Lecture 23: Computed Tomography

Machine Learning and Imaging

BME 548L
Roarke Horstmeyer

deep imaging

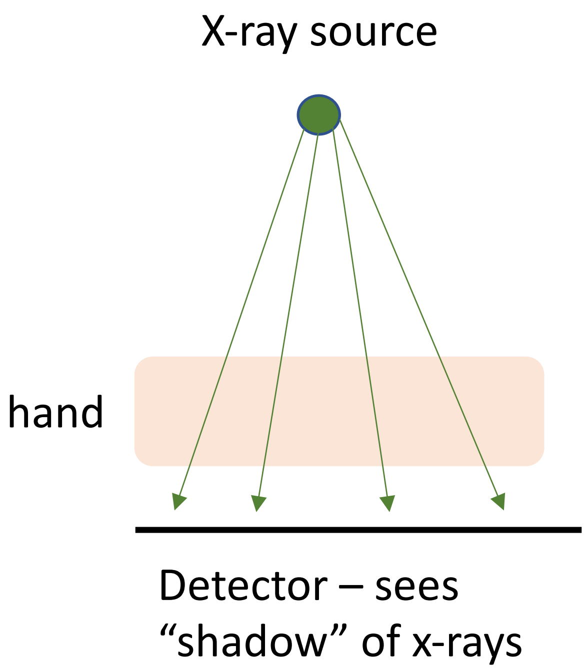
Reminder: Please complete course evaluations here

duke.evaluationkit.com

Any input and feedback is greatly appreciated!!

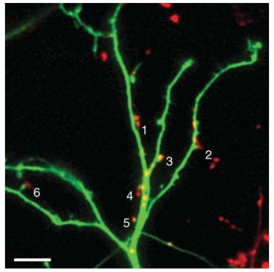
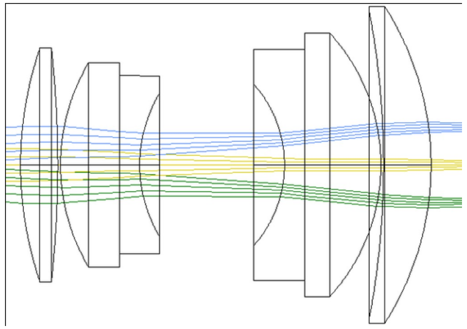
What is computed tomography?

Standard X-ray



How can we model radiation?

- Interpretation #1: Radiation (*Incoherent*)
- Model: Rays

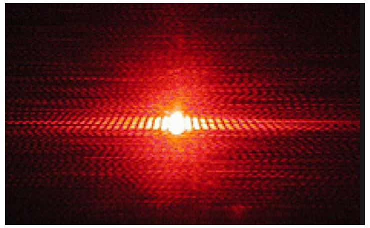




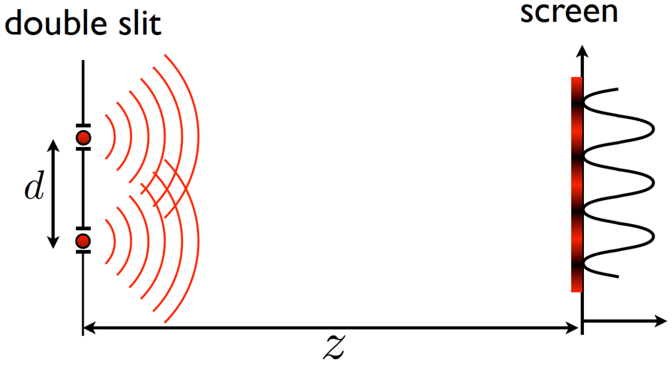
- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

Most X-rays: Incoherent radiation (treat as ray)

- Interpretation #2: Electromagnetic wave (*Coherent*)
- Model: Waves





- Complex field
- Models Interference

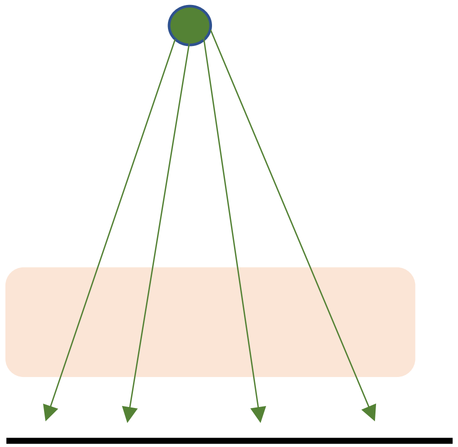
$$E_{tot} = E_1 + E_2$$

What is computed tomography?

Standard X-ray

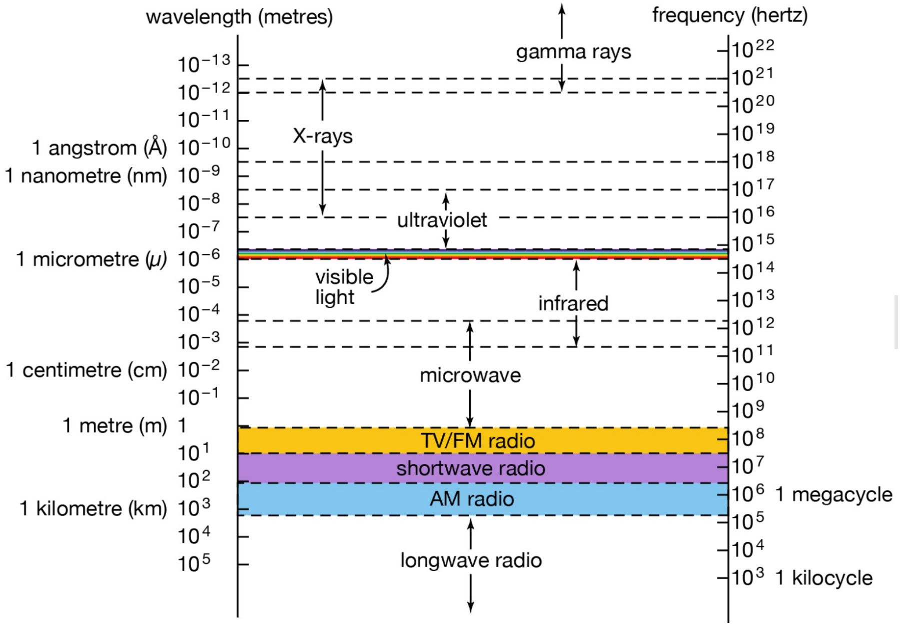
X-ray source

hand



Detector – sees "shadow" of x-rays

X-rays: EM radiation with very short wavelength

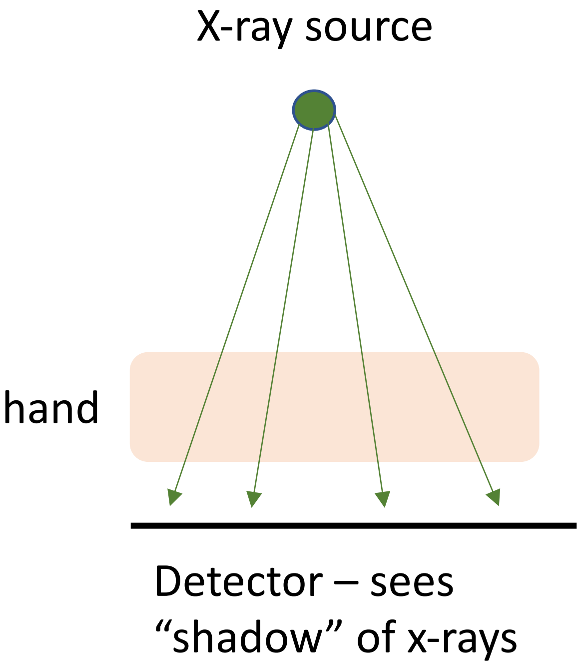


Typically reported as energy: E=hv

e.g., 100 kEV x-ray source

What is computed tomography?

Standard X-ray



Intensity *I* of a beam of X-ray satisfies Beer’s Law:

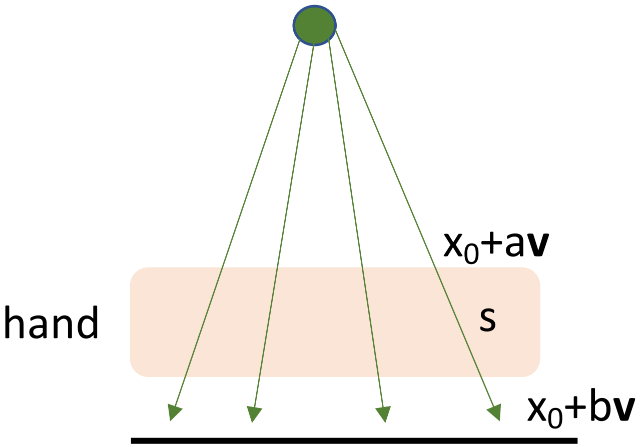
$$\frac{dI}{ds} = -\mu(\boldsymbol{x})I.$$

Material	Attenuation coefficient in Hounsfield units
water	0
air	−1000
bone	1086
blood	53
fat	−61
brain white/gray	−4
breast tissue	9
muscle	41
soft tissue	51

What is computed tomography?

Standard X-ray

X-ray source



Detector – sees "shadow" of x-rays

Intensity *I* of a beam of X-ray satisfies Beer's Law:

$$\frac{dI}{ds} = -\mu(\boldsymbol{x})I.$$

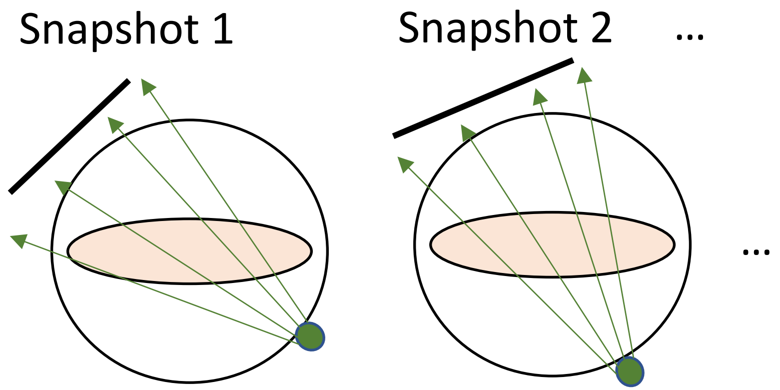
$$m(s) = \mu(\boldsymbol{x}_0 + s\boldsymbol{v})$$

Between the points $\boldsymbol{x}_0 + a\boldsymbol{v}$ and $\boldsymbol{x}_0 + b\boldsymbol{v}$, the intensity of the beam is attenuated by

$$I_b / I_a = \exp\left[-\int_a^b m(s)\,ds\right].$$

Under some approximations,

$$I_b\,(x+z) / I_a\,(x) = \exp(-\mu z)$$

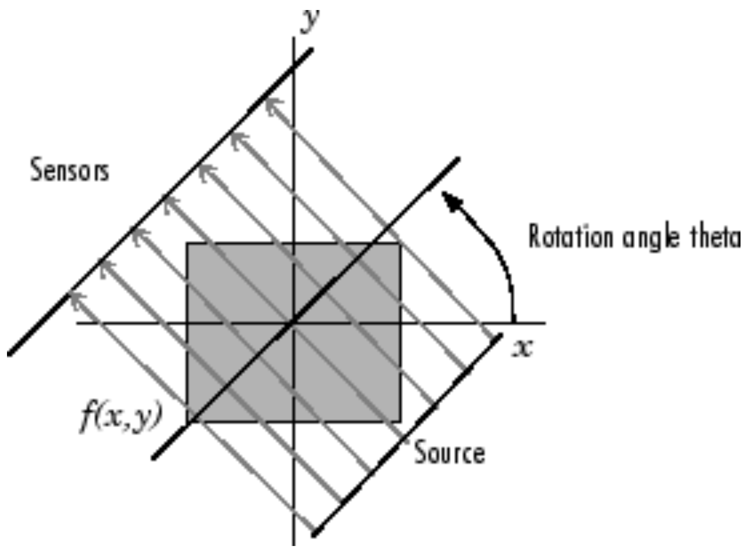


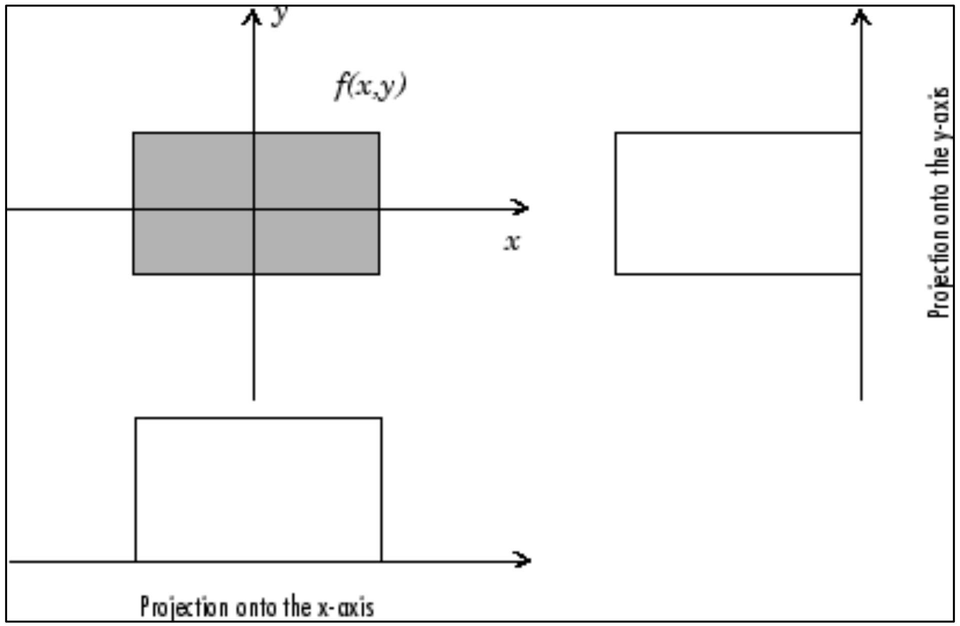
What is computed tomography?

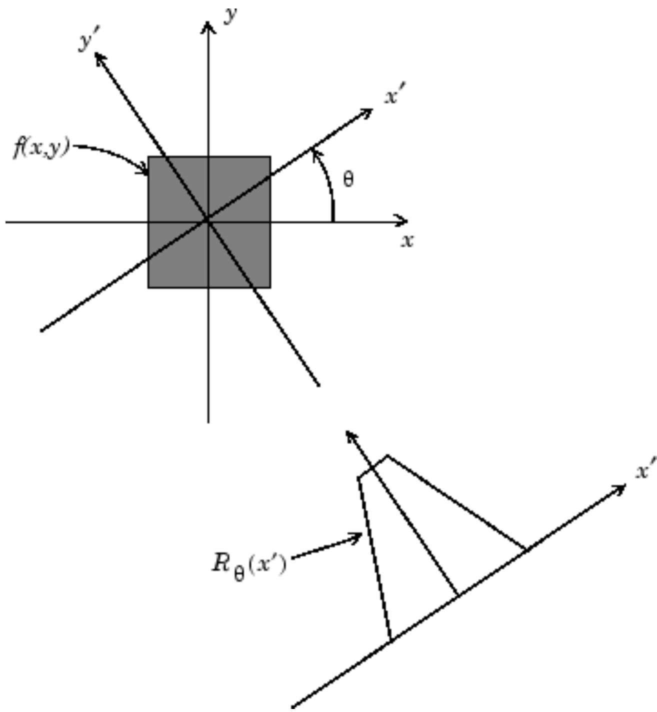
Standard X-Ray

Computed tomography: X-Ray projector that spins!

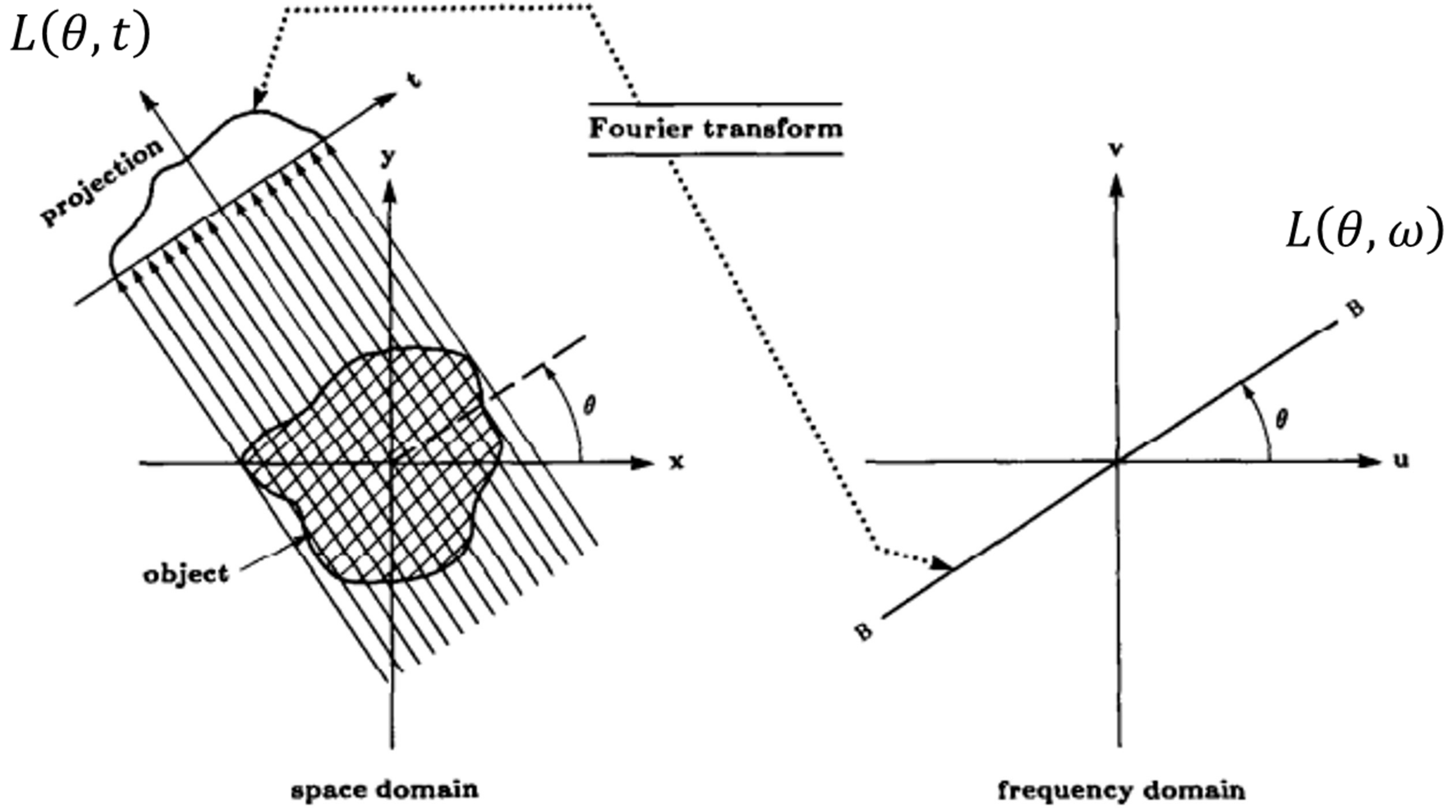
The radon transform – from "shadows" to 3D shape







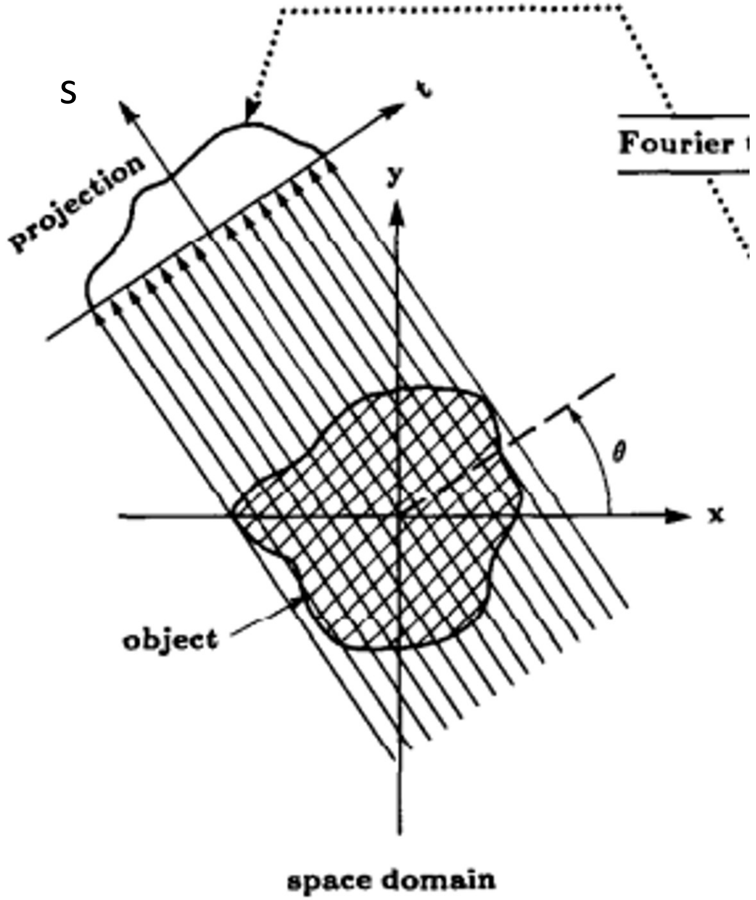
Key idea – the Fourier transforms of projections are really useful



Fourier slice theorem, in a nutshell

- If we have an imaging object $f(x, y)$, and we measure the projection $p_\theta(t)$ of it at an angle θ.
- Then, the Fourier transform of the projection along line $L(\theta, t)$,
- Is a slice of the object's spectrum
- $F(u, v) = \iint f(x, y) e^{-j2\pi(ux+vy)} dxdy$
- Along the line $L(\theta, \omega)$, where
- $L(\theta, \omega) = \{(u, v) \in \mathbb{C} \times \mathbb{C}: u\cos\theta + v\sin\theta = \omega\}$
- $L(\theta, t) = \{(x, y) \in \mathbb{R} \times \mathbb{R}: x\cos\theta + y\sin\theta = t\}$

Derivation of Fourier slice theorem

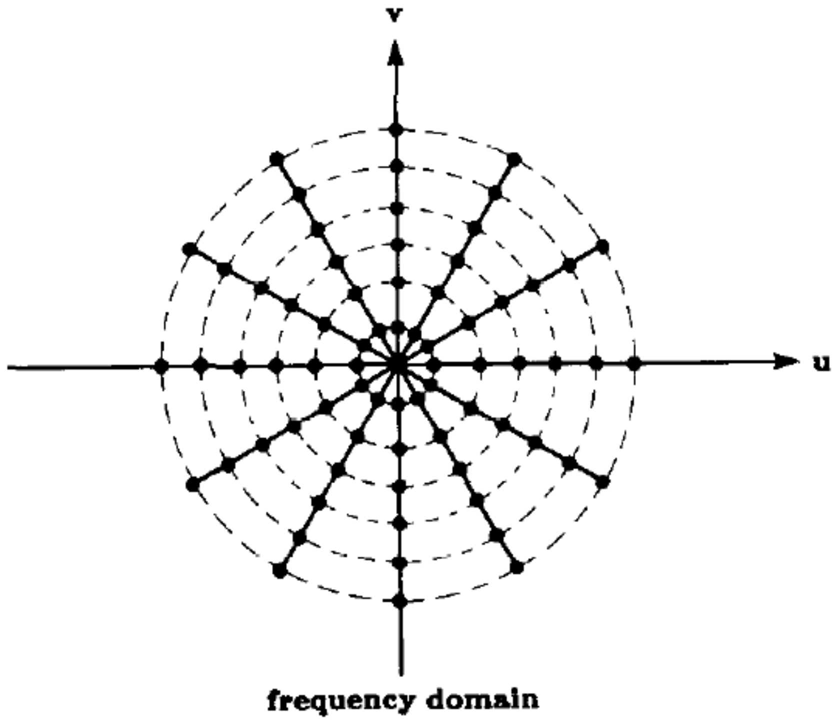


- We work in the transformed coordinate
  - $t = x\cos\theta + y\sin\theta$
  - $s = -x\sin\theta + y\cos\theta$
- The projection measurement along $P_\theta(t) = \int f(t, s)ds$
- Whose Fourier transform is $S_\theta(\omega) = \int P_\theta(t)e^{-j2\pi\omega t}dt$
-

Derivation of Fourier slice theorem

- Substituting the projection expression into the Fourier transform equation
- $S_\theta(\omega) = \iint f(t,s) ds\, e^{-j2\pi\omega t} dt$
- Since $t = x\cos\theta + y\sin\theta$, and both $\{x, y\}$ and $\{s, t\}$ are defined along $(-\infty, \infty)$, we can substitute x and y for t
- We have $S_\theta(\omega) = \iint f(t,s) ds\, e^{-j2\pi\omega(x\cos\theta + y\sin\theta)} dt = F(\omega\cos\theta\,, \omega\sin\theta)$.
- Where $F(u,v)$ is the 2D Fourier transform of $f(x,y)$

K space sampling



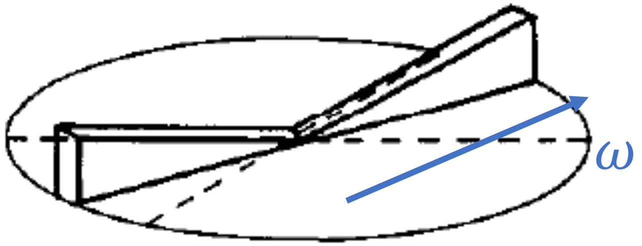
Note that high frequencies are sampled much less densely than low frequencies

Filtered back-projection algorithm

- Note that to invert the process, we need to do Frequency domain interpolation, which is not desired.
- The filtered back-projection algorithm is more popularly used, which contains following step:

Filtered back-projection algorithm

- Fourier transform the projection measurement
- Filter each frequency component using a "wedge" function shown on the right
- 2D Inverse Fourier transform each slice and sum in the space domain

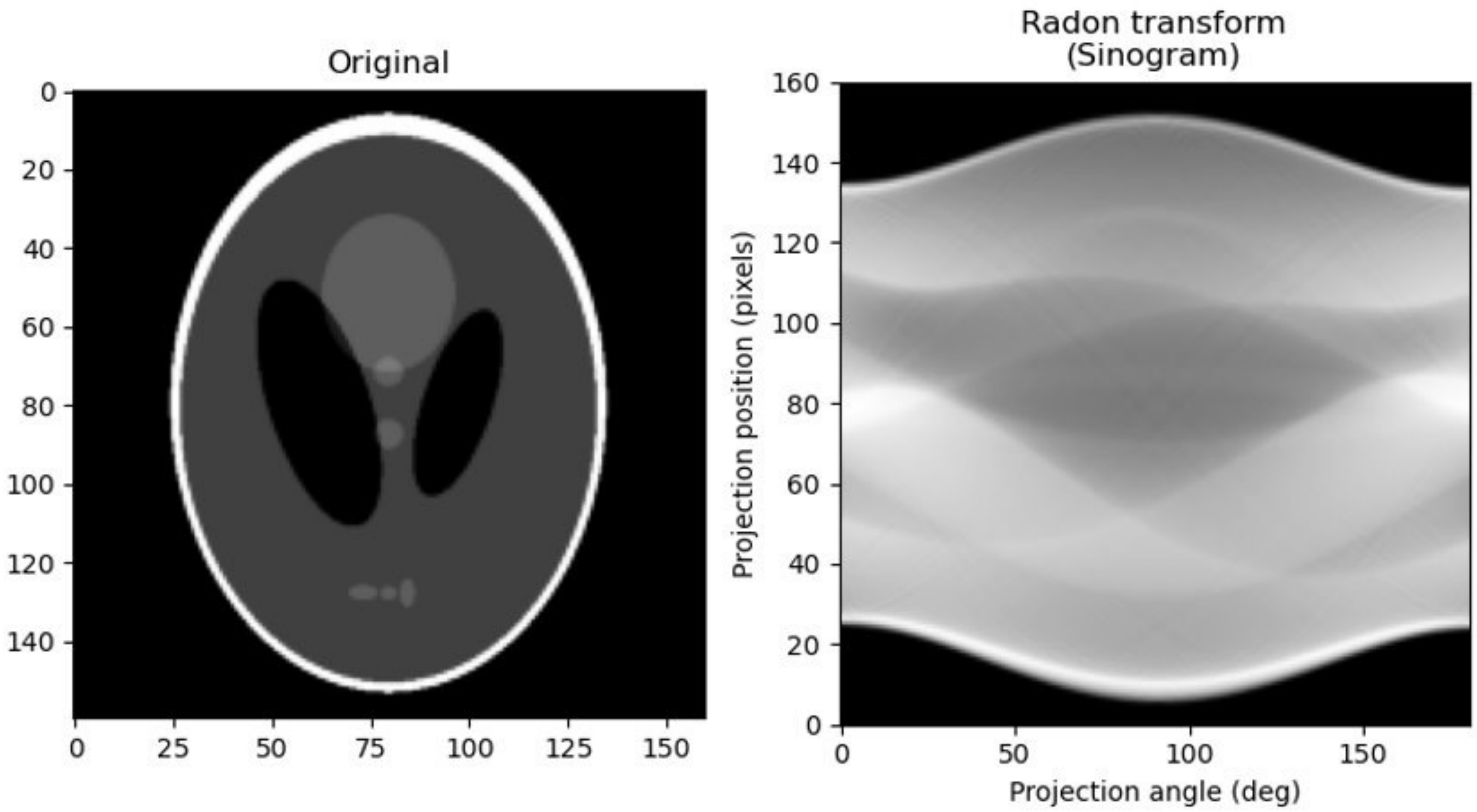


Derivation

- We write the inverse Fourier transform in polar coordinate
- $f(x,y) = \int_0^{2\pi}\int_0^{\infty} F(\omega,\theta) e^{j2\pi\omega(x\cos\theta + \sin\theta)} \omega d\omega d\theta$
- Let's split into tow parts: 0-180 degrees, and then the rest:
- $f(x,y) = \int_0^{\pi}\int_0^{\infty} F(\omega,\theta) e^{j2\pi\omega(x\cos\theta + \sin\theta)} \omega d\omega d\theta + \int_0^{\pi}\int_0^{\infty} F(\omega,\theta + \pi) e^{j2\pi\omega(x\cos\theta + \sin\theta)} \omega d\omega d\theta$
- Using $F(\omega,\theta+\pi) = F(-\omega,\theta)$, and again $t = x\cos\theta + y\sin\theta$
- We write $f(x,y) = \int_0^{\pi} F(\omega,\theta)|\omega| e^{j2\pi t} d\omega d\theta$

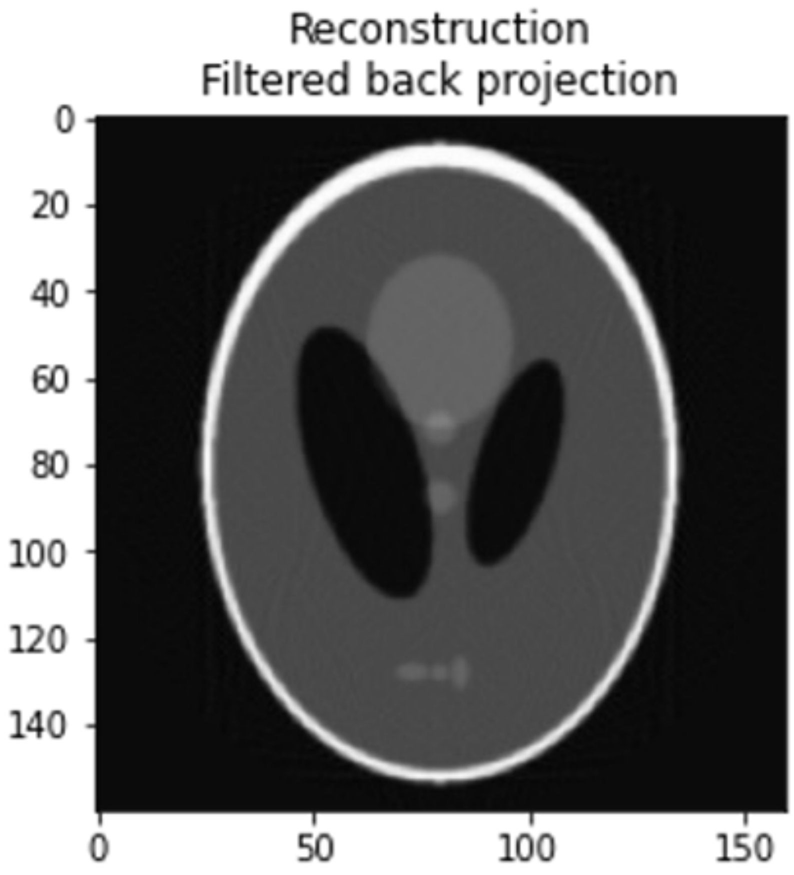
"Wedge filter" Sum in space

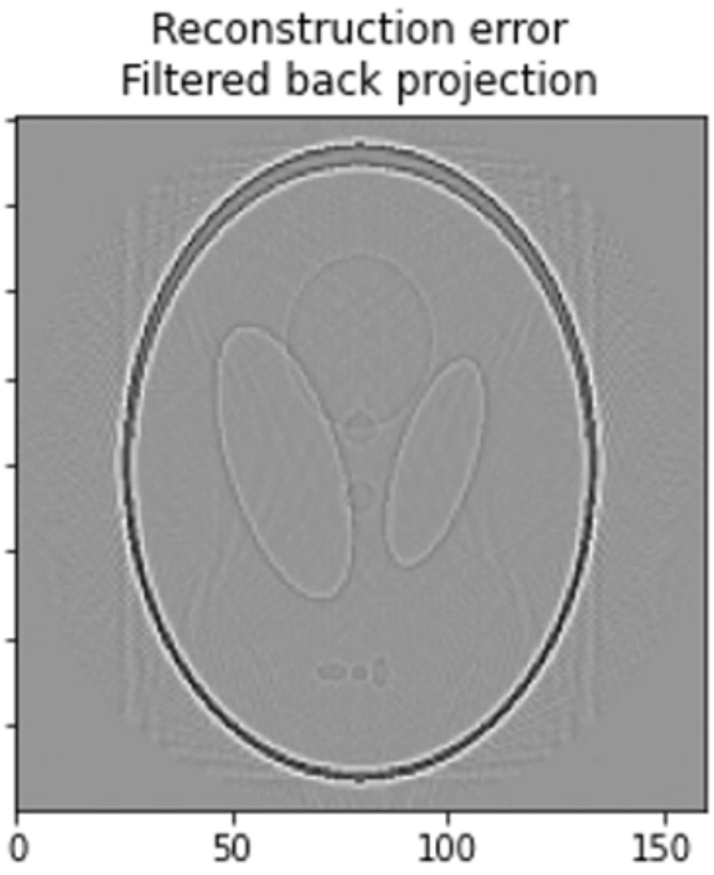
Reconstruction from 180 measurements



https://scikit-image.org/docs/stable/auto_examples/transform/plot_radon_transform.html

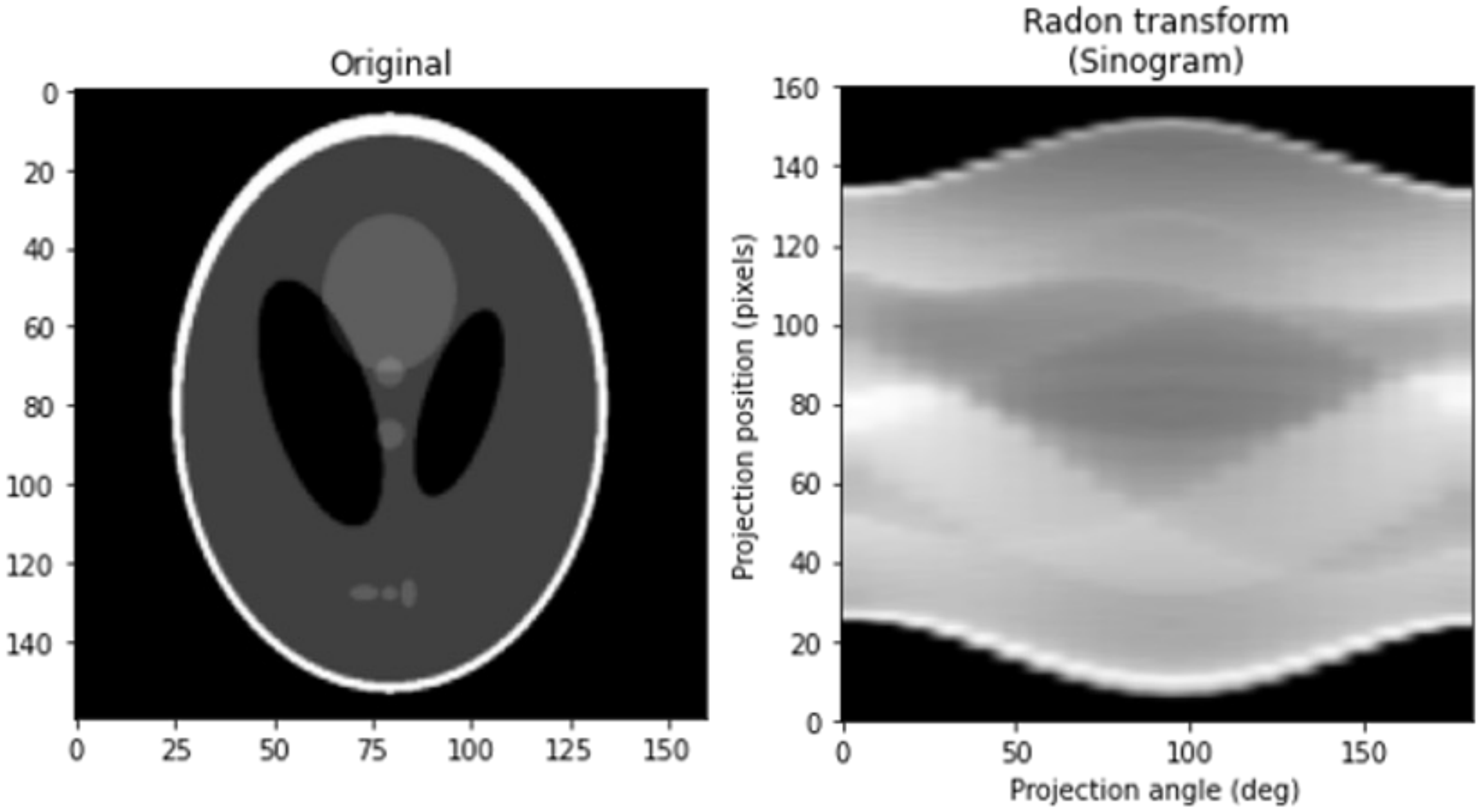
Reconstruction from 180 measurements



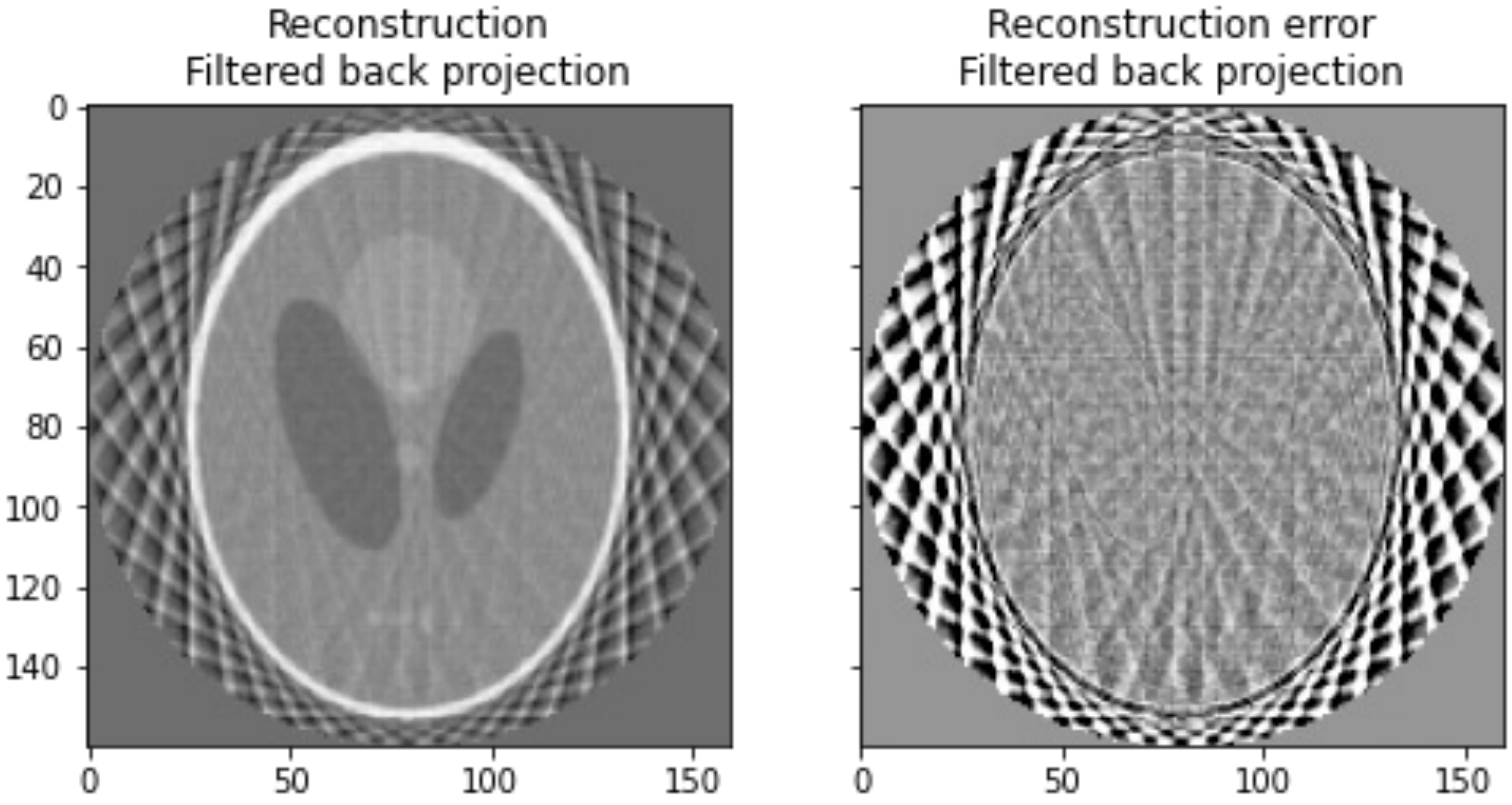


FBP rms reconstruction error: 0.0298

Reconstruction from 22 measurements



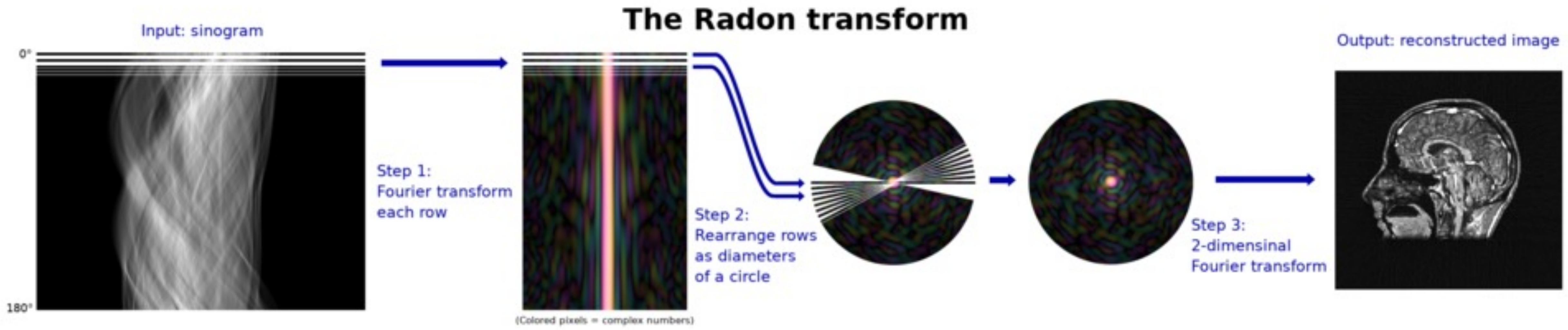
Reconstruction from 22 measurements



FBP rms reconstruction error: 0.117

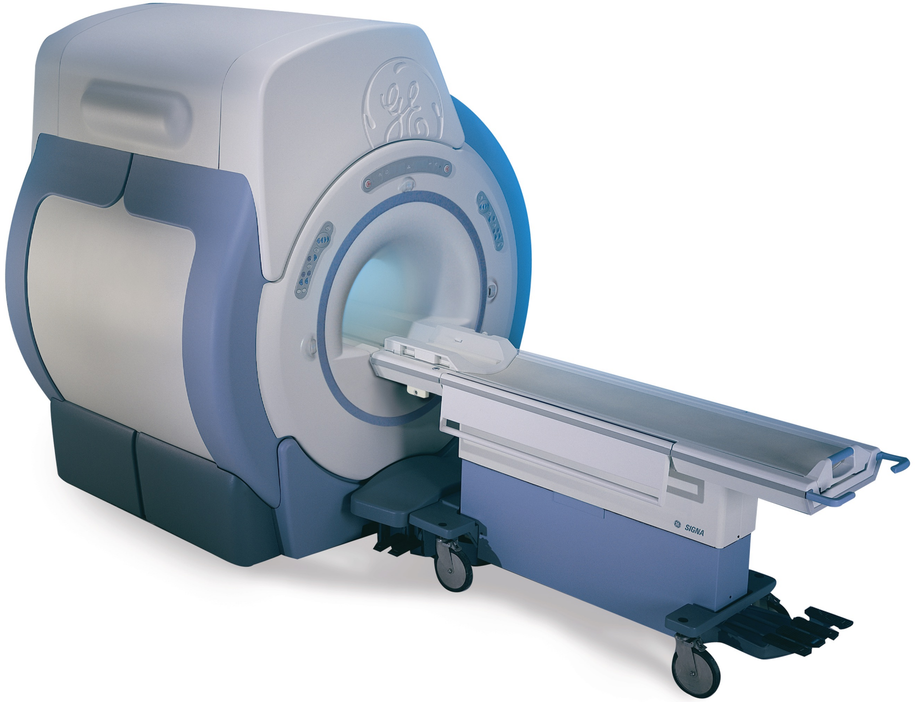
Code

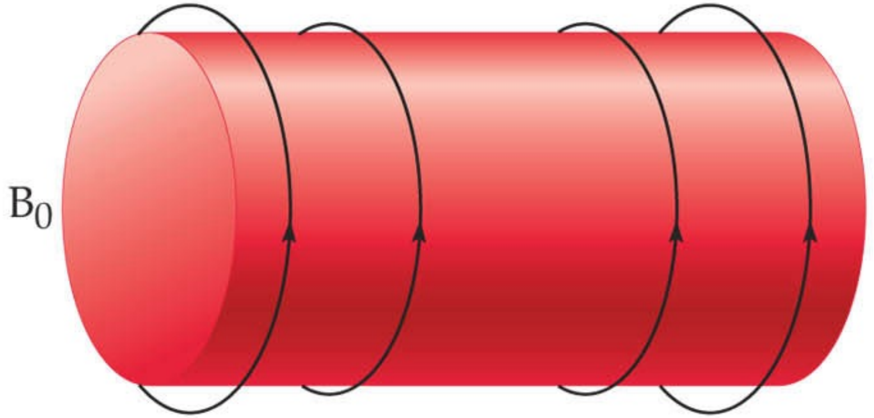
```
image = shepp_logan_phantom()
subsample = 8
theta = np.linspace(0., 180., max(image.shape)//subsample, endpoint=False)
sinogram = radon(image, theta=theta)
reconstruction_fbp = iradon(sinogram, theta=theta, filter_name='ramp')
```


The Radon transform
Input: sinogram
0°
180°
Step 1:
Fourier transform
each row
(Colored pixels = complex numbers)
Step 2:
Rearrange rows
as diameters
of a circle
Step 3:
2-dimensinal
Fourier transform
Output: reconstructed image

(f)MRI

1. Magnetic: Static Magnetic Field Coils
2. Resonance: Radiofrequency Coil
3. Imaging: Gradient Field Coils



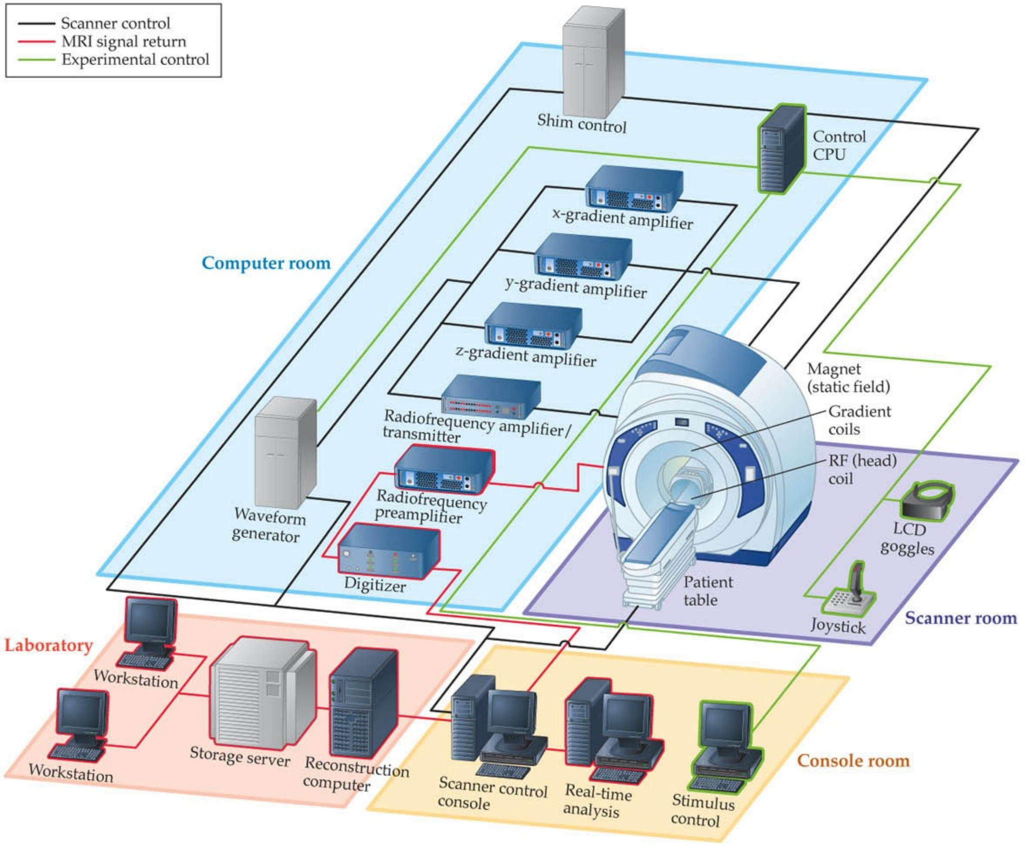


The scanner contains large parallel coilings of wires.
These generate the main magnetic field (B_0), which gives the scanner its field strength (e.g., 3T).

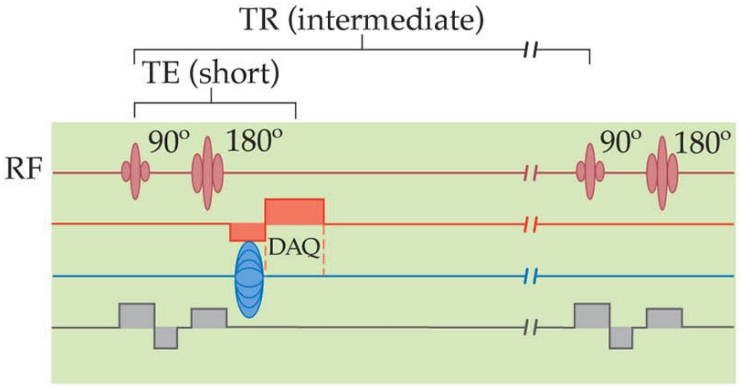
Slides adopted from: https://www.biac.duke.edu/education/courses/fall08/fmri/handouts/2008_Week1_Introduction.ppt

(f)MRI

Resonance: Send radiofrequencies in at different times/locations



FUNCTIONAL MAGNETIC RESONANCE IMAGING, Figure 2.2 © 2004 Sinauer Associates, Inc.



naging

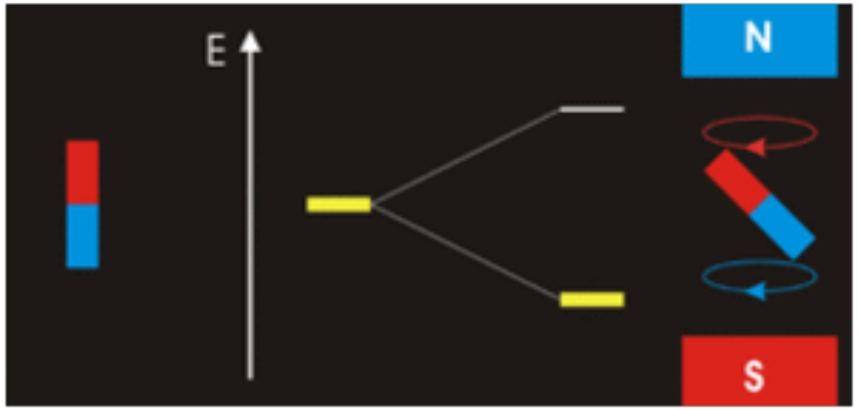
Tissue can be characterized by two different relaxation times – T1 and T2. T1 (longitudinal relaxation time) is the time constant which determines the rate at which excited protons return to equilibrium. It is a measure of the time taken for spinning protons to realign with the external magnetic field. T2 (transverse relaxation time) is the time constant which determines the rate at which excited protons reach equilibrium or go out of phase with each other. It is a measure of the time taken for spinning protons to lose phase coherence among the nuclei spinning perpendicular to the main field.

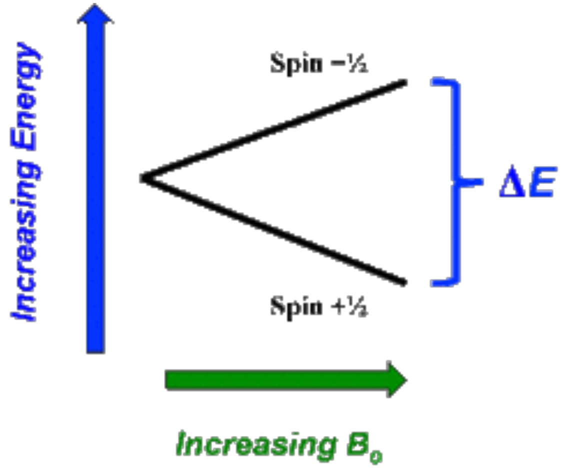
Slides adopted from: https://www.biac.duke.edu/education/courses/fall08/fmri/handouts/2008_Week1_Introduction.ppt

Physical explanation of fMRI

Protons have spin, act like a magnetic dipole

When placed in a magnetic field, dipole tends toward a low and high energy state





$\Delta E = \gamma\ h\ Bo$ γ = *gyromagnetic ratio*

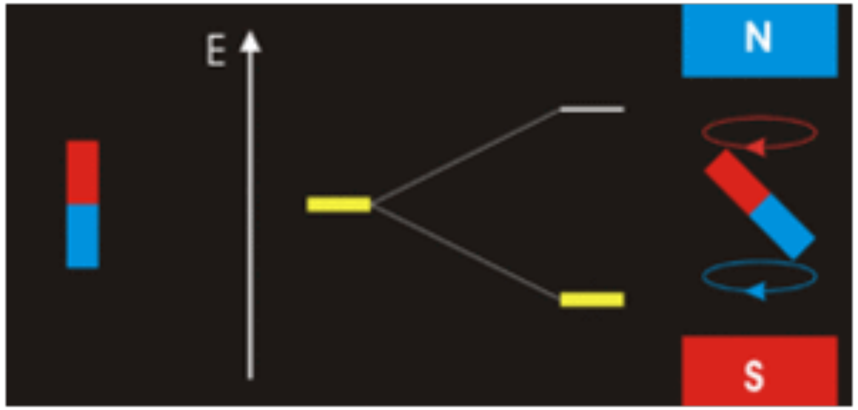
$E = h\ f_o = \gamma\ h\ Bo$

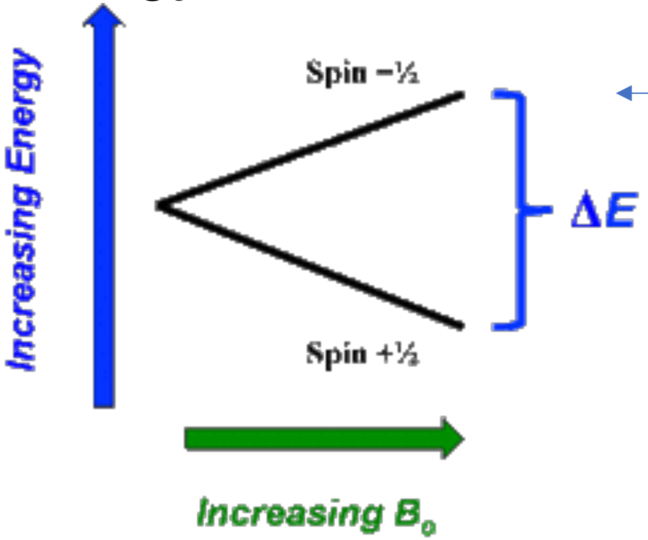
MRI resonance frequency (f_o) is simply the gyromagnetic ratio (γ) times the magnetic field strength (B_o)

Physical explanation of fMRI

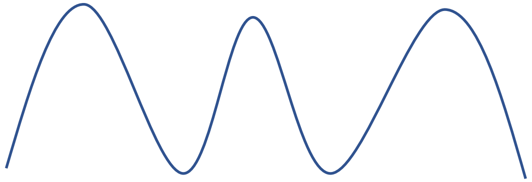
Protons have spin, act like a magnetic dipole

When placed in a magnetic field, dipole tends toward a low and high energy state

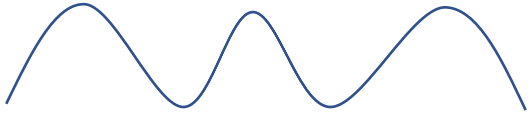




Send in radio waves with frequency f ~ ∂E



measure dynamics



MRI sampling in a nutshell

deep imaging

We suppose that the object to be scanned is placed within the uniform, strong background field, B0 = (0, 0, b0), for a time that is long compared to T1. This polarizes the spins, leading to an equilibrium magnetization:

$M_0 = \rho(x)B_0(x)$.

$\rho(x)$ = density of interest

Only magnetization with a non-zero transverse component will precess in the background field, producing a measurable signal. The measured signal takes the form:

$$s(t) \propto \int_D \overline{\rho}(x, y)dxdy,$$

where D is the projection, onto the xy-plane, of the object between $z = z_0$ and $z = z_1$, and

$$\overline{\rho}(x, y) = \int_{z_0}^{z_1} \rho(x, y, z)dz.$$

https://www2.math.upenn.edu/~cle/notes/Lect2.pdf

We can do much better than the signal described in the previous slide, as this is little more than the total spin density within the selected slice. If, while acquring the RF-signal, we turn on a gradient field of the form

$$\boldsymbol{G}_\phi = (\star, \star, \langle g(\cos\phi, \sin\phi, 0), \boldsymbol{x}\rangle),$$

then the measured signal takes the form

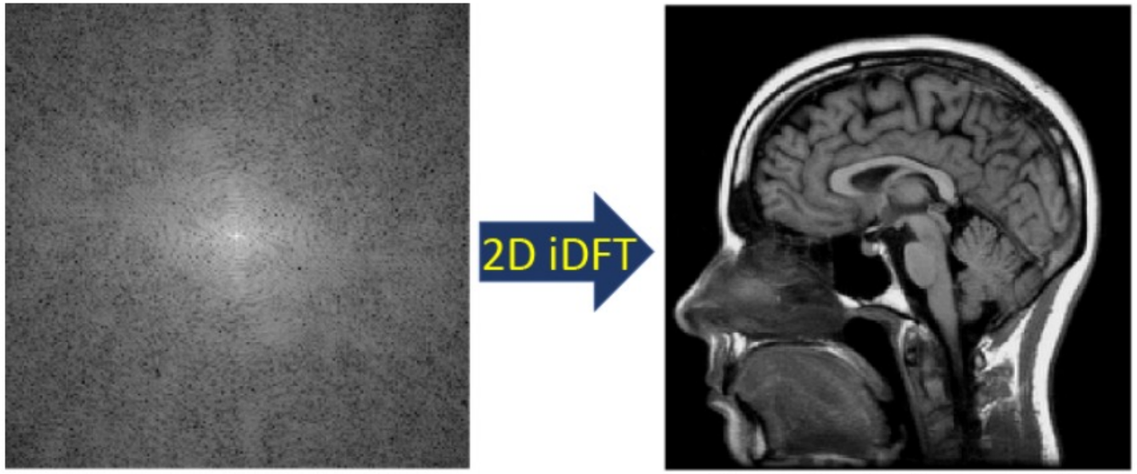
$$s(\phi; t) \propto \int_D e^{i\gamma g t \langle(\cos\phi, \sin\phi), (x,y)\rangle} \overline{\rho}(x, y) dx dy,$$

which is the Fourier transform of $\overline{\rho}$ at the spatial frequency

$$\boldsymbol{k}(t) = -\gamma g t (\cos\phi, \sin\phi).$$

15.2 MRI Reconstruction

Reconstruction of MR images from MR raw data, which is acquired in the Fourier space (termed “k-space” in MRI lingo), can be as simple as a DFT in 2D (for 2D MR imaging, see illustration in Figure 15.1) or a DFT in 3D (for 3D MR imaging).

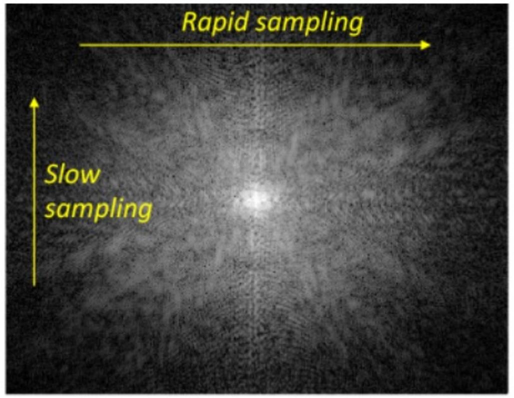


Samples measured as a 2D grid in Fourier space

Reconstructed image results from iDFT of sampled data

https://www2.math.upenn.edu/~cle/notes/Lect2.pdf

https://qiml.radiology.wisc.edu/wp-content/uploads/sites/760/2020/10/notes_015_dft_recon.pdf



In conventional MRI, we can assume that we sample nearly instantaneously along one direction of Fourier space (horizontal in this case, where samples are taken one line at a time), and slowly along the other direction (vertical in this case). In the presence of physiological motion, this leads to motion artifacts along the slow-sampling direction.

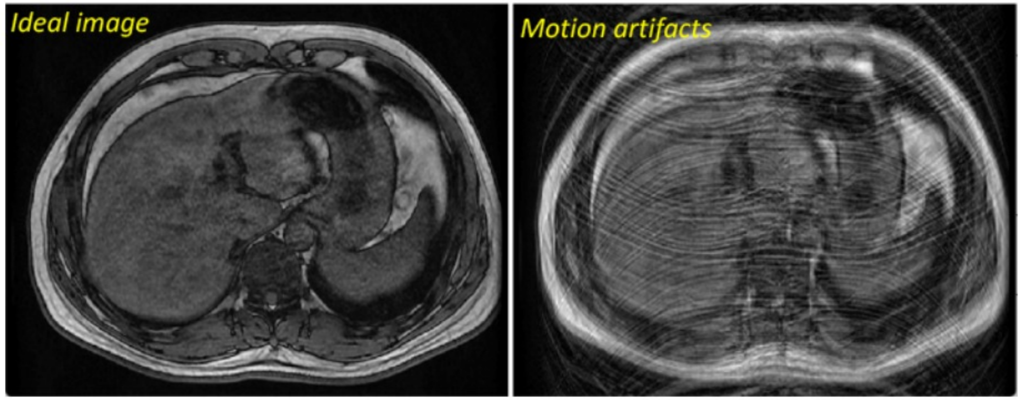
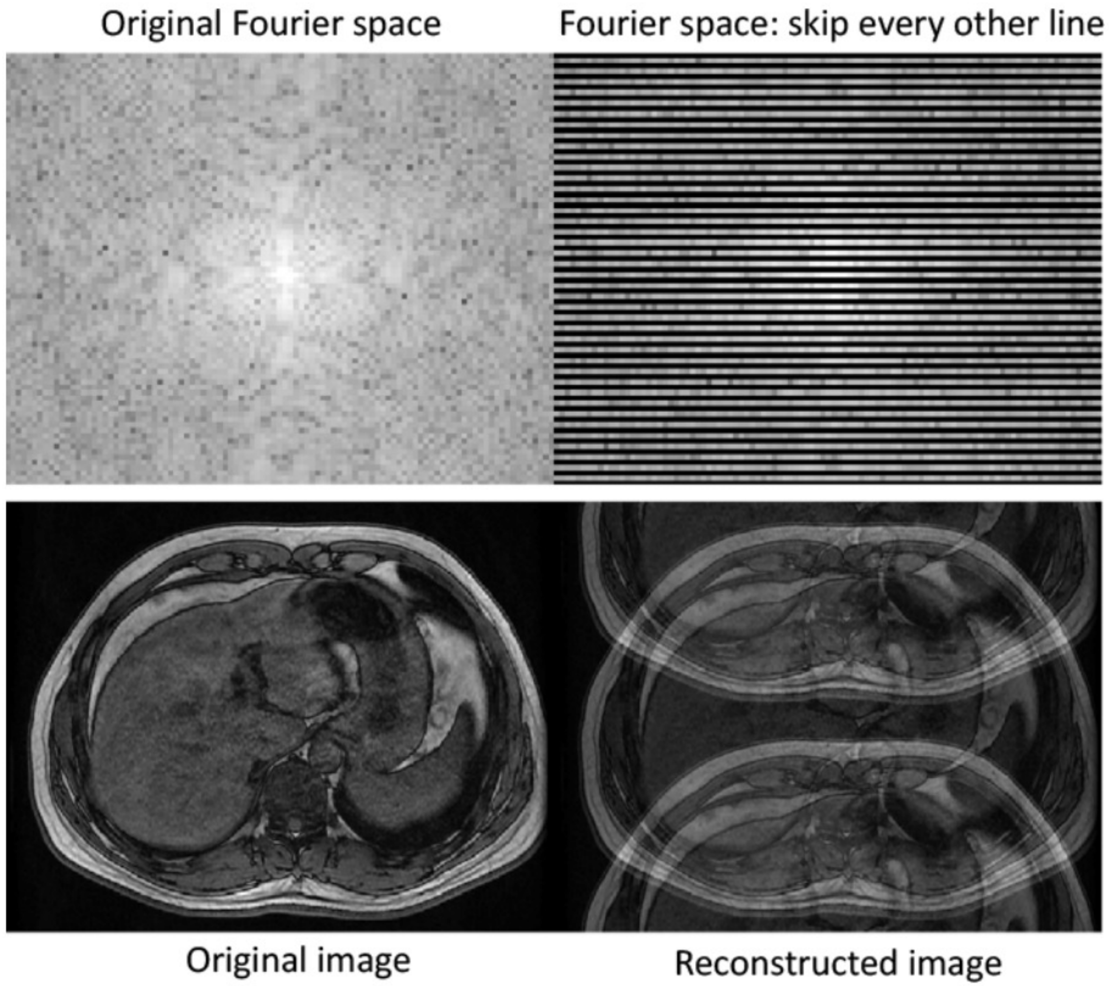


Figure 15.2: Physiological motion in MRI leads to motion artifacts along those directions that are sampled slowly. This effect is due to an inconsistency in the lines of Fourier space acquired at different stages of motion. In other words, different sampled lines in Fourier space correspond to images that are shifted with respect to each other. When we perform the inverse DFT (in 2D in this case) to reconstruct our image, these inconsistencies lead to deleterious motion artifacts observable in image space.

https://www2.math.upenn.edu/~cle/notes/Lect2.pdf



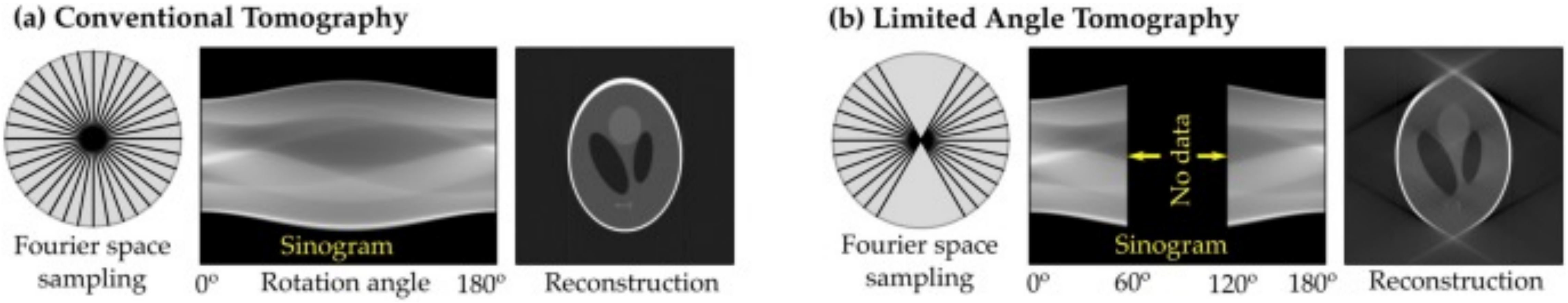
https://www2.math.upenn.edu/~cle/notes/Lect2.pdf

deep imaging

Examples of Deep Learning + Hardware Optimization with CT/MRI

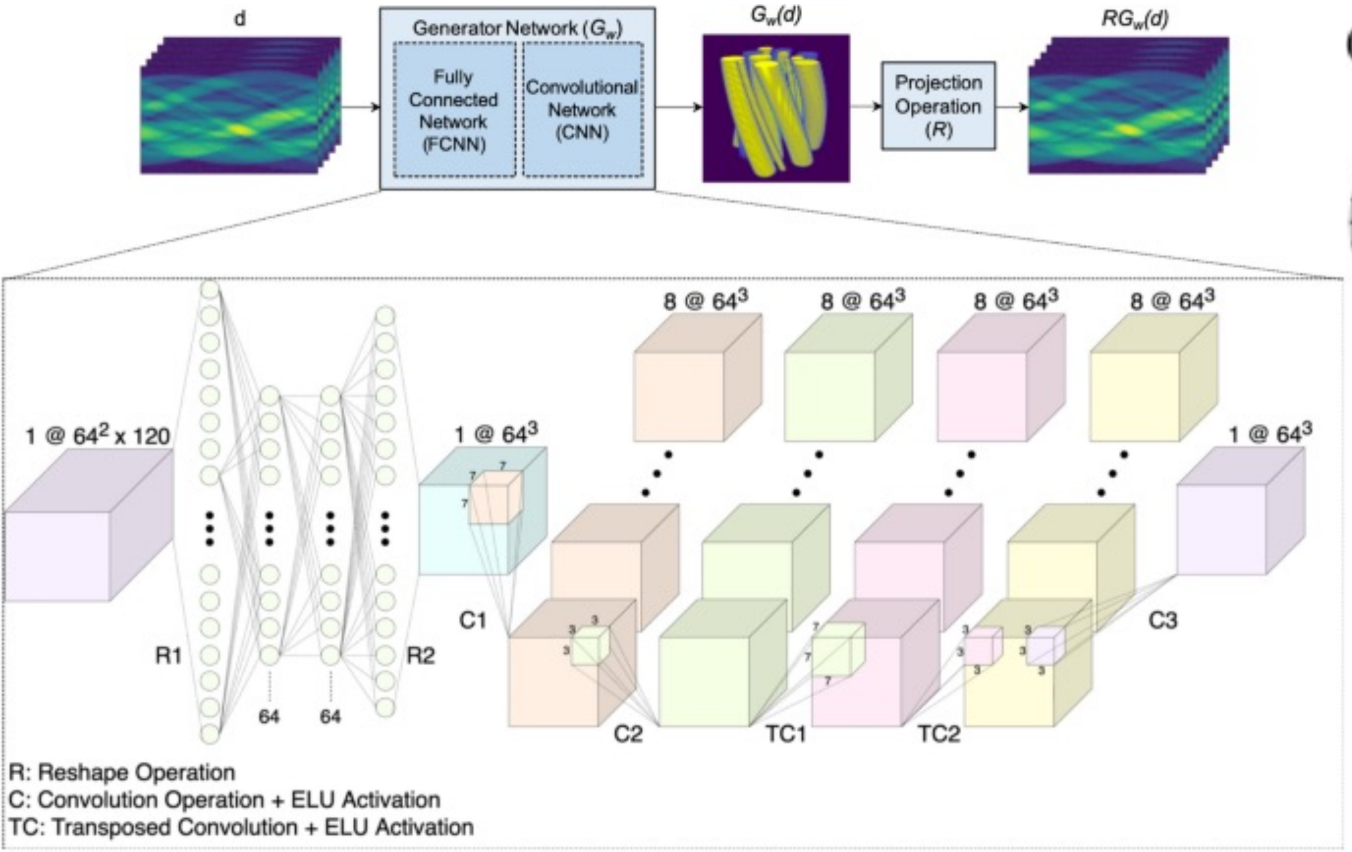
Limited-angle computed tomography with deep image and physics priors

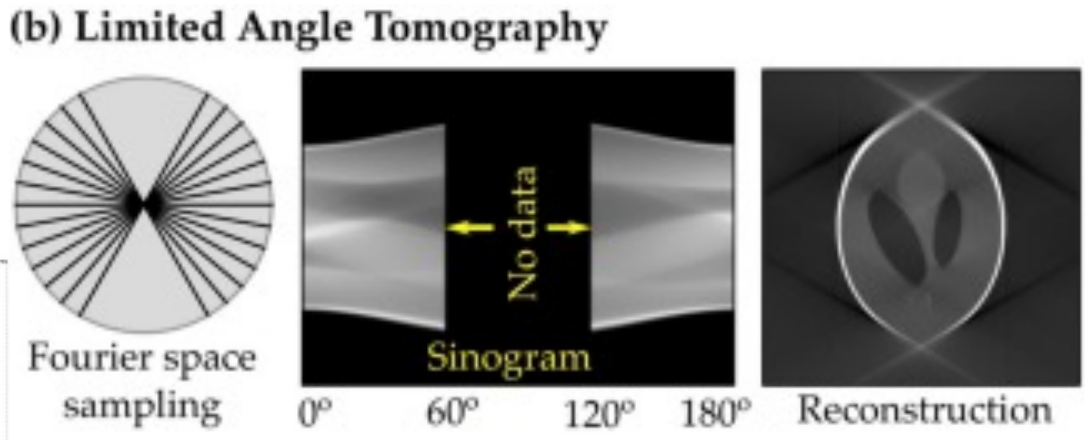
S. Baructu et al., Nat. Sci. (2023)



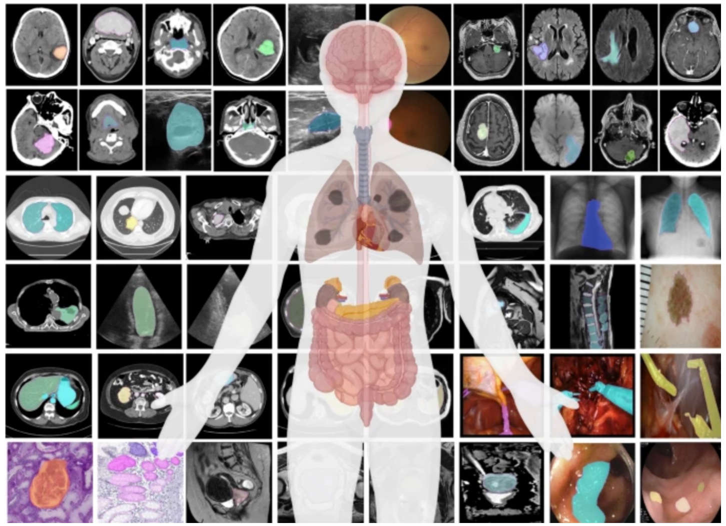
Limited-angle computed tomography with deep image and physics priors

S. Baructu et al., Nat. Sci. (2023)



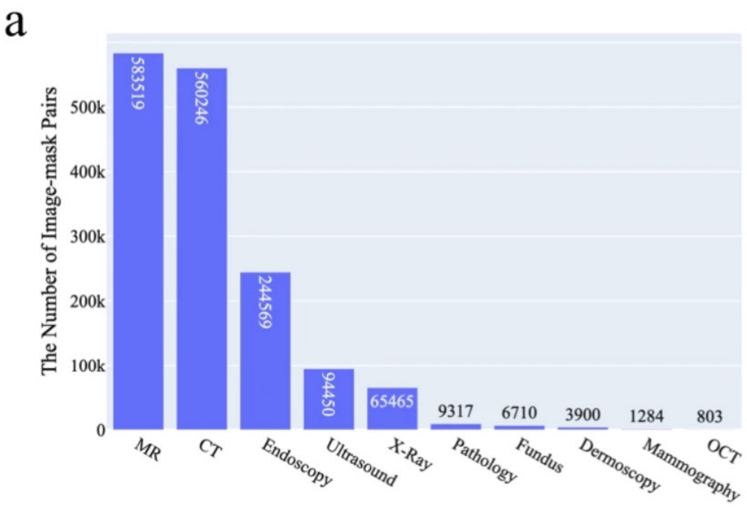


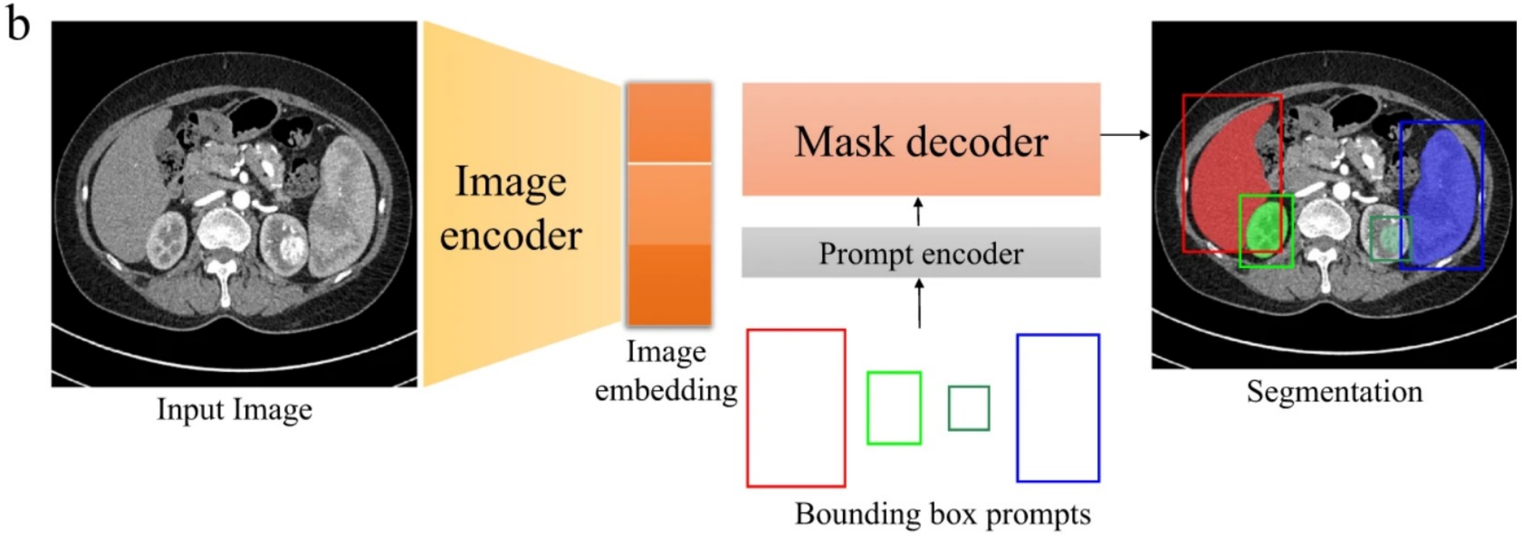
Segment Anything – for medical images



Segment anything - A. Krillov et al., 2023

Segment Anything – for medical images
J. Ma et al., Nature Communications (2024)





Learning a Variational Network for Reconstruction of Accelerated MRI Data

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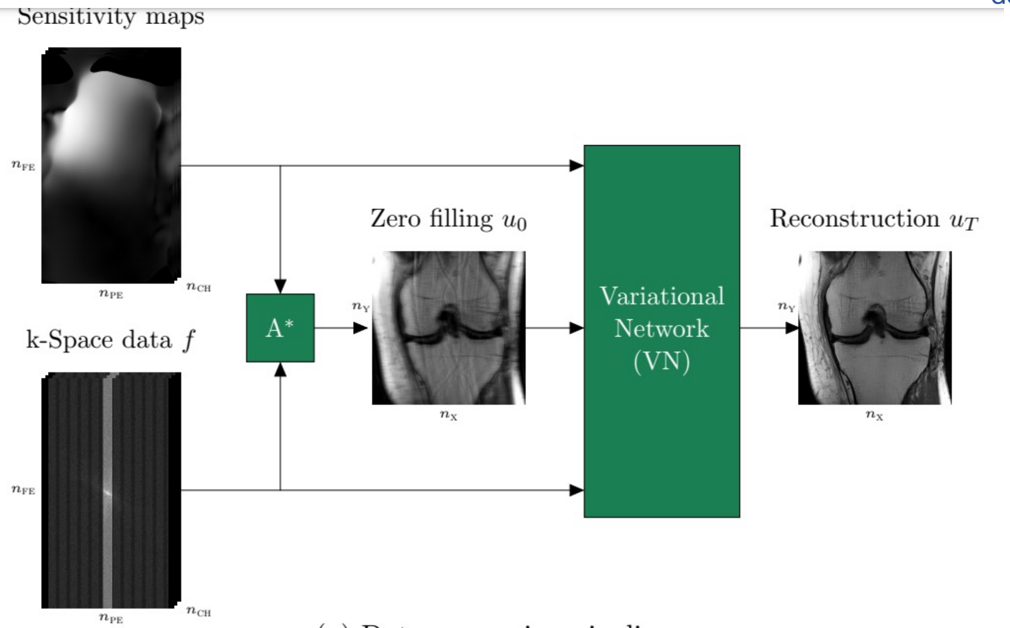
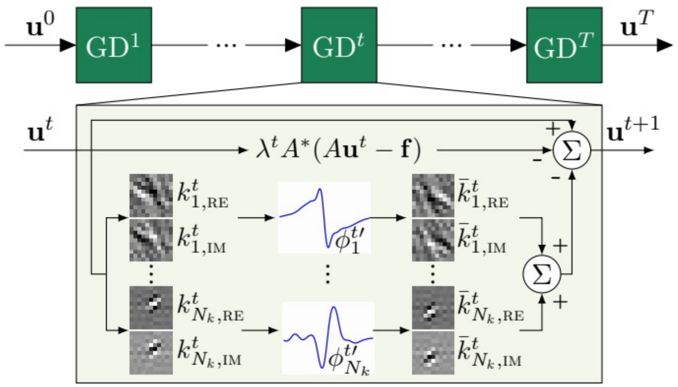


(a) Data processing pipeline



(b) Structure of the variational network (VN)

Facebook Fast MRI Challenge:
https://ai.facebook.com/blog/using-reinforcement-learning-to-personalize-ai-accelerated-mri-scans/

Reducing Uncertainty in Undersampled MRI Reconstruction with Active Acquisition

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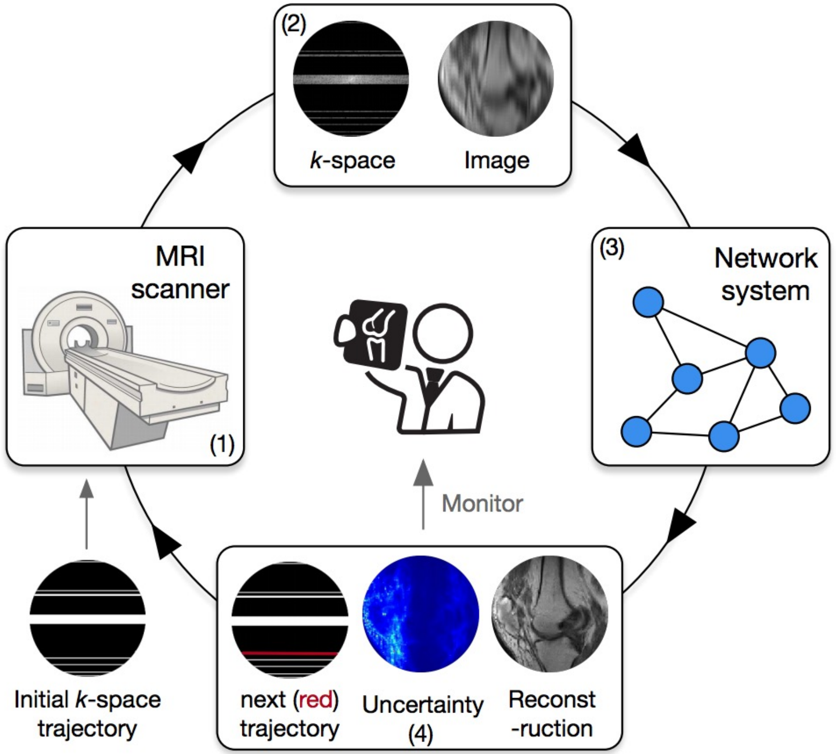

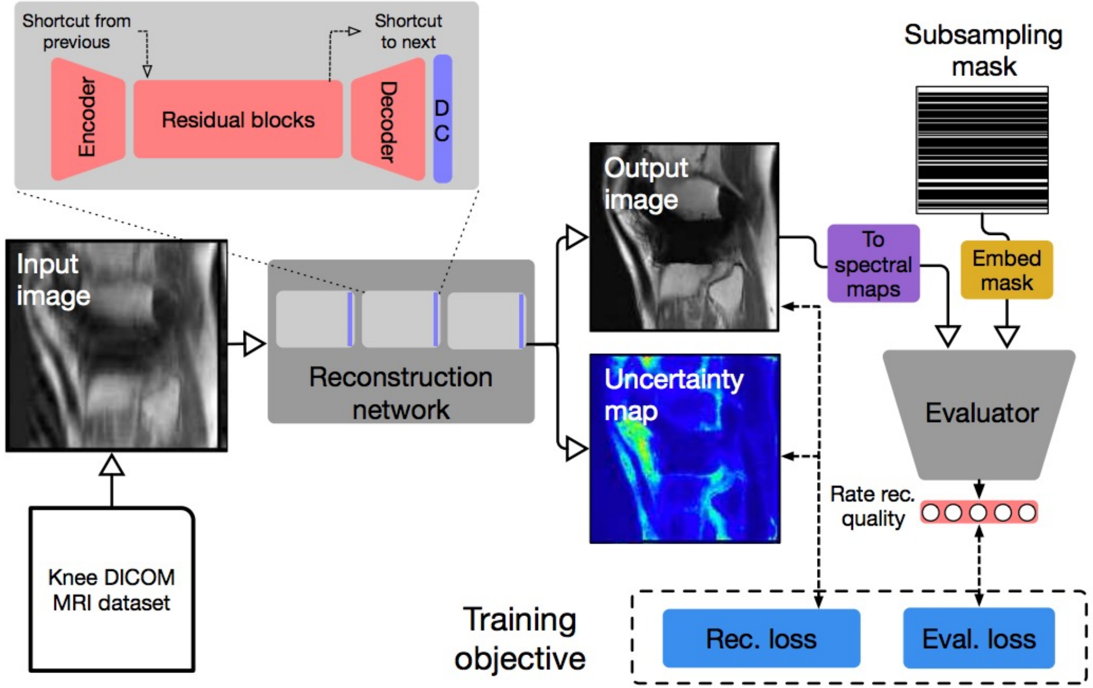

Figure 3: The training pipeline of the proposed method.