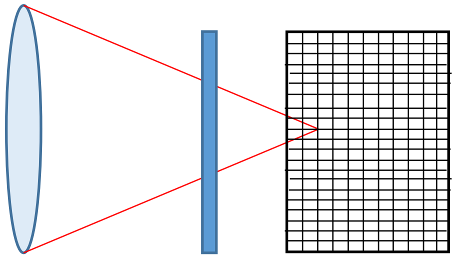


Training data:
[$I_0(x, y, \lambda)$, y]
I_0: 100 x 100 x 30
Label y: 1x3

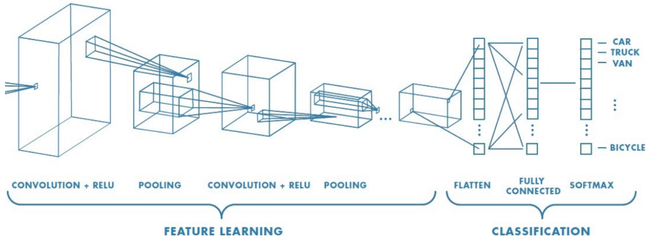
$$I_s(x, y) = \sum_{\lambda} W_0(\lambda)\, I_0(x, y, \lambda)$$

Physical Layer

Example #2: Optimized color filter for a grayscale camera



$[I_s(x, y), y]$



Training data:

$[I_0(x, y, \lambda), y]$

I_0: 100 x 100 x 30

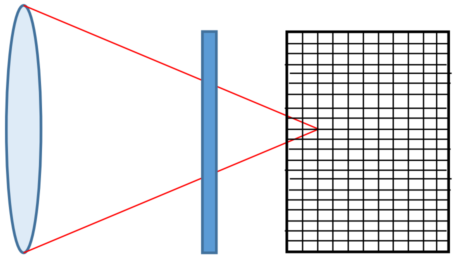
Label y: 1x3

$$\boxed{I_s(x, y) = \sum_{\lambda} W_0(\lambda)\, I_0(x, y, \lambda)}$$

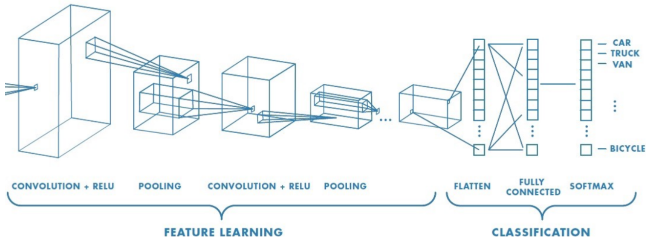
Physical Layer

$$\text{Task} = \mathbf{W_n} \ldots \mathrm{ReLU}[\mathbf{W_1}\, \mathrm{ReLU}[\mathbf{W_0}\, I_s] \ldots]$$

Example #2: Optimized color filter for a grayscale camera



$[I_s(x, y), y]$



Training data:
$[I_0(x, y, \lambda), y]$
I_0: 100 x 100 x 30
Label y: 1x3

$$I_s(x, y) = \sum_{\lambda} W_0(\lambda)\, I_0(x, y, \lambda)$$

Physical Layer

Task = $\mathbf{W_n} \ldots \mathrm{ReLU}[\mathbf{W_1}\, \mathrm{ReLU}[\mathbf{W_0}\, I_s] \ldots]$

```
multispectral_data = tf.placeholder(tf.float32, [None, num_colors, image_size])
veg_labels = tf.placeholder(tf.float32, [None, 3])
filter_weights = tf.truncated_normal([num_colors, 1], stddev = 0.1)
filtered_images = tf.einsum('aij,jk->aik', multispectral_data, filter_weights)

#Now, train CNN of your choice using [filtered_images, veg_labels]
#Output of training/testing will be CNN weights, classification performance AND filter_weights!
```

Example implementation with Tensorflow 1.0 code

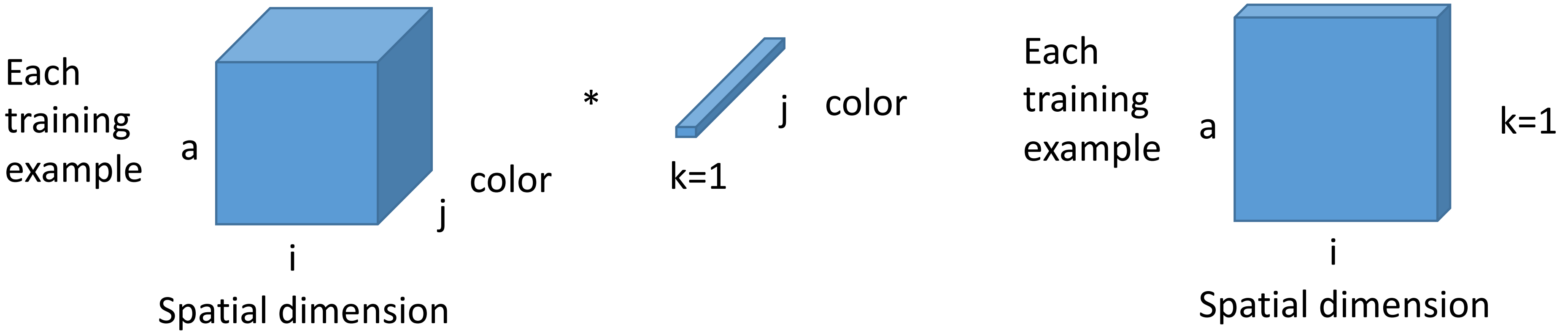
Tensorflow: operations to sum along 3rd (or higher) dimension

Option 1: Einsum (shown as Tensorflow 1.0 code, and also applicable in Tensorflow 2.0)

```
filtered_images = tf.einsum('aij,jk->aik', multispectral_data, filter_weights)
```

Option 2: tf.reduce_sum

```
filtered_images = tf.reduce_sum(multispectral_data * filter_weights, axis=2)
```



Option 3: Locally connected conv2D with a 1x1 filter size

https://github.com/keras-team/keras/blob/master/keras/layers/local.py#L183

Pause to take a look at:

weighted_image_sum_example.ipynb

Summary: simple physical layers for incoherent imaging

deep imaging

- Deal with sample/image intensities I, real and non-negative
- Effect of illumination is element-wise multiplication $I_e(x,y) = \mathbf{S}\, I_0(x,y)$

λ

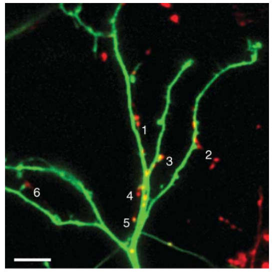
- Imaging systems blur the object via point-spread function matrix **H**

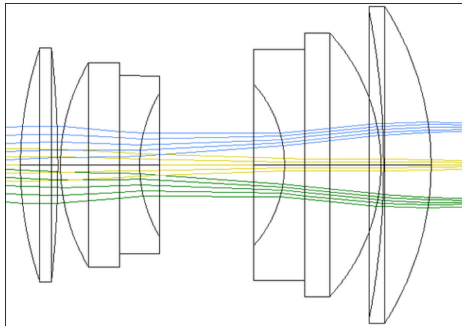
$I_b(x,y) = \mathbf{H}\, I_0(x,y)$

- Discrete pixels down-sample the object via $I_d(x,y) = \mathbf{D}\, I_0(x,y)$
- Add noise into measurement $I_N(x,y) = \mathbf{D}\, I_0(x,y) + \mathbf{N}$
- Different colors add linearly $I_s(x, y) = \Sigma\, I_0(x, y, \lambda)$

What is light and how can we model it?

- Interpretation #1: Radiation (*Incoherent*)
- Model: Rays

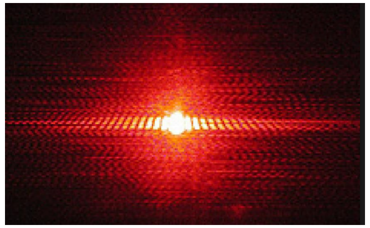


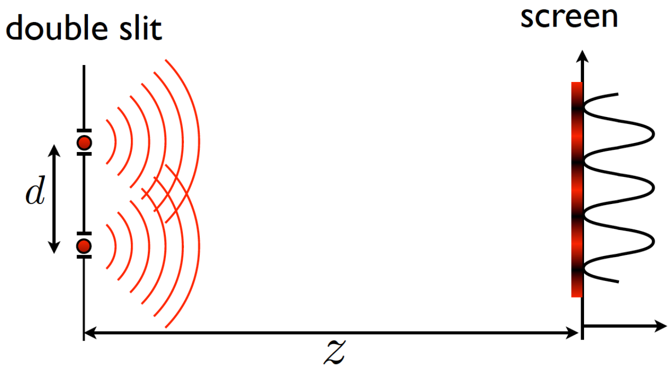


- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

- Interpretation #2: Electromagnetic wave (*Coherent*)
- Model: Waves





- Complex field
- Models Interference

$$E_{tot} = E_1 + E_2$$

Next class: Modeling coherent radiation as a wave